Crack propagation analysis due to fatigue using the Stable Generalized Finite Element Method (SGFEM)

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Abstract. This work aims to present a combination of the stable generalized finite element method (SGFEM) and of the displacement correlation method (DCM) for analyzing 2D crack propagation problems due to fatigue. The SGFEM allows to model fracture mechanics problems without excessive refinement and/or concern of the mesh-to-crack alignment. Additionally, this method maintains the numerical conditioning under control. The DCM, in turn, is computationally efficient when compared to traditional energy based methods (like J integral) and herein is even improved by means of a linear least square extrapolation. The maximum circumferential tension criterion and well-known propagation laws available in literature are adopted for considering the crack propagation. Experimental and numerical benchmark examples are proposed and the obtained results are compared against results provided by a commercial software widely adopted in the aeronautics industry.

Keywords: crack propagation, fatigue, GFEM.

1 Introduction

In the past, during a certain period, the aeronautics industry did not yet see the importance of considering cyclic loads, focusing excessively on static loads. However, unexpected structural failures began to appear even when dealing with loads below the critical threshold. Consequently, fatigue concepts like safe life and fracture mechanics features like stress intensity factors started to be included on the structural analysis at early aircrafts design stage.

A fundamental aspect related to fatigue analysis involves estimating how cracks can propagate. This requires the development of precise and robust numerical methods, which often entail high computational costs. The Finite Element Method (FEM) is still widely used in this context; however, classical FEM can have some drawbacks. For instance, achieving an accurate representation of the stress field near the crack tip requires a very fine mesh. Consequently, these requirements can reduce productivity.

Since the beginning of this century, improved versions of the FEM have been developed, generally providing extremely satisfactory results. It is the case of the eXtended Finite Element Method (XFEM) (Moes et al. [1] and Belytschko and Black [2]) and the Generalized Finite Element Method (GFEM) (Duarte et al. [3] and Strouboulis et al. [4]). Both methods - *following Belytschko et al. [5], the GFEM and the XFEM are treated here as equivalent numerical methods* - allow to model fracture mechanics problems without excessive refinement and/or concern of the mesh-to-crack alignment. Despite these positive features such methods present certain drawbacks, for example, lack of reproducibility of the enrichment functions and ill conditioning. The basic premise of the Stable Generalized Finite Element Method (SGFEM) (Babuška and Banerjee [6] and Gupta et al.

[7]) is precisely to overcame these drawbacks.

In the context of fatigue crack growth, the Stress Intensity Factor (SIF) is one of the main features to be analyzed. Through it, it is possible to evaluate important features like propagation angle and the crack growth rate using crack propagation laws. Thus, it is required a robust and accurate technique to extract this quantity. In this work the Displacement Correlation Method (DCM) is chosen for the extraction of SIF. Although there are others techniques to extract SIF's, the DCM was preferred due to its application simplicity that results in low computational cost.

In this context, this work aims to present a combination of the SGFEM and of the DCM for analyzing 2D crack propagation problems due to fatigue. However, G/XFEM results is also investigated. The reference solutions used to assess the accuracy of the results are mostly provided by the commercial software NASGRO®.

This paper is outlined as follows. The next section presents the fundamental aspects of this work such as: G/XFEM and SGFEM formulations, DCM details and crack propagation criteria. Section 3 shows two numerical examples of 2-D fatigue problems used to evaluate the performance of the combination SGFEM and DCM . Finally, the main conclusions of this work are presented in Section 4.

2 Fundamental aspects

This section addresses some fundamental aspects implemented during the development of this work. First of all, a brief review about the G/XFEM and SGFEM formulation are presented. After that, the SIF computation by means of the DCM is shortly explained. At last, basic concepts related to crack propagation and fatigue are discussed.

2.1 G/XFEM and SGFEM

Both G/XFEM and SGFEM essentially create a global approximation space which is composed by the classical FEM approximation space (S_{FEM}) added by the enrichment functions spaces (S_{ENR}). The difference between them is established in manner how the enrichment functions space is build. In the G/XFEM case, S_{ENR} is defined by:

$$S_{\mathrm{ENR}}^{G/XFEM} = \sum_{j=1}^{n} N_j\left(\mathbf{x}\right) \sum_{i=2}^{q_j} L_{ji}\left(\mathbf{x}\right) \mathrm{b}_{ji} \tag{1}$$

where N are the FEM partition of unity shape functions, n is the total number of nodes, q_j is the number of enrichment functions attached to initial degree of freedom j, b_{ji} are nodal parameters associated with each component $N_j(\mathbf{x})L_{ji}(\mathbf{x})$ and $L_{ji}(\mathbf{x})$ is one component of the enrichments functions set (traditionally $L_{j1}(\mathbf{x}) = 1$).

