DETERMINATION OF CRITICAL PLANES IN MULTIAXIAL FATIGUE WITH ESTIMATION OF SHEAR STRESS AMPLITUDE IN MECHANICAL COMPONENTS THROUGH DIFFERENT FAILURE CRITERIA

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Abstract. Industrial mechanical components are susceptible to fatigue failures under multiaxial loading conditions, which can compromise their structural integrity. These failures can be validated through experimental data and predicted using computational approaches that determine maximum normal and shear stresses based on failure criteria and fatigue strength analysis models. This study presents the Susmel and Lazzarin Failure Criterion and the Findley Failure Criterion, along with fatigue analysis models such as the Socie Method, Maximum Circumscribed Rectangle Method, and Maximum Variance Method, to analyze critical failure planes in the steel alloys 34Cr4, 25CrMo4, 42CrMo4V, and 30NDC16. The results indicate that lower error margins in various fatigue models correspond to more reliable material failure predictions. Based on the experimental data analyzed, 20% of the results fall within a statistically consistent band for conservative models, except for synchronous Ax and out-of-phase FFase loadings in the Socie Model and Maximum Variance Method. Each fatigue model and stress condition presents specific characteristics that influence the reliability of fatigue strength predictions.

Keywords: Critical Plane; Multiaxial Fatigue; Variable Amplitude Loading; Findeley Criterion.

1 INTRODUCTION

Multiaxial fatigue is a critical phenomenon in mechanical components characterized by failures under cyclic loading conditions. The fatigue resistance of materials determined through experimental stress measurements can also be predicted using computational analyses based on well-established failure criteria such as those proposed by Susmel and Lazzarin (2002) and Findley (1956). This study employs fatigue models including the Socie Method, the Maximum Variance Method, the Maximum Circumscribed Rectangle Method and the Moment of Inertia Method to predict fatigue failure in mechanical components. This fatigue failure prediction for mechanical components through known fatigue models in engineering implemented in computational routines can present relative reliability in prediction since not all models have parameters that can depending on the loading characteristics accurately predict material failure and provide reliable statistical responses.

This work justifies the application of these failure criteria and fatigue models to predict the failure of mechanical components by identifying the relationship between maximum shear stress amplitudes and maximum normal stresses that promote crack propagation and penetration in these materials. Predicting material failure in industrial and construction materials helps identify their resistance. The objective of this work is to identify the critical failure planes of materials especially metallic alloys and the error index of failure prediction of these materials by identifying the maximum shear stress amplitude and maximum normal stress. Thus indicating the reliability of the application of these fatigue models according to the characteristics of the materials, the proper conduct of the referenced experimental tests, and other test parameters.

2 BASIC CONCEPTS

Critical plane models are based on the mechanisms of crack growth during their early stages of propagation. In the high-cycle fatigue regime experimental observations have shown that small cracks occur on planes where a combination of shear and normal stresses is most severe [1]. Let $\boldsymbol{T}(t)$ be the stress tensor referenced in the base xyz representing the histories of stress components acting at a material point of a shaft subjected to normal and torsional stresses. According to Cauchy's Theorem, the stress vector $\boldsymbol{t}(t)$ in a material plane Δ, defined by the unit vector $\boldsymbol{n}$, normal to the plane where n is referred to the xyz system by the angles θ and ϕ defined by:

$$\boldsymbol{n} = [n_x \quad n_y \quad n_z]^{\mathrm{T}} = [sin(\theta)\cos(\phi) \quad \sin(\theta)\sin(\phi) \quad \cos(\theta)]^{\mathrm{T}} \tag{1}$$

The stress vector $\boldsymbol{t}(t)$ can be decomposed into normal and shear stress components, $\boldsymbol{\sigma}_n(t)$ and $\boldsymbol{\tau}(t)$, respectively

$$\boldsymbol{\sigma}_n(t) = [(\mathbf{T}(t)\boldsymbol{n})\boldsymbol{n}]\boldsymbol{n} = [\boldsymbol{n}^{\mathrm{T}}\boldsymbol{T}(t)\boldsymbol{n}]\boldsymbol{n};\ \boldsymbol{\tau}(t) = \boldsymbol{t}(t) - \boldsymbol{\sigma}_n(t) = \boldsymbol{t}(t) - [\boldsymbol{n}^{\mathrm{T}}\boldsymbol{t}(t)\boldsymbol{n}]\boldsymbol{n} \tag{2}$$

When considering multiaxial fatigue analysis with constant amplitude stress componentes there are several criteria for identifying critical planes based on the maximum amplitude of the shear stress $\tau_{a_{max}}$.

