Application of Nonlinear Optimization in Flight Path Reconstruction of a Sub-scale Aircraft

Clecio Fischer[1], Éder Alves de Moura[2], Leonardo Murilo Nepomuceno[3], Vítor Paixão Fernandes[3], Roberto Gil Annes da Silva[3], Luiz Carlos Sandoval Góes[3]

[1]*Dept. of Mechanical Engineering, Aeronautics Institute of Technology and Federal Institute of Education, Science and Technology of São Paulo*
Praça Marechal Eduardo Gomes, 50, 12228-900, São José dos Campos/SP - Brasil
clecio@ifsp.edu.br
[2]*Dept. of Electrical Engineering, Federal University of Uberlandia*
Praça Marechal Eduardo Gomes, 50, 12228-900, São José dos Campos/SP - Brasil
edermoura@gmail.com
[3]*Dept. of Mechanical Engineering, Aeronautics Institute of Technology*
Praça Marechal Eduardo Gomes, 50, 12228-900, São José dos Campos/SP - Brasil
leaobravo@gmail.com, vitorpaixao.eng@gmail.com, roberto.gil.silva@gmail.com, lcsgoes@gmail.com

Abstract. One key challenge in working with free-flight test data is ensuring its compatibility with the dynamic systems of sub-scale unmanned aircraft. This is crucial to avoid errors in measurements, bias, or scaling issues that could compromise the integrity of the estimated results. We refer to this initial processing step as data compatibility checking, a rigorous process that forms the foundation of our research. Typically, this verification process involves Flight Path Reconstruction (FPR) predicated on estimating the aircraft's flight kinematics. The predominant methodologies encompass the stochastic approach, using the Extended Kalman Filter (EKF), and the deterministic approach, employing the Output Error Method (OEM). In this research, we propose to approach the problem from the perspective of a nonlinear optimizer to estimate the reconstruction of data derived from free-flight tests of a sub-scale aircraft. The nonlinear optimizer we employ in this research was designed to verify the authenticity of the data representing the aircraft's actual flight. It does this by comparing the measurements of attitude angles, accelerations, and velocities with the expected kinematic behavior of the aircraft. This rigorous evaluation process ensures that the reconstructed data is as close to the actual flight as possible, enhancing the reliability of our results. The sub-scale model deployed in these tests was engineered to emulate the dynamic characteristics of a full-scale aircraft, utilizing the Froude number criterion for appropriate scaling. The chosen aircraft for this project was a Cessna 177B, and the sub-scale aircraft was developed by the Aeronautical Systems Laboratory (LSA) at the Aeronautics Institute of Technology (ITA).

Keywords: Estimation, flight path reconstruction, free-flight test.

1 Introduction

Parameter estimation in aerospace generally assumes that the measured data contain sufficient information about the cause-effect relationship, minimizing the influence of systematic errors such as scale factors, zero offsets and time delays. Various techniques can be used to account for the presence of noise and systematic errors. Least squares methods are particularly sensitive to these errors, while methods such as the output error and Kalman filter methods allow estimation of noise statistics and correction of systematic errors. However, these approaches can introduce unwanted correlations that affect the accuracy of the estimates, so independent data verification is advisable [1, 2].

Before using the raw flight-test data, it is essential and efficient to check if the recorded data are consistent. This task is accomplished through Flight Path Reconstruction (FPR), which ensures the compatibility of measured channels by validating kinematic relationships. These methods are crucial for applications involving complex flight dynamics and significant nonlinearities due to factors such as aerodynamics, varying speeds, and changing environmental conditions.

For flight test programs, the compatibility check of the recorded data is essential. This process utilizes kinematic relationships to ensure that measurements are consistent and error-free. The goal is to ensure that the data used to identify the aerodynamic model is coherent and accurate. For example, measured angles of attack and sideslip must match values reconstructed from measured linear accelerations and angular velocities [1].

