FLOW OVER A STATIC 2D SAVONIUS ROTOR USING IMMERSED BOUNDARY AND FOURIER PSEUDOSPECTRAL METHODS

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ABSTRACT

Savonius rotors are vertical-axis wind turbines characterized by their "S" shape, low speed, low noise, and effective startup capabilities. Composed of two semi-cylindrical blades, the rotation is initiated by the interaction of the fluid with the blade surfaces, generating combined torques. To enhance understanding of this turbine's behavior, this study presents a simplified model of the flow around the rotor using the Fourier Pseudospectral Method coupled with the Immersed Boundary Method. The model assumes an incompressible, Newtonian fluid with constant properties, no heat transfer or gravitational effects, and a two-dimensional analysis. The results, based on Re_D = 200, evaluate the flow over the static rotor and show that the advancing blade has a higher drag coefficient at startup compared to the returning blade. This behavior suggests the rotor's direction of rotation under the pre-established flow conditions in an induced rotation.

Keywords: Savonius Rotor, Vertical Axis Turbine, Computational Fluid Dynamics, Immersed Frontier, Pseudospectral Fourier.

INTRODUCTION

Wind turbines convert wind energy into electrical power, with various models and sizes available. The primary distinction between them lies in the rotor axis orientation, with Vertical Axis Wind Turbines (VAWT) and Horizontal Axis Wind Turbines (HAWT). The Savonius turbine, a VAWT model initially developed by Savonius [1], is recognized for its S-shaped blades, which operate with both concave and convex surfaces to capture and convert wind energy [2]. Despite its characteristic design, variations have been developed to improve efficiency [3][4]. Research on Savonius turbines has focused on various aspects, including the analysis of overlap rates in freely rotating turbines [5], blade flexibility [6][7], and the effects of both fixed and flexible blades [8]. Additionally, studies have explored elliptical [9] rotor blades and overlapping [7] stages through 3D simulations.

Although 3D simulations and turbulence models are commonly used to analyze Savonius turbines [2][3][6][7], two-dimensional models have proven to be effective representations of their behavior [10][11]. Numerical methods such as the finite volume method (FVM) and finite element method (FEM) are often employed for these simulations. However, this research introduces a methodology combining the Fourier

Pseudospectral Method (FPM) with the Immersed Boundary Method (IBM) [13].This approach has being validated for flow over immersed geometry [13] and offers significantly lower computational costs for two-dimensional analyses [14].

This study aims to simulate the flow over a static Savonius rotor using the Fourier Pseudospectral Method coupled with the Immersed Boundary Method (IMERSPEC).

MATERIALS AND METHODS

Pseudospectral Fourier Method

The pseudospectral Fourier method transforms the primitive variables from physical space to spectral space using the direct Fourier transform [15]. Thus, the equation which describes the fluid dynamic behavior of the flow over the rotor is governed by the continuity equation, Eq. (1), transformed into spectral space,

$$ik_i \hat{u} = 0, \qquad \textit{Eq. (1)}$$

where *û(k,t)* is the velocity field in spectral space, *k* is the wave vector, and i is the imaginary unit $\sqrt{-1}$. The Navier-Stokes equation transformed into Fourier spectral space, Eq. (2),

$$\frac{\partial \hat{u}_i}{\partial t} + ik_i(\widehat{u_i u_j}) = - ik_i\hat{p} - \nu k^2 \hat{u}_i + \hat{f}, \qquad \textit{Eq. (2)}$$

where k^2 is the square of the wave number vector *k* and is calculated as $k^2=k_jk_j$.

Immersed Boundary Method (IBM)

The immersed boundary method employs two independent domains: the Lagrangian domain (Γ) for the immersed geometry in the fluid, and the Eulerian domain (Ω) for the fixed Cartesian plane representing the flow [14].

The source term can be written as,

$$\hat{f}_i(x,t) = \sum_{\Gamma} D_h(x - X)F_i(X,t)\Delta s^2, \qquad \textit{Eq. (3)}$$

influenced by the Lagrangian force (F_i), the spacing between the points defining the geometry (Δs), and the distribution function,

$$D_h(x - X) = \frac{1}{\Delta x^2} W_h\left(r_x\right) W_h\left(r_y\right), \qquad \textit{Eq. (4)}$$

where $r_x = \frac{x-X}{\Delta x}$, $r_y = \frac{y-Y}{\Delta y}$, Δx and Δy are the spacings between points in the Eulerian mesh in *x* and *y*, respectively, and W_c is the weight function,

$$W_c(r) = \begin{cases} 1 - \frac{1}{2}|r| - |r|^2 + \frac{1}{2}|2|^3 & \text{if} \quad 0 \le |r| < 1 \\ 1 + \frac{11}{6}|r| + |r|^2 - \frac{1}{6}|r|^3 & \text{if} \quad 1 \le |r| < 2 \\ 0 & \text{if} \quad 2 \le |r| \end{cases} \qquad \textit{Eq. (5)}$$

computed with respect to the previously indicated *r*.

More details on the combination of the Pseudospectral Fourier method and the immersed boundary method, leading to the IMERSPEC methodology, can be found in the work of Mariano [13] and Nascimento [14].

RESULTS AND DISCUSSION

The simplified physical model of the static Savonius turbine consists of the combination of advancing (concave perpendicular to the flow) and returning (convex perpendicular to the flow) blades that form the 'S' shape profile. The Eulerian domain consists of lengths $L_x=16D$ in the x-axis and $L_y=8D$ in the y-axis, with 3D in length for the wake zone (ZB) and *D* in length for the frontal zone (ZF). The geometry is located at a distance of *8D* in length and *4D* in height. The simulation parameters are shown in Table 1.

Table 1. Simulation parameters.