On the other hand, in the SGFEM case, S_{ENR} is defined by:

$$S_{\mathrm{ENR}}^{SGFEM} = \sum_{j=1}^{n} N_j\left(\mathbf{x}\right) \sum_{i=2}^{q_j} \tilde{L}_{ji}\left(\mathbf{x}\right) \mathrm{b}_{ji} \tag{2}$$

Therefore, the unique difference between G/XFEM and SGFEM is that in the last one the enrichment functions become set as:

$$\tilde{L}_{ji} = L_{ji} - I_{\omega_j}\left(L_{ji}\right) \tag{3}$$

where $I_{\omega j}$ (L_{ji}) is the piecewise finite element interpolant of the original enrichment function L_{ji} on the support ω_j:

$$I_{\omega_j}\left(L_{ji}\right)\left(\mathbf{x}\right) = \sum_{k=1}^{ne} N_k\left(\mathbf{x}\right) L_{ji}\left(\mathbf{x}_k\right) \tag{4}$$

where $\mathbf{x}_k$ is the coordinate vector of node k of element e (which is the element that contains the $\mathbf{x}$ position) and ne is the number of nodes of the element e.

More information about G/XFEM and SGFEM can be consulted in references cited in previous section.

Furthermore, it is necessary to highlight that the search for a stable and optimally convergent GFEM continues being investigated (see Bento et al. [8]).

2.2 SIF's extraction based on DCM

The computation of the SIF's by means of DCM is given by following equations:

$$K_I = \sqrt{\frac{2\pi}{r}}\frac{\mu}{\left(\kappa+1\right)}CTOD$$
$$K_{II} = \sqrt{\frac{2\pi}{r}}\frac{\mu}{\left(\kappa+1\right)}CTSD \tag{5}$$

where μ is the shear modulus, CTOD (Crack Tip Opening Displacement) is the difference between displacements in perpendicular direction of the upper and lower crack faces and CTSD (Crack Tip Sliding Displacement) is the difference between displacements in parallel direction of the upper and lower crack faces.

A linear least square extrapolation was proposed by Gupta et al. [9] with the objective of increasing the accuracy of the DCM technique. In this work this same technique is adopted. Basically, the DCM will be applied in different distances from crack tip to extract the SIF for modes I and II in different positions (see Figure 1). It was adopted by default an extraction of 20 equally spaced sampling points.

2.3 Fatigue crack growth

In fatigue crack growth problems, basically two parameters are required for the numerical simulations: crack increment and propagation direction. The first one was hereby defined as a constant crack increment small enough to guarantee an accurate estimate of the crack propagation. The second one was defined by criterion of maximum tangential stress (Erdogan and Sih [10]). According this criterion, the crack increment direction, θ_c, can be calculated as:

$$\theta_c = 2arctg\frac{1}{4}\left[\frac{K_I}{K_{II}} + \sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8}\right] \quad \text{if } K_{II} \geq 0$$
$$\theta_c = 2arctg\frac{1}{4}\left[\frac{K_I}{K_{II}} - \sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8}\right] \quad \text{if } K_{II} < 0 \tag{6}$$

For the next step, it is needed to have a way to define the relationship between the SIF's and the crack growth rate. Several relations were developed to connect the crack growth rate (da/dN) with the variation of SIF's (ΔK) related to the maximum and minimum load applied at the structure. One of the most popular was introduced by Forman and Mettu [11]. This relation is adopted hereby and can be written as:

$$\frac{da}{dN} = C\left[\left(\frac{1-f}{1-R}\right)\Delta K\right]^n \frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1-\frac{K_{\max}}{K_c}\right)^q} \tag{7}$$

where R is the stress ratio defined by the quotient between the minimum and maximum applied load, ΔK_{th} is the threshold SIF, K_c is the fracture toughness, C, n, p and q are material constants defined empirically and f is the crack opening function defined by Newman [12].