In general, there are two main approaches to implement FPR: a rigorous approach based on the Extended Kalman filter (EKF) and a simpler approach based on the Output Error Method (OEM). The OEM is commonly used in practice as it is suitable for calibrating flow variables such as angle of attack and slip angle, which are crucial for parameter estimation and flight simulation [3].

The process involves checking the state of the aircraft at various points in time, with the state typically including position, velocity, acceleration, orientation (pitch, roll, yaw), and sometimes higher-order derivatives. Flight Path Reconstruction gathers data from onboard sensors, such as GPS and IMU, and applies a nonlinear optimization method to estimate the aircraft's state. The model parameters are adjusted to minimize estimation error, and the estimated states are used to reconstruct the flight path. This reconstructed flight path provides a detailed and accurate representation of the aircraft's trajectory, which is essential for navigation, control, and analysis purposes.

This paper will present the results obtained from the compatibility checking of flight test data collected from a dynamically scaled model of a Cessna 177B. The nonlinear technique applied involves minimizing a cost function using the Levenberg-Marquardt algorithm. The Levenberg-Marquardt method is an effective approach for addressing the numerical challenges encountered in the OEM procedure. It combines the best features of the unconstrained Gauss-Newton method and the steepest descent technique, which follows the gradient direction. Due to the optimized combination of these two search directions, the Levenberg-Marquardt method offers a broader convergence region.[1]

This scaled model was designed to simulate flight and provide a dynamic representation of the actual aircraft, enabling the validation of aerodynamic parameter estimation methodologies and the analysis of flight test data.

2 Development

The development of this project begins with the analysis of data from the technical report *Flight Test Data for a Cessna Cardinal* (Kohlman, 1974) [4], which provides flight performance data for the Cessna 177B Cardinal. To replicate these flight tests and validate the methodology for creating sub-scale aircraft that accurately reflect the dynamics of full-scale aircraft, this study examines the development of a sub-scale model. The analysis determined that the most effective scaling method is to use the Froude number (*Fr*), calculated by [5]:

$$Fr = \frac{V^2}{lg} \tag{1}$$

where V is the velocity, l is the characteristic dimension, and g is the acceleration due to gravity.

The project steps included analyzing the flight test data from the Cessna 177B and selecting relevant test points. The wingspan of 2.4 meters and scale factor n is 22.2% compared to the full-size aircraft's wingspan of 10.82 meters [6].

The model was constructed from composites such as fiberglass and carbon fiber, along with balsa wood and marine plywood. The prototype, shown in Fig. 1. The flight test conditions for the sub-scale aircraft were set for São José dos Campos (SJC) with an altitude of approximately 46 meters above ground level. Applying these conditions the weight calculated was 14.60 kg, and the test speed as 27.48 m/s.

Figure 1. Sub-scale Cessna 177 Aircraft

To perform flight path reconstruction, it is necessary to acquire flight data through an onboard acquisition system. For this activity, sensors were employed to collect various flight parameters, including anemometric data, aircraft attitude, and input and output variables from the control surfaces.

The data acquisition system implemented for this project comprises a Pixhawk Cube Orange controller, which integrates multiple sensors, including accelerometers and gyroscopes. These sensors are capable of measuring Euler angles, vibrations, and velocities. The system also includes a GPS module, an airspeed sensor, telemetry for real-time monitoring during flight tests, a communication interface between the radio control and the Pixhawk, an engine RPM measurement system, and a power supply and control system for the servomotors. The system utilizes ArduPilot software, an open-source platform, configured to record relevant variables at a sampling frequency of 100 Hz.

In addition to the Pixhawk system, an air data boom is utilized to measure angles of attack and sideslip, see Fig. 2 This setup incorporates a vane aligned with the airflow, which is connected to a potentiometer. The output voltage from the potentiometer is captured by an Arduino Nano, which records the data onto a memory card at a constant sampling rate of 100 Hz.

In Fig. 3 illustrates the integration of this system within the sub-scale Cessna 177B aircraft.

Figure 2. Anemometric sensors for measuring angles of attack and sideslip installed on the Cessna 177B subscale model.