3 FAILURE CRITERIA

3.1 Susmel and Lazzarin Criterion

The MWCM is a bi-parametrical critical plane approach whose formalisation takes as a starting point the assumption that fatigue damage under both variable and constant amplitude (CA) loading reaches its maximum value on that plane experiencing the maximum shear stress amplitude [2]. This mathematically defined as:

$$\tau_{n,a}(\theta^c, \phi^c) + \kappa\rho(\theta^c, \phi^c) \geq \lambda \tag{3}$$

Susmel, Tovo and Lazzarin (2005) also defined the variable ρ of Eq. 4 which represents the influence of normal stress on fatigue strength, limiting the use of the model which applies up to a limit value, ρ_{lim} , above which it has no meaning and therefore the MWCM cannot be applied. The values of ρ and ρ_{lim} are obtained as follows [3]:

$$\rho = \frac{\sigma_{n,máx}}{\tau_a};\ \rho_{lim} = \frac{\sigma_0}{2\tau_0 - \sigma_0} \tag{4}$$

3.2 Findley Criterion

Despite its age and the development of many newer critical plane methods, the Findley method is still widely used today [4]. The Findley damage parameter f is accumulated based on shear stress amplitude τ and the maximum occurring normal stress over the load time history β.

The Findley Criterion combines shear stress amplitude τ_a and the maximum normal stress $\sigma_{m.máx}$ on the critical plane. Findley (1956) demonstrated that a linear relationship between τ_a e $\sigma_{m.máx}$ provides good correlation with experimental data [5]:

$$[\tau_a(\theta^c, \phi^c) + \kappa\sigma_{m.máx}(\theta^c, \phi^c)] \geq \lambda \tag{5}$$

where the κ and λ parameters which are material parameters derived from two fatigue strengths under different loading conditions. Typically, if the uniaxial and torsional endurance limits (f_{-1} and t_{-1}) are used to calibrate, the constants κ and λ are calculated as follows:

$$\kappa = \frac{1-0.5r}{\sqrt{r-1}}\,;\, \lambda = \frac{\sigma_{-1}}{2\sqrt{r-1}} \tag{6}$$

4 FATIGUE MODELS

4.1 Maximum Circumscribed Rectangle Method

The axes of the rectangular prismatic hull represent the amplitudes of the stress history along its directions, allowing a proper characterization of non-proportional loadings and resulting in a criterion well suited for any periodic stress paths [6].

It is claimed here that the equivalent shear stress amplitude which correctly characterizes fatigue damage under multiaxial loadings is given by the Maximum Rectangular Hull (MRH) of the shear stress vector path ψ in a material plane Δ. The halves of the sides of a retangular hull with orientation φ bounding the shear stress path ψ can be defined (Eq. 7) [7].

$$a_i(\varphi) = \frac{1}{2}\left[\max_t \tau_i(\varphi, t) - \min_t \tau_i(\varphi, t)\right] \; i = 1,2 \tag{7}$$

For each φ – oriented rectangular hull one can define is its amplitude as

$$\tau_a = \max_\varphi \sqrt{a_1^2(\varphi) + a_2^2(\varphi)} \tag{8}$$

4.2 Moment of Inertia Method

The MOI method calculates alternate and mean components of complex non-proportional load histories. The shear-shear diagrams are used for critical-plane approaches, where the moment of inertia I_{CM} of the stress history centroid is determined by (Eq. 9). The load history is first represented in a 2D deviatoric subspace whose metric should be proportional to the maximum shear or to the von Mises equivalent stress or strain. Therefore, for critical-plane approaches the shear-shear diagrams are appropriate since their metric (the distance between two stress or strain states) can be used in the calculation of the maximum shear range $\Delta\tau_{máx}$ [8]. For tension–torsion histories using an invarian based approach the σ_x x $\tau_{xy}\sqrt{3}$ stress diagram is a good choice since its metricis proportional to the von Mises equivalent stress $\left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2}$.