At last, for the examples simulated in next section, it must be highlighted that all calculations are discrete extracting SIF's at half length of the crack increment. Therefore, for each propagation step:

$$\Delta N = \Delta a \cdot f\left[\Delta K\left(a_{avg}, \Delta\sigma\right), R\right] \tag{8}$$

where a_{avg} is defined according to Figure 1 below.

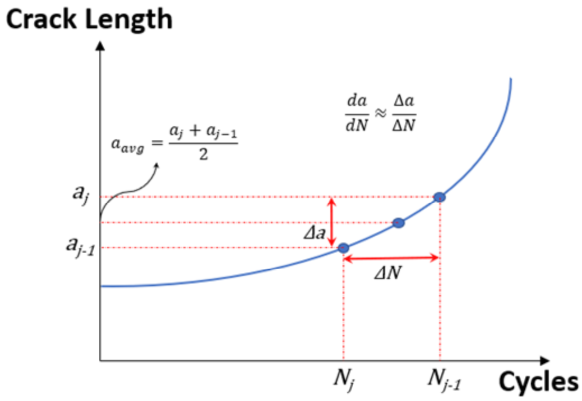


Figure 1. Crack length *versus* number of cycles

3 Numerical examples

In this section two numerical examples are investigated to demonstrate the accuracy and efficacy of the combination SGFEM and DCM to analyze fatigue crack growth. The reference solutions are provided by the commercial software NASGRO® and by experimental values obtained from Zhang et al. [13].

3.1 Plate with hole and single edge crack

This problem consists of an aluminum square plate (H = W = 200 mm) with a hole (D = 40 mm) located at its center. The thickness plate (t) is 2 mm. There is an initial crack (a_0 = 10 mm) located at the hole edge and the plate is submitted to a cyclic uniaxial load with constant amplitude (σ_{max} = 50 N and σ_{min} = 0) (see Figure 2). The mechanical and fatigue parameters of aluminum alloy 2024 T3 needed to perform the simulation are also depicted in Fig. 2. At last, it is assumed that the plate is under plane stress state and the crack step (Δa) utilized in crack propagation corresponds to 5 mm.

Mechanical and fatigue parameters
(Aluminum alloy 2024 T3)

$E = 72397.5MPa$

$\nu = 0.30$

$\Delta K_{th} = 110.43MPa\sqrt{mm}$

$K_c = 3820MPa\sqrt{mm}$

$C = 1.46 \cdot 10^{-9}, n = 2.89, p = 0.96 \text{ and } q = 3.76$

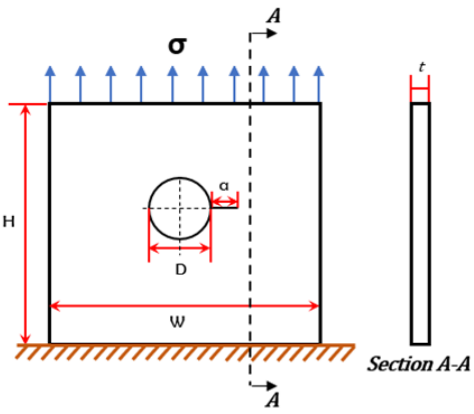
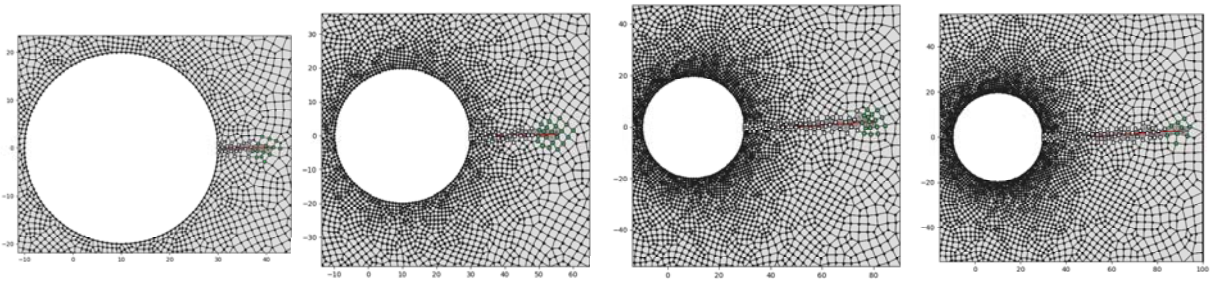