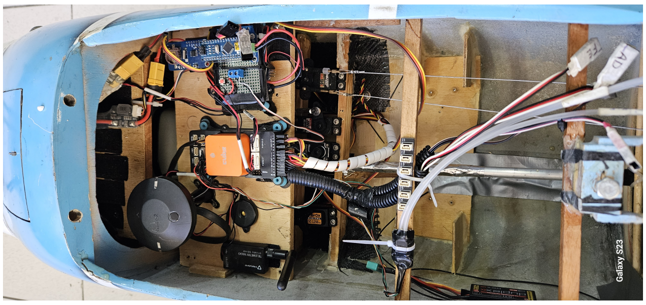

Figure 3. Electronic system installed on the Cessna 177B subscale aircraft.

2.1 Nonlinear Optimization

The estimation of the parameters and the states is done using a combination of sensor data, the mathematical models of the aircraft dynamics and an optimization strategy to obtain optimal parameters that minimizes a given criterion of optimality, usually formulated as a filtering technique or by an global optimization algorithm. To reconstruct the flight path, filtering techniques are used to estimate the state vector $x(t)$ from the noisy measurements $y(t)$, as the use of the Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF). In the optimization framework, the objective is to minimize the difference between the estimated states obtained from the model and the observed data from sensors and this involves defining a cost function or objective function that quantifies this difference. The cost function typically incorporates terms for both the measurement errors and the process noise.

Optimization techniques are often employed to minimize the difference between the estimated and observed trajectories and are, usually formulated as a filter estimation or with by algorithms of search. a This involves solving a minimization problem where the objective function represents the error or cost associated with the estimation. Since analytical solutions to the nonlinear differential equations are usually not feasible, numerical integration methods (like Runge-Kutta) are used to solve these equations over time.

The Gauss–Newton method for nonlinear optimization is a second-order algorithm widely used in the time-domain system identification of aerospace vehicles. It is a modification of the Newton method, also known as the Newton–Raphson method. To fully understand this, it is necessary first to comprehend the base Newton method. The necessary condition for minimizing the likelihood function with respect to the unknown parameters is given by:

$$\frac{\partial J(Q)}{\partial Q} = 0 \tag{2}$$

The Levenberg–Marquardt method provides a robust approach to addressing the numerical issues discussed earlier. It effectively combines the strengths of two established techniques: the unconstrained Gauss–Newton method and the steepest descent method. The Gauss–Newton method is known for its rapid convergence when near the solution, while the steepest descent method is more reliable in regions far from the solution due to its gradual movement in the direction of the gradient [1].

By blending these two methods, the Levenberg–Marquardt method achieves a broader convergence region. The update formula for this method is expressed as:

$$Q_{i+1} = Q_i + \Delta Q \qquad (F + \lambda I)\Delta Q = G \tag{3}$$

where λ is the Levenberg–Marquardt (LM) parameter. This parameter can be adjusted to control the balance between the steepest descent direction and the Gauss–Newton direction. As λ approaches 1, the method behaves more like steepest descent. Conversely, as λ approaches 0, it resembles the Gauss–Newton method more closely.

The computation of F and G remains consistent with the Gauss–Newton method. The critical task is to determine an appropriate value for λ that optimally interpolates between these two approaches. Various strategies exist for calculating the LM parameter λ, with the control strategy outlined being both straightforward and effective in practical applications [1].

The state estimation problem reduces to simple numerical integration. Under this assumption, the model for estimating instrumental errors is given by:

$$\begin{gathered} \dot{x}(t) = f[x(t), u(t), \Theta], \\ x(t_0) = x_0 \end{gathered} \tag{4}$$

$$y(t) = g[x(t), \Theta] \tag{5}$$

$$z(t_k) = y(t_k) + v(t_k) \tag{6}$$

Where $z(t_k)$ is the vector of observed variables $y(t_k)$, with the estimation error $v(t_k)$.

