Comparative Analysis of Evolutionary Methods in Topological Optimization of Truss Structures: Progressive Directional Selection versus Evolutionary Structural Optimization

Nayro Silva Noronha Cavalcante [1], Márcio André Araújo Cavalcante [2], Luiz Carlos Lima Véras [3]

[1]*Center of Technology, Federal University of Alagoas*
Av. Lourival Melo Mota, 57072-900, Alagoas, Brazil
nayro.cavalcante@ctec.ufal.br
[2]*Campus of Engineering and Agricultural Sciences, Federal University of Alagoas*
BR-104, Rio Largo, 57100-000, Alagoas, Brazil
marcio.cavalcante@ceca.ufal.br
[3]*Center of Technology, Federal University of Alagoas*
Av. Lourival Melo Mota, 57072-900, Alagoas, Brazil
luiz.veras @ctec.ufal.br

Abstract. The main difference between evolutionary methods and traditional structural topological optimization methods is that the former may not always converge to an optimized solution and can be challenging to implement. This work presents two heuristic and evolutionary methods, which aim to analyze truss structures obtained by the topological optimization process using the PDS and ESO methods, employing a ground structure type structure as the initial design domain. The Progressive Directional Selection (PDS) method is based on selecting a certain number of elements, established by the user, that most contribute to supporting the loads applied in an initial design domain. The selection stages involve a progressive number of selection steps, each eliminating a certain number of less efficient elements. The Evolutionary Structural Optimization (ESO) method removes elements that contribute less to supporting loads in an initial domain through a rejection rate. If removing elements with the same rejection rate value is no longer possible, an evolution rate value can be added to remove more inefficient elements. The optimized structures obtained by both methods efficiently support the applied loads. The PDS method stands out for reaching optimized truss structures with a specific number of elements that ESO cannot obtain, but the former stands out for its shorter processing time.

Keywords: structural topology optimization, trusses, ground structure, progressive directional selection.

1 Introduction

Man has always sought inspiration in nature to solve problems that appear in society, and it is not different in engineering. The morphology of animals or plants can inspire us, but understanding how life works and how the environment changes is essential for solving complex problems. Through his theory, the English naturalist Charles Darwin concluded that the living beings currently inhabiting the planet result from successive evolutions of more primitive living beings that inhabited the Earth thousands of years ago. Darwin's theory of natural selection suggests that living beings constantly struggle for survival, and those who survive are not necessarily the strongest, but the best adapted to environmental changes (Darwin, 1859). This theory helps explain what occurs in nature and serves to understand and solve problems in other fields of knowledge (Dennett, 1995; Deb, 2001).

In structural engineering, it is essential to design cost-effective and safe structures. Topology optimization is a valuable tool for achieving this objective and is already being utilized by some designers. A new topological structural optimization approach is presented in this work, using the theory of natural selection as inspiration, more specifically, directional natural selection, called Progressive Directional Selection (PDS), proposed by Véras and Cavalcante (2020). The method consists of removing elements (individuals) from an initial design domain formed by several elements (population) that contribute less to support the loads acting on the structure.

Xie and Steven (1997) developed a method called Evolutionary Structural Optimization (ESO), which involves removing inefficient materials from a structure to generate an optimized solution. Even though the method is easy to implement computationally, it has limitations, as the removed parts cannot be restored, leading to convergence and mesh dependence issues. To address these problems, Huang and Xie (2007) introduced an extended method called Bidirectional Evolutionary Structural Optimization (BESO), which allows the addition and removal of material from the initial design domain.

According to Munk et al. (2015), structural topological optimization has two main approaches. The first approach is gradient-based, involving the derivation of mathematical models by calculating design variable sensitivities. The second approach is not based on gradient, which uses heuristic or stochastic approaches for material addition or removal without relying on sensitivity functions. Gradient-based methods are computationally complex and may not always yield solutions. On the other hand, non-gradient-based methods offer a simpler alternative with comparable results to gradient-based methods.

This work aims to compare the results obtained by evolutionary topological optimization methods, PDS versus ESO, in two-dimensional truss structures, where the initial design domain is a grid of points interconnected by bars, known as a ground structure.

2 Progressive Directional Selection

The Progressive Directional Selection (PDS) method was developed by Véras and Cavalcante (2020) as a new approach for the topological optimization of two-dimensional continuous elastic structures. Like other topological optimization methods, PDS aims to achieve the best structure topology through optimal material distribution within a defined domain, considering an objective function and constraints.

The initial design domain is considered as the starting population of individuals, with each element in that domain being treated as an individual. When the acting loads and constraints are imposed, each element needs to contribute so that the structure can support these loads. The elements that contribute the least to the structure are gradually eliminated, similar to what occurs in nature. This process results in the structure evolving into a configuration that efficiently supports the loads.