Figure 2. Aluminum alloy properties and plate with hole edge crack (geometry and boundary conditions)

Regarding to enrichment strategy in this case singular enrichment functions were applied at nodes inside a circular region with radius equal to 3.0 mm for the first propagation step and 5.0 mm for the other cases. For the elements nodes crossed by the crack segments, Heaviside enrichment functions were applied when G/XFEM was used and linear Heaviside was applied when SGFEM was used. Furthermore, with the objective of improve the SIF results, for both cases, linear polynomial enrichment functions were applied only at the elements nodes crossed by the crack segments. More information about these enrichment functions can be consult in Gupta et al.

[7].

Figure 3 depicts some steps of the crack propagation. Clearly, due to loading direction, as expected, the crack shows a tendency to propagate in horizontal direction. In this same figure is possible to see that the mesh is composed by bilinear quadrilateral elements.



Step 1 Step 4 Step 9 Step 11

Figure 3. Some crack propagation steps for plate with hole edge crack. Green nodes are enriched by singular functions, white nodes are enriched by Heaviside functions (constant and linear) and also linear polynomial functions.

Figure 4 presents two fundamental features of fatigue analysis. On the left, it is shown an comparison between the SIF values extracted during the crack propagation. The reference values were acquired from NASGRO®. Noticeably, the SGFEM results are more accurate than the G/XFEM results. More specifically, the relative errors associated to SGFEM remain below to 5% for all propagation, whereas the relative errors associated to G/XFEM reach until almost 30% near to the end of the propagation.

From the SIF values is possible to evaluate the crack growth rate. As can be seen on the right of Fig. 4, the estimated number of cycles by the SGFEM is almost half of the estimated number by the G/XFEM due to considerable difference between G/XFEM and SGFEM results.

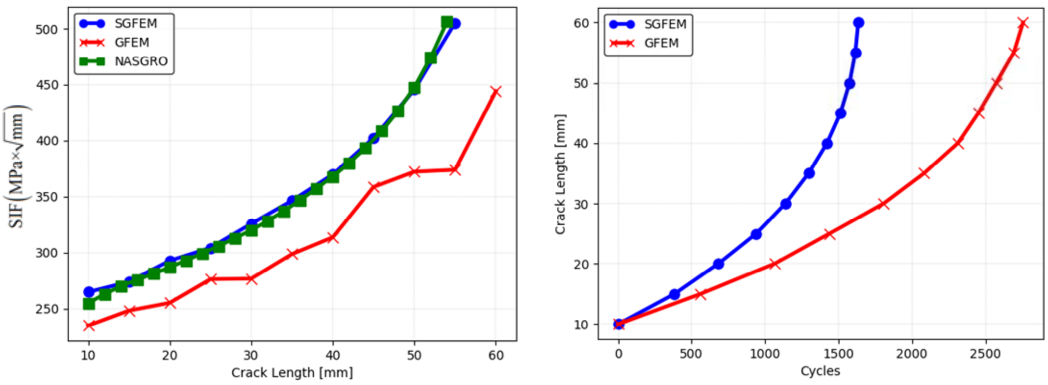


Figure 4. SIF results comparison (left) and number of cycles *versus* crack length (right) for plate with hole edge crack.

3.2 Plate with two holes and single edge crack

This last problem refers to an aluminum rectangular plate (H = 60 mm and W = 91 mm) with two holes (D = 4 mm) equally spaced of plate center. The distance between the hole centers (B) corresponds to 25.4 mm. The thickness plate (t) is 2 mm. One inclined initial crack (a_0 = 5 mm and α_0 = 45°) emerges from each hole and the plate is submitted to a cyclic uniaxial load with constant amplitude (σ_{max} = 50 N and σ_{min} = 0) (see Figure 5). The mechanical and fatigue parameters of aluminum alloy 7075 T6 required to perform the simulation are also illustrated in Fig. 5. Once again, by hypothesis, it is assumed that the plate is under plane stress state and the crack step (Δa) used in crack propagation corresponds to 1.25 mm. Finally, the enrichment strategy adopted in this example is the same explained in previous example.