For this system model, the state vector for data compatibility is given by:

$$x = [u \quad v \quad w \quad \phi \quad \theta \quad \psi \quad h]^T \tag{7}$$

In the vector x defined by Eq. 7: u represents the velocity component of the aircraft along the x-axis (longitudinal axis), measured in meters per second (m/s); v denotes the velocity component along the y-axis (lateral axis), also measured in meters per second (m/s); w is the velocity component along the z-axis (vertical axis), measured in meters per second (m/s); the angles ϕ, θ, and ψ correspond to the roll, pitch, and yaw angles of the aircraft, respectively, describing its orientation in three-dimensional space in degrees; and, lastly, h indicates the altitude of the aircraft, representing its height above sea level, measured in meters (m).

The input vector u is defined by:

$$u = [a_x \quad a_y \quad a_z \quad p \quad q \quad r]^T \tag{8}$$

where the terms a_x, a_y, and a_z represent the linear acceleration components of the aircraft along the x, y, and z axes, respectively, measured in meters per second squared (m/s²). These components describe the acceleration in the longitudinal, lateral, and vertical directions. The terms p, q, and r denote the angular rates of the aircraft about the x, y, and z axes, respectively, measured in degrees per second (deg/s). These rates reflect the aircraft's rotational motion or spin around each axis.

The observation vector y is defined by:

$$y = [V \quad \alpha \quad \beta \quad \phi \quad \theta \quad \psi \quad h]^T \tag{9}$$

the term V represents the velocity of the aircraft, typically in meters per second (m/s), indicating the speed at which the aircraft is moving. The angles α and β denote the angle of attack and the sideslip angle, respectively, measured in degrees, describing the orientation of the aircraft relative to the airflow.

The unknown parameters are:

$$\Theta = [\Delta a_x \quad \Delta a_y \quad \Delta a_z \quad \Delta p \quad \Delta q \quad \Delta r \quad K_\alpha \quad \Delta\alpha \quad K_\beta \quad \Delta\beta]^T \tag{10}$$

The terms Δa_x, Δa_y, Δa_z, Δp, Δq, and Δr refer to errors or deviations in the measurements from acceleration sensors and gyroscopes. The parameters K_α and K_β are gain factors used to adjust the sensor response with respect to the angle of attack and the sideslip angle, respectively. The errors $\Delta\alpha$ and $\Delta\beta$ refer to discrepancies in the measurements of the angle of attack and the sideslip angle.

Additionally, it may be necessary to estimate the initial conditions:

$$x_0 = [u_0 \quad v_0 \quad w_0 \quad \phi_0 \quad \theta_0 \quad \psi_0 \quad h_0]^T \tag{11}$$

3 Results

After conducting the free-flight tests, the data were processed and segmented based on different maneuver types used to excite the flight modes. These maneuvers included short-period tests with doublet inputs on the elevator, Dutch Roll tests with doublet inputs on the rudder, and Bank-to-Bank maneuvers excited by the ailerons. The maneuvers were divided into sections to perform the parameter identification process, as described in Eq. 10. Three concatenated maneuvers in sequence were used, each with its own identification of initial conditions. Fig. 4 presents the input data used for identification, where the first segment corresponds to an input on the elevator, the intermediate part to inputs applied to the ailerons, and the final part to the input on the rudder.