When considering the stress history as a unit mass thread the center of mass is defined as the mean stress τ_m which can be decomposed into the directions of the vectors e_a and e_b and defined as $\tau_{m_{e_a}}$ and $\tau_{m_{e_b}}$ [10]. Thus, for the determination of the moment of inertia I_{CM} of the centroid of the entire stress history:

$$I_{CM} = \frac{1}{P}\int\left(\frac{dp^2}{12} + \tau_{c,A}^2 + \tau_{c,B}^2\right)dp - (\tau_{m,e_A}^2 + \tau_{m,e_B}^2) \tag{9}$$

With the value of I_{CM} calculated and applying the Von Mises relationship for determining stresses in ductile metals it is possible to find the maximum stress amplitude on an arbitrary plane of the stress history:

$$\tau_a = \frac{\Delta\sigma_{MISES}}{2} = \sqrt{3I_{CM,h}} \tag{10}$$

4.3 Maximum Variance Method

The MVM posits that the critical plane aligns with the plane experiencing the maximum variance of resolved shear stress. The shear stress variance is calculated as (Eq. 12). In the case of metallic materials, initiation occurs along the critical plane where the resolved shear stress variance is maximum [10].

The Maximum Variance Method assumes that a plane where the variance of the equivalent stress. In general, in a multivariate analysis it is necessary to know not only the measures of variance of the random variables but also the quantification of the dependence. These quantities are presented in the form of a matrix called the covariance matrix determined through the basic definition [11].

The Maximum Variance Method posits that the critical plane aligns with the material plane undergoing the highest variability in shear stress. A complex system of time-variable forces resulting in a stress state at origin point whose components vary randomly over the time interval [0, T]. According to the MVM, the critical plane can be determined also in such circumstances by directly locating that plane containing the direction MV experiencing the maximum variance of resolved shear stress $\tau_{MV}(t)$. As soon as the orientation of the critical plane is known the mean value of the shear stress relative to the critical plane takes [2].

$$\tau_m = \frac{1}{T}\int_0^T \tau_{MV}(t)\,dt \tag{11}$$

the variance of stress signal $\tau_{MV}(t)$ being

$$Var[\tau_{MV}(t) = \frac{1}{T}\int_0^T [\tau_{MV}(t) - \tau_m]^2\,dt \tag{12}$$

5 COMPARASION OF EXPERIMENTAL DATA AND ERROR INDEX

5.1 Metal Alloys and other Mechanical Components

In this topic, the analysis of various materials is highlighted such as: 34Cr4, 25CrMo4, 42CrMo4 and 30NCD16 in Table 1. In which this emphasizes the yield strength σ_y; ultimate tensile strength σ_u; tensile fatigue limit σ_{-1} and torsional fatigue limit τ_{-1} of the aforementioned materials; κ and λ are material parameters.

Table 1 – Mechanical properties of the tested materials

					FINDLEY	
	σ_y (MPa)	σ_u (MPa)	σ_{-1} (MPa)	τ_{-1} (MPa)	κ	λ (MPa)
34Cr4-b	657	795	343	204	0,193002	207,76
34Cr4-c	657	795	410	256	0,256856	264,31
34Cr4	700	858	415	256 - 259	0,256209	267,37
25CrMo4-a	660	780	361	228	0,272772	236,33
25CrMo4-b	660	780	340	228	0,362954	242,55
42CrMo4-a	888	1025	398	265	0,351557	280,90
42CrMo4-b	1003	1142	485	315	0,313298	330,10
30NCD16-a	1020	1160	660	410	0,249878	422,61
30NCD16-b	1080	1200	690	428	0,247859	440,95
30NCD16-c	1020	1080	585	405	0,416667	438,75

From the experimental data, the phase diagram and stress history are known for synchronous, out-of-phase and non-zero mean stress loadings of these materials.

Table 2 – Data set obtained from literature

Data Set	Material	Mean Stress	Loadings		REF
1	34Cr4	$\sigma_m = 0$	Ax	EFase	[12]
2	25CrMO4	$\sigma_m = 0$	Ax	EFase	[13]
3	42CrMo4V	$\sigma_m = 0$	Ax	EFase	[13]
4	34Cr4	$\sigma_m \neq 0$	ASinc	FFase	[13]

5	25CrMO4	$\sigma_m \neq 0$	ASinc	FFase	[13]
6	42CrMo4V	$\sigma_m = 0$	ASinc	FFase	[13]
7	34Cr4	$\sigma_m = 0$	ASinc	EFase	[6]
8	25CrMO4	$\sigma_m \neq 0$	ASinc	EFase	[14]
9	30NDC16	$\sigma_m = 0$	ASinc	EFase	[6]
10	34Cr4	$\sigma_m \neq 0$	ASinc	FFase	[12]
11	25CrMO4	$\sigma_m \neq 0$	ASinc	FFase	[13]
12	42CrMo4V	$\sigma_m \neq 0$	ASinc	FFase	[13]