Mechanical and fatigue parameters
(Aluminum alloy 7075 T6)

$E = 71700 MPa$

$\nu = 0.30$

$\Delta K_{th} = 103.31 MPa\sqrt{mm}$

$K_c = 1717.4 MPa\sqrt{mm}$

$C = 9.686 \cdot 10^{-12}, n = 3.00, p = 0.5 \text{ and } q = 1.00$

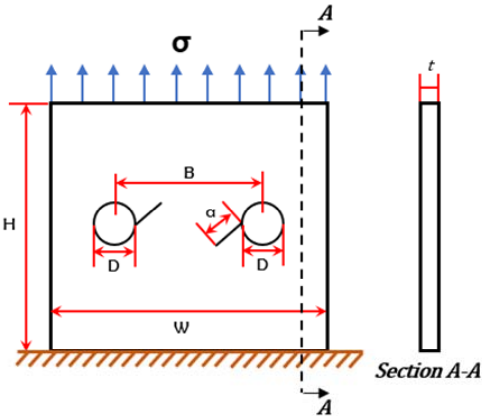


Figure 5. Aluminum alloy properties and plate with double hole edge crack (geometry and boundary conditions)

Figure 6 shows a comparison between the crack path provided by SGFEM and the path provided by experimental data obtained from Zhang et al. [13]. Since it was not observed significant differences in the crack path for G/XFEM and SGFEM, it was plotted only the results provided by SGFEM. As it can be observed, both path curves present a similar trend to get closer to the opposite hole.

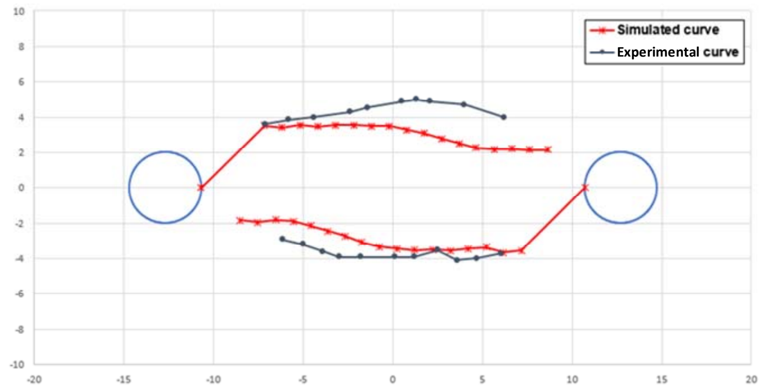


Figure 6. Crack path comparison between the simulation and experimental data for plate with double hole edge crack

Figure 7 presents the crack propagation rate provided by G/XFEM and SGFEM considering both cracks. Remarkably, it is possible to observe that the SGFEM has provided more conservative values than the G/XFEM. Furthermore, the SIF values extracted from SGFEM generated a crack propagation rate very similar to the experimental data.

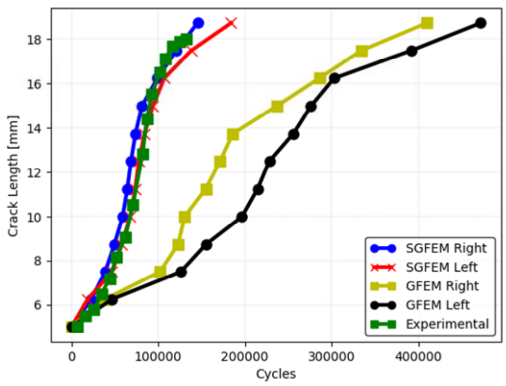


Figure 7. Number of cycles versus crack length (right) for plate with double hole edge crack

4 Conclusions

According to obtained results is possible to establish the following findings:

i) Despite its simplicity when compared to techniques based on energy the DCM can provide accurate SIF's.

ii) In general, the SGFEM provides results more accurate than the G/XFEM for fatigue crack growth problems.

iii) The estimated safe life for fatigue computed by the SGFEM is ordinarily lower than the estimated by the G/XFEM.

As a suggestion for a future work, it can be update the SGFEM formulation now considering new modifications in order to become the method more stable and optimally convergent.

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