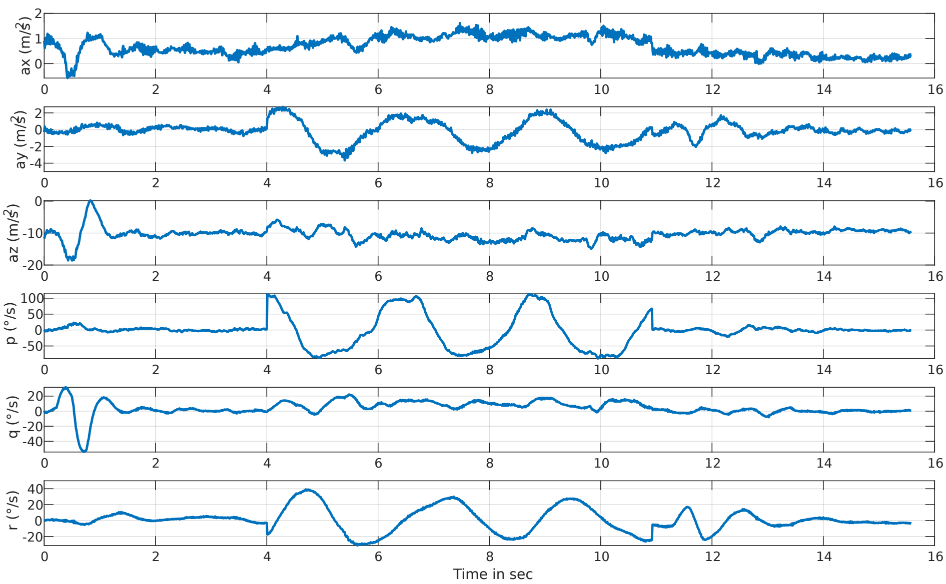

Figure 4. Temporal records of input variables for verifying data compatibility.

In the Fig. 5, the behavior of the parameter estimates for each iteration is shown. Here, the determinant of R converged to 7.5516e-19 after 17 iterations. Additionally, the autocorrelation of the estimates was checked, with the criterion being that the correlation should be analyzed if it exceeded 0.9.

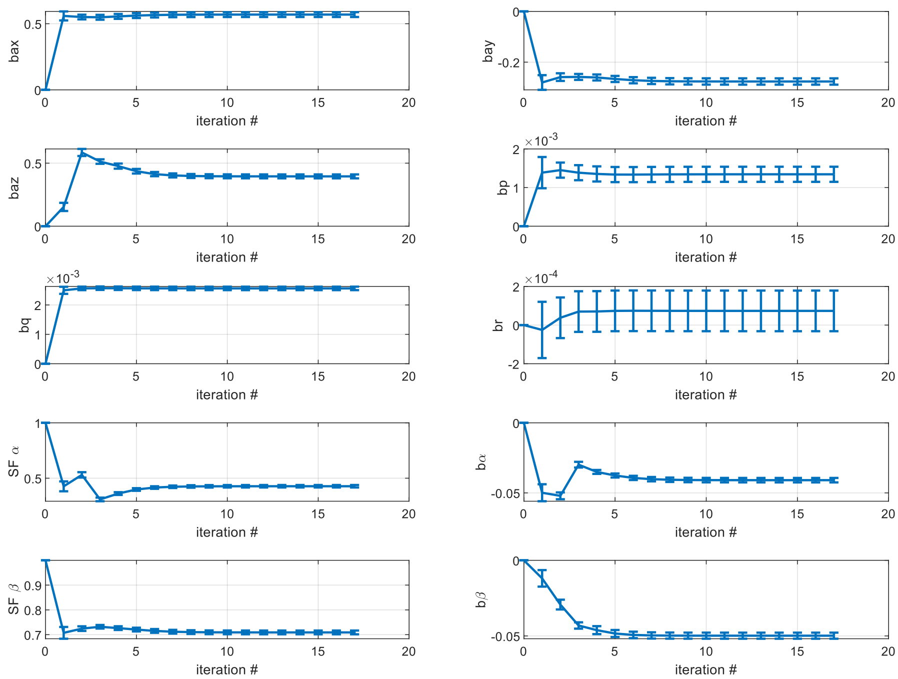

Figure 5. Convergence of parameter estimates with error bounds.

In the Fig. 6, the results of the measured and estimated variables from the data compatibility check process are presented. It is observed that the variables with the greatest divergence are the angle of attack (α) and the sideslip angle (β). This is because these parameters are difficult to collect accurately during free-flight conditions, which can lead to discrepancies between the true values and the measured data. Additionally, the sensors that measure airflow can experience vibrations due to the engine's vibrations being transmitted throughout the aircraft, further affecting the accuracy of the measurements.

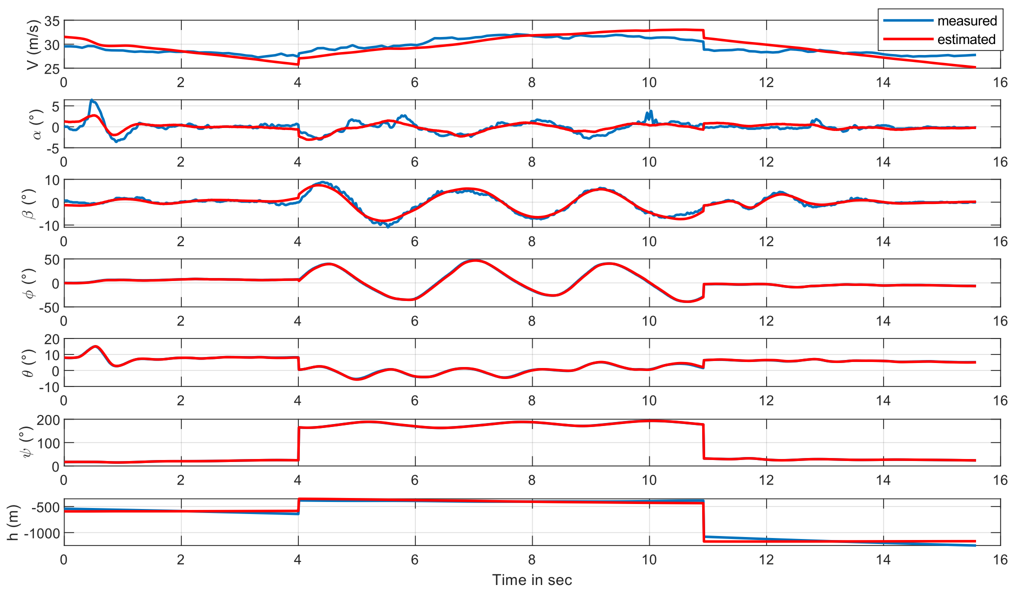

Figure 6. Data measured in flight tests of the Cessna 177B subscale model and estimated by the nonlinear optimizer

In Tab. 1 presents the estimated errors for each of the output variables.

Table 1. Bias parameters estimated

1	Δa_x	0.5691	m/s^2	6	Δr	0.0042	deg
2	Δa_y	-0.2762	m/s^2	7	K_α	0.4274	-
3	Δa_z	0.3952	m/s^2	8	$\Delta\alpha$	-2.3458	deg
4	Δp	0.0770	deg	9	K_β	0.7090	-
5	Δq	0.1468	deg	10	$\Delta\beta$	-2.8562	deg

4 Conclusions

The application of nonlinear optimization in the flight path reconstruction of a sub-scale aircraft model has been shown to be an effective approach to validate and improve the accuracy of flight test data. This study focused on the use of optimization algorithms, in particular the Levenberg-Marquardt method, to minimize errors and ensure the compatibility of the measured data with the expected kinematic behavior of the aircraft.

The results obtained show that the proposed technique allows a faithful reconstruction of the flight path. The data analysis showed that the variables with the greatest divergence were the angle of attack and the sideslip angle, which illustrates the difficulty of accurately measuring these parameters under free flight conditions.

The study also highlights the importance of rigorous calibration and verification of the sensors, particularly those measuring angles and velocities, to ensure the integrity of the data. Nonlinear optimization was found to be essential for fitting the model parameters and reducing the observed deviations, which contributes significantly to the reliability of the trajectory estimates.

This research reaffirms the need for advanced data processing techniques in aerospace and provides a robust method that can be adapted for future flight studies and the development of dynamic aircraft models. The practical application of these results could lead to improvements in aircraft design and control and increase the accuracy of simulations and performance analysis.

In conclusion, this work highlights the potential of nonlinear optimization in aircraft engineering and the importance of careful validation of experimental data for the advancement of flight technologies.

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