# Mathematical Model Applied for the Simulation of Electric **Bus Performance**

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# **ABSTRACT**

This research paper aims to present a mathematical model developed and applied using software to evaluate the performance of electric buses. The purpose of developing this model is to simulate for real the battery's charge behavior in these buses, making it easier to install fast chargers at strategic points on the highway network.

**Keywords:** Bus; Electric; Mathematic; Mobility; Charge.

# INTRODUCTION

At first, it is worth pointing out that the use of electric vehicles has began popularized in casual ways in families' daily lives. However, as time goes by, electric buses arises in big Brazilian cities such as São Paulo, Curitiba, Salvador and Goiânia on a trial basis. The continued use brought them closer to the final goal, which is to reduce pollutant emissions aligned with the idea of sustainability growth.

Though, since the change from combustion-engine fleets to electric ones, there still exists a principal challenge to face in electric vehicles: the batteries. It is possible to note that researchers have been developing ways to expand the battery's capacity and, as a consequence, the autonomy of electric vehicles. This article has a very clear objective to present a different perspective that searches to understand battery's charge behavior and her connection with weight and bus route. This approach allows strategic stands of electric chargers during the route,

resulting in a great vehicle autonomy besides, as a bonus, helps pass around energy expending all over the covered area, by that means bringing down the impact on the cities' electrical distribution network.

## MATERIALS AND METHODS

In this present study, has been first analyzed the parameters that make a difference to the electric bus' movement, such as its aerodynamics, the system of brake's regeneration, kinetic energy and anothers measures to be demonstrated. The basis to get the opening equations was the lecture of Professor Michael O'Keefe [2], who discusses how it would be possible to reproduce vehicles' consumption energy. In the picture below, it is possible to see the block's diagram used.

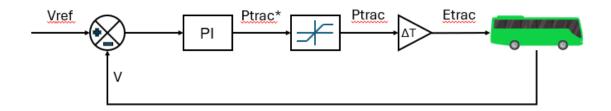


Figure 1. Block's diagram

This diagram was the basis for deduction of the following equation:

$$Etrac_k = Ptrac_k * \Delta T$$
 Eq. (1)

Expanding Equation 1, it was possible to have Equation 2.

$$Etrac_k = Eair_k + Escroll_k + Ekinetics_k + Epotential_k - Ebrake_k$$
 Eq. (2)

To understand Equation 2, it is necessary to take in each type of energy.

- Drag Energy (E<sub>air</sub>) is created through the fluid's movement, in this case, air. The head points that guides this energy are fluid's denseness, drag's coefficient, cross-sectional area and the speed of objects' movement.

$$E_{air} = \rho * C * A * v_k^3 * \Delta T$$
 Eq. (3)

Roll Energy  $(E_{Rol})$  embrace the energy formed by a rolling object over an area. This energy is a combination of rotating and translating kinetic energies.

$$E_{Rol} = RRCo * m * g * v_k * \Delta T$$
 Eq. (4)

- Kinetic energy ( $E_{\text{kinetics}}$ ) is the energy of an object's movement. This energy depends on two factors: mass and speed.

$$E_{kinetics} = \frac{1}{2} * m * (v_{k+1}^2 - v_k^2)$$
 Eq. (5)

- Potential energy (E<sub>potential</sub>) is the energy stored by the object's position. In this case, it will be gravitational and relative height to the plane.

$$E_{potential} = m * g * (h_{k+1} - h_k) Eq. (6)$$

$$h_{k+1} - h_k = v_k * \Delta T * \sin \theta \text{ Eq. (7)}$$

 Brake energy, or braking energy, is the energy that dissipates when a vehicle slows down, conversioning into heat, for example. Has an important detail: this energy can be recovered and reused by the electric vehicle's motor to rise charge.

Based on these energies, after mathematics and physics handlings, a positive solution was found and applied in MATLAB software. For the parameters related to electric buses, were used statistics from Brazilian's company "Eletra" and the battery', CATL (Contemporary Amperex Technology Co. Limited) and LFP (Lithium Iron Phosphate).

#### **RESULTS AND DISCUSSION**

The main results earned are shown in following figures.

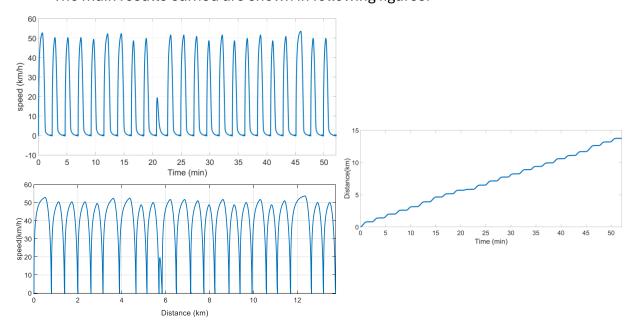


Figure 2. Graphs relating electric bus' time, distance and speed.

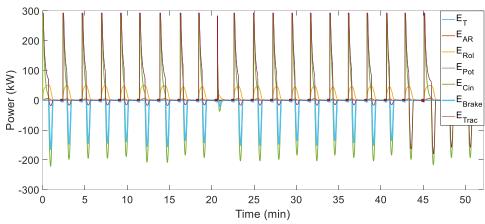


Figure 3. Graphs relating electric bus' time and power.

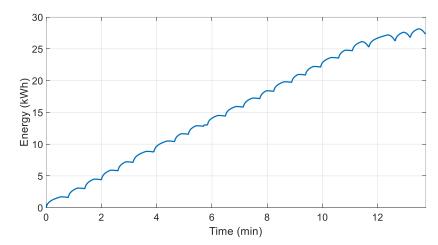


Figure 4. Graphs relating electric bus' energy and time.

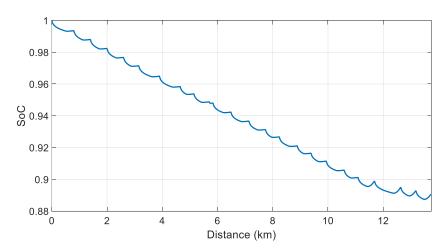


Figure 5. Graphs relating electric bus' battery charge to the distance traveled.

In Figure 2, the time is related to speed, distance, and energy, allowing view important aspects such as the vehicle's efficiency, performance, and range. For example, at higher speeds, the vehicle requires more energy to overcome resistance and friction. In other words, understanding these factors helps create opportunities for optimizing the vehicle's performance.

Meanwhile, in figure 3 relates energy to time and let us to observe energy expeding over time, reaching a peak and looking for balance. In figure 4 is relating battery's charge to distance (declining linearly until has a energy's return from the system itself), allowing visualizes energy's feedback during electric's vehicle operation.

Thereby, its can be noted that data's simulation were successfully gets and represents all the parameters. Its also possible observes that the current code is in early progress stages and was used for a short time period. However, it'll be improved for simulates a hole day of bus' operation, intending to understand battery' discharge in particular routes at many times of the day.

## CONCLUSION

So ending this present article, we have a deeply understanding of the battery discharge process in electric buses been essential for the effective carrying out of fast charging points. This study allows strategics placement of charging stations, minimizing the impact on the electrical grid and preventing potential overloads that may happen amplify energy demand. Also, detailed planning of fast charging contributes to maximizing the operational efficiency of electric buses, improving the availability and reliability of public transportation services. By aligning charging practices with the battery discharge profile, its possible to optimize both batteries lifespan and overall vehicle performance, promoting a more sustainable integration of electric buses into urban infrastructure.

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