Parameters	Symbols	Values
Placement points	$N_x\, x\, N_y$	$1024\ x\ 512$
Diameter (m)	*D*	2
Reynolds number	Re_D	200
Courant number	CFL	0,1
Dimensionless final time	t_f^*	300
Uniform velocity at inlet (m/s)	U_∞	1,00

With the separation of boundary layers around the static Savonius turbine, most of the flow in contact with the geometry is directed towards the advancing blade, creating a complex interaction with the geometry. Downstream of the geometry, recirculations initially develop symmetrically, indicating the formation and evolution of low-pressure zones and the overall flow structure around the turbine.

Figure 1 presents the vorticity field ω_z, with scale values defined by the range $-5 \le \omega_z \le 5$. In this field, white coloration represents negative recirculations, while dark blue indicates positive recirculations. Initially, in the counter-rotating structures downstream of the flow, at $t^*=5$ and $t^*=10$, it is observed that the negative recirculation increases in size, pushing the positive structure towards the shear layer. Subsequently, between $t^*=10$ and $t^*=25$, the first releases of counter-rotating structures occur alternately, forming the wake, which can be observed from $t^*=25$ to 110.

The temporal evolution of the drag coefficient and lift coefficient is calculated separately for the advancing and returning blade. The mean coefficients are calculated from time $t^*=200$, as shown in Table 2. It is noted that the drag coefficient is higher for the advancing blade. Meanwhile, the lift coefficient becomes higher for the returning blade. This indicates a greater contribution from the advancing blade to counterclockwise rotation, adding to the torque generated by the horizontal and vertical forces.

Table 2. Average values of drag coefficients and lift coefficients.

Blade	C_D	C_L
Returning	2,7122	1,9280
Advancing	3,6747	0,3595

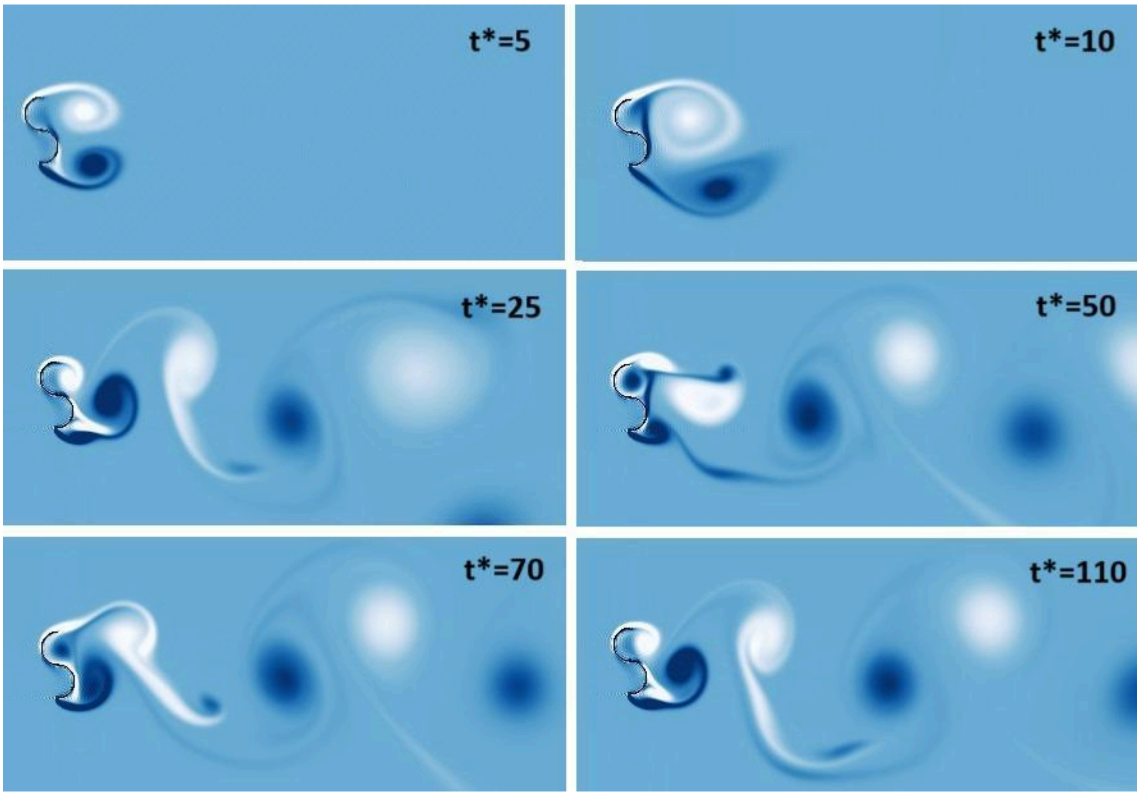

Figure 1. Temporal evolution of the vorticity field $-5 \leq \omega_z \geq 5$.

CONCLUSION

In this work, the Fourier pseudospectral method coupled with the Immersed Boundary Method, which together form the IMERSPEC methodology, was used to study the fluid dynamic behavior of a simplified model of the static Savonius turbine.

The shape of each semi-cylinder induces recirculations downstream of the rotor, generating a wake that, upon reaching steady-state, indicates higher drag coefficients for the advancing blade. This suggests that the concave geometry makes the most significant contribution to the counterclockwise rotation.

ACKNOWLEDGMENT

The authors express their gratitude to Eletrobras, the Research and Technological Development Program (P&D) of ANEEL, the Goiás State Research Support

Foundation (FAPEG), the Center of Excellence in Hydrogen and Sustainable Energy Technologies (CEHTES) and the Graduate Program in Mechanical Engineering (PPGMEC) for the financial support and infrastructure provided for the development of this research.

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