When directional selection acts on a population in nature, a specific characteristic can ensure the survival of those individuals. The PDS method optimizes the structure by minimizing the objective function, such as flexibility, strain energy, or average von Mises stress, and determines which structural elements will remain until the end of the selection process. The process is simple: once the desired final volume of the structure is known, the initial idea is to gradually remove elements from an initial configuration as many times as necessary throughout the process. At each selection stage, the number of removal steps increases, and the number of elements removed per step decreases until this process results in the same set of individuals, i.e., the same selection (Véras e Cavalcante 2020).

Consider Figure 1 as an illustrative example of this process. Suppose a building floor where the designer needs to define the position of ten columns, knowing there are twenty possibilities for their location. In this scenario, the initial population of twenty columns will undergo a progressive directional selection. Once the loads are applied to the structure, the first selection stage consists of removing the ten least contributing columns in a single step to obtain the ten selected columns.

The process of removing elements is divided into several stages to gradually exclude them from the initial set. For instance, the second stage consists of two steps: the load is applied, and the five least contributing columns are removed. Then, the load is applied again to the remaining fifteen elements, and the five least contributing columns are removed, resulting in ten columns at the end of the stage. The third stage involves three steps for removing elements, with 4, 3, and 3 columns removed in each subsequent step.

The final population of columns in each stage can be different due to the rearranging of internal forces. The maximum number of selection stages is equal to the total number of elements to be removed, which is ten in the case of the building floor with twenty columns. The process continues until the selected elements in the last stage match those in the previous stage, indicating convergence. In the case of the building floor, the positions of the columns at the end of the third stage match those in the second stage, demonstrating convergence. Generally, the user will determine the number of consecutive stages (n) with repetition of the selected elements through the convergence criterion C_n, which defines when the process must stop.

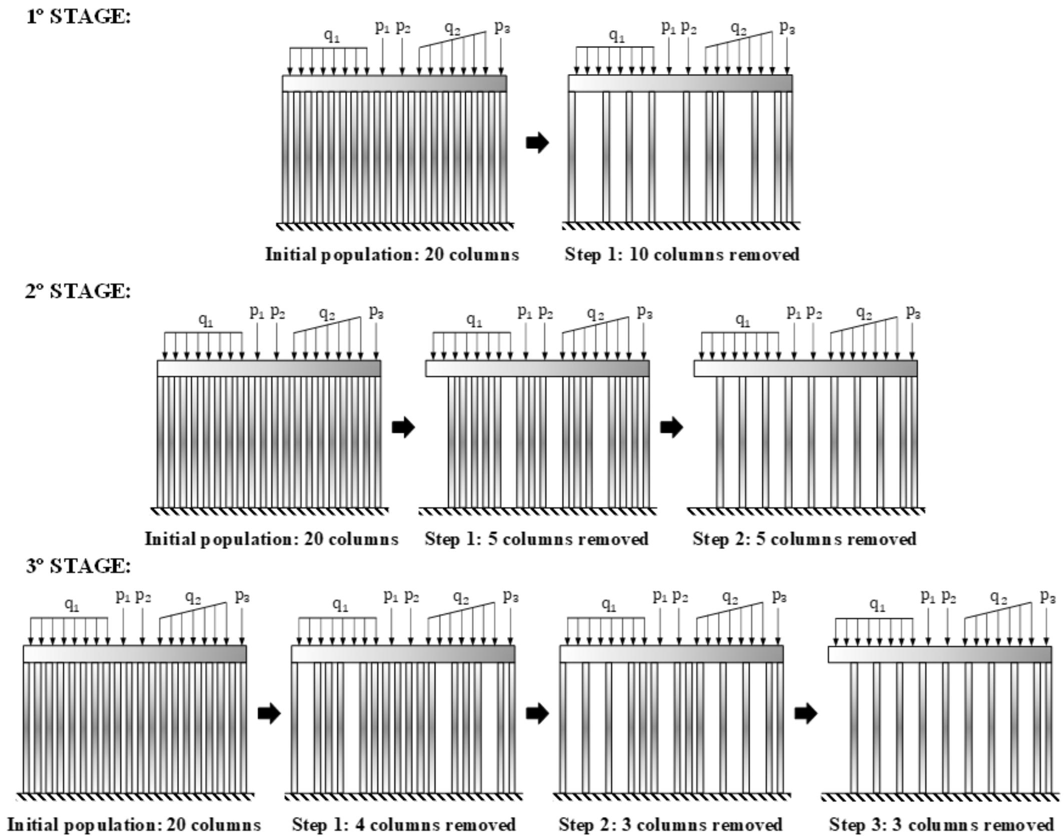


Figure 1. Hypothetical example of the PDS application on a building floor with an initial population of twenty columns.

3 Evolutionary Structure Optimization

Xie and Steven proposed the Evolutionary Structure Optimization (ESO) method in the early 1990s, and it has been widely used since then to solve topology optimization problems. The method is based on the simple concept of gradually removing inefficient material from a structure. ESO can be a valuable tool for engineers and architects looking for structurally efficient forms in the conceptual phase of the project (HUANG and XIE, 2010).

In the case of removing elements with low-stress levels, it is necessary to determine the stress levels in any part of the structure through an analysis that can be carried out using the finite element method. Ideally, all parts of the structure need to be close to safety values, making the structure fully tensioned. Therefore, a rejection criterion can be established based on local stress levels, in which elements that present a low-stress level are assumed to be underused and are removed during the optimization process.

The stress level of each element can be determined by comparing the effective von Mises stress σ_e^{vm} of each element and the maximum effective von Mises stress across the entire structure σ_{max}^{vm}. After finite element analysis, the elements that satisfy the relationship below:

$$\frac{\sigma_e^{vm}}{\sigma_{max}^{vm}} < RR_i \tag{1}$$

are removed from the model, where RR_i is the current rejection ratio.

The finite element analysis cycle, together with the removal of elements, is repeated for the same value of RR_i until a steady state is reached, which means that no more elements can be removed with the current rejection rate. At this stage, an evolution ratio, ER, is added to the rejection ratio, that is:

$$RR_{i+1} = RR_i + ER \tag{2}$$

As the rejection ratio increases, the process occurs again until a new steady state is reached. This evolutionary process continues until a desired optimized solution is obtained.

4 Methodology

4.1 Construction of the ground structure

The design domain consists of a regular mesh of points with a prescribed density, forming the initial framework for structural analysis. The nodal mesh is connected by a pattern of elements linked to these nodes. This configuration, referred to in the literature as a ground structure, comprises bars connected through the nodes of the nodal mesh. Zegard and Paulino (2014 apud Zhao, 2014) introduced an efficient method to generate the ground structure without overlapping bars. They defined various levels of connectivity to enhance structural optimization. At level 1, each node connects to its immediate neighbors, establishing a basic connectivity pattern (Figure 3a). At level 2, the connectivity extends to the neighbors' neighbors, increasing the complexity and potential stiffness of the structure (Figure 3b). Finally, at level 3, each node connects to all other nodes within the mesh, maximizing connectivity and potentially optimizing material distribution and load transfer across the structure (Figure 3c). This multi-level connectivity approach facilitates the exploration of optimized topologies, balancing the demand for material and structural performance.

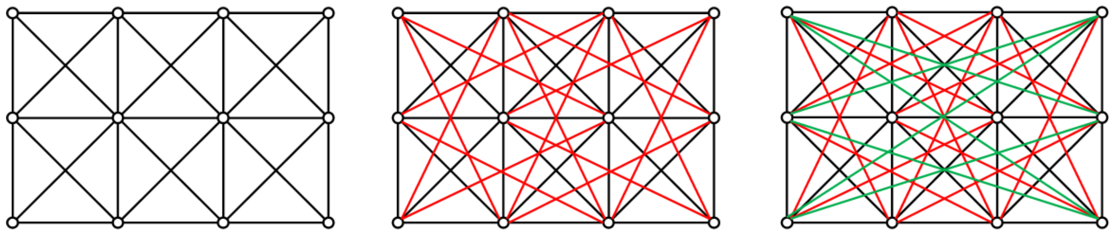


Figure 2. Ground structures with different levels of connectivity (Zhao, 2014).

The ground structure used in the examples is rectangular, a common choice due to its simplicity and effectiveness in structural analysis. First, the dimensions of the initial design domain are determined, specifying the width L and the height H of the rectangle. These dimensions define the boundary within which the structural optimization will occur. Next, the numbers of divisions in the horizontal direction (NEH) and the vertical direction (NEV) are determined. This step is crucial as it sets the resolution of the mesh, with a finer mesh typically leading to more complex topologies at the cost of increased computational effort. The grid of points is then established based on these parameters, and the ground structure is formed with a total number of nodes equal to (NEH+1) ×(NEV+1).

4.2 Two-dimensional truss optimization based on PDS and ESO

Each ground structure bar is modeled as a two-dimensional truss element, with each element having two degrees of freedom per node. Loads and supports are specified within the initial design domain alongside the elements' cross-sectional area A and longitudinal modulus E. In this study, all elements within the initial design domain share the same cross-section and material properties. Subsequently, each element's