Legend: Ax – loading synchronous; ASinc – loading asynchronous; EFase – loading in-phase; FFase – loading out-of-phase

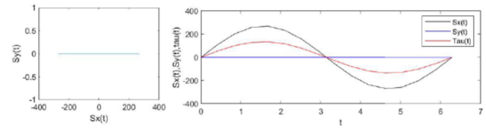

Figure 1 – (a) Phase diagrams and (b) load histories on 25CrMO4 in Test 2

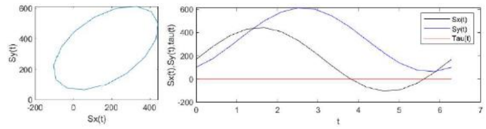

Figure 2 – (a) Phase diagrams and (b) load histories on 25CrMO4 in Test 5

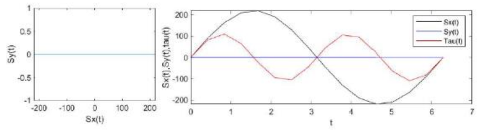

Figure 3 – (a) Phase diagrams and (b) load histories on 25CrMO4 in Test 8

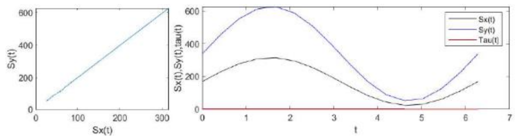

Figure 4 – (a) Phase diagrams and (b) load histories on 25CrMO4 in Test 11

5.2 Comparison of Experimental Data and Error Index

The comparison involves analyzing experimental data for diverse materials under different loading conditions. The error index IE is used to evaluate the accuracy of shear stress amplitude predictions (Eq. 13):

$$IE = \frac{\max\{\tau_a + \kappa\sigma_{nmax}\} - \lambda}{\lambda} \tag{13}$$

Thus, the following situations can be stated: $IE < 0\%$ - Non-conservative model; $IE > 0\%$ - Conservative model; and $IE = 0\%$ - Exact prediction. The conservative model is explained by its prediction of failure when, in reality, failure will not occur.

6 RESULTS

6.1 Determination of the Error Index in the Calculation of Stress Amplitude and Fatigue Models

The Susmel and Lazzarin Criterion and the Findley Criterion show that the error index in determining shear stress amplitude do not vary significantly. That is, for multiaxial stresses with zero mean stress in Figure 5, the accuracy of the prediction models is around 20%. This is considered an acceptable value although subject to further evaluation regarding the statistical behavior of other fatigue prediction parameters. It is highlighted that initially with in-phase EFase and synchronous Ax loadings, the results practically coincide. However, as the analysis progresses, asynchronous ASinc and out-of-phase FFase loadings compromise the accuracy of result predictions.

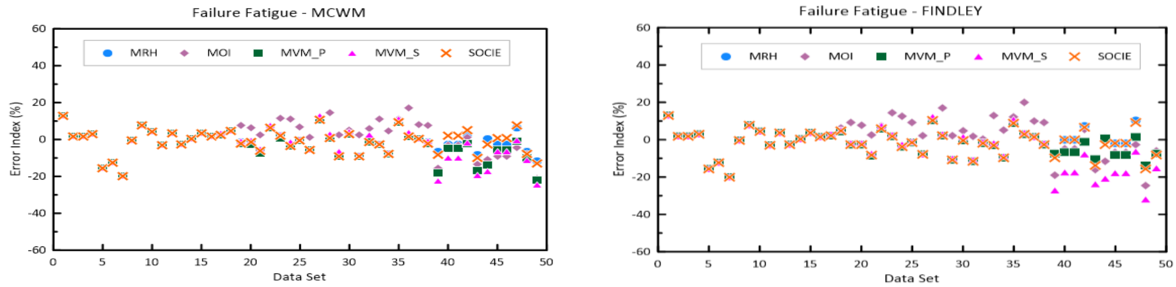

Figure 5 – Error index of failure predictions considering tests carried out under zero mean stress conditions

Considering loadings with non-zero mean stress in Figure 6, there is initially an equivalence in results for synchronous Ax and in-phase EFase loadings. It is noted that using the Socie Method there is a dispersion in the prediction of shear stress amplitudes compared to the predictions of other fatigue models especially in the range of out-of-phase FFase synchronous and Ax loadings. In regions where the loadings are synchronous Ax and in-phase EFase, both failure criteria present coincident predictions. According to the analysis of fatigue model prediction accuracy, in the cases presented, only the moment of inertia model both in Susmel and Lazzarin and in Findley deviates from the error index observed in the other models.

This may result in lower reliability in predicting shear stress compared to the other models. By contrast, the Socie model shows consistent results within the 20% error range for both in-phase EFase and out-of-phase FFase, as well as synchronous Ax and asynchronous loadings ASinc.

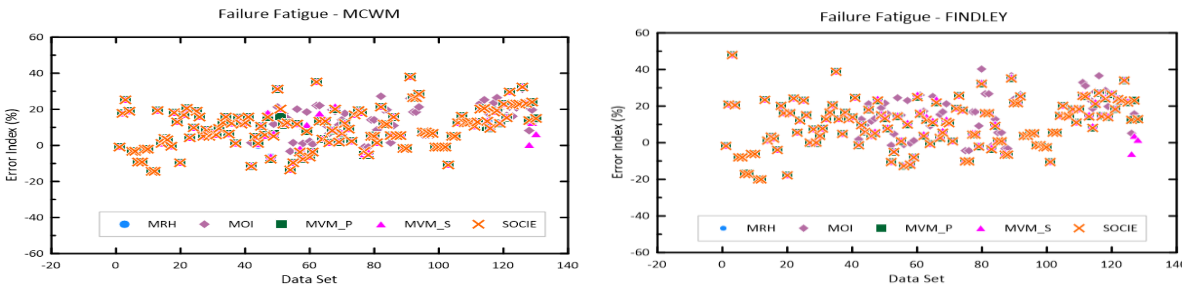

Figure 6 – Error index of failure predictions considering tests carried out under non-zero mean stress conditions

6.2 Determination of Critical Planes in the Calculation of Stress Amplitude and Fatigue Model

The results show that under zero mean stress conditions all fatigue models applied to the experimental data present similar results under in-phase EFase and synchronous Ax loading conditions. This is also noted under out-of-phase FFase and asynchronous ASinc loading conditions.

In the characteristic range of out-of-phase FFase and synchronous Ax loadings as well as in-phase EFase and asynchronous Asinc loadings the results of applying the fatigue models show slight divergences in indicating the crack growth plane as can be observed in Figure 7.

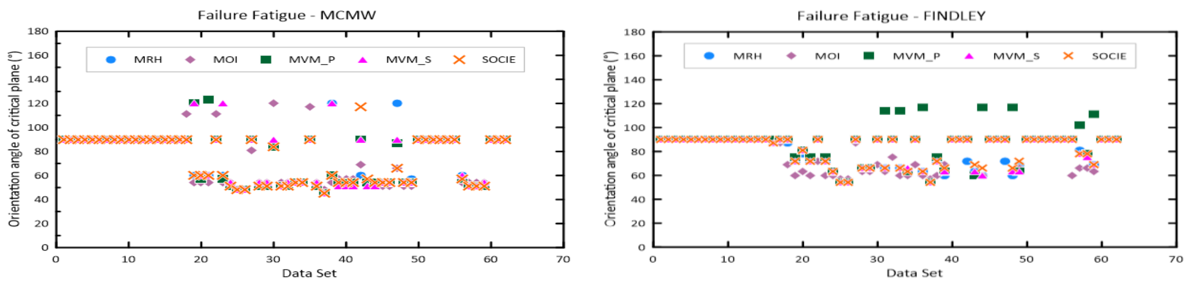

Figure 7 – Critical planes orientations for fatigue models

7 CONCLUSION

The applied fatigue models and failure criteria of Susmel and Lazzarin and Findley provide satisfactory and consistent results for predicting fatigue strength and identifying critical failure planes under specified multiaxial loading conditions. Thus, they can predict the fatigue strength of mechanical components by identifying the critical failure planes of materials subjected to specified and known multiaxial loadings. Each fatigue model and stress condition has specific characteristics that influence the reliability of fatigue strength predictions and confirmation of experimental results and material failure.

For future work, it is suggested to study more specific non-harmonic loadings and correlate the approach of the obtained data with analytical statistical parameters. And also address the application of other failure criteria such as Smith-Watson-Topper and Fatemi and Socie as well as other fatigue models like Minimum Circumscribed Circle Method (MCC) and the application of Genetic Algorithms for satisfactory computational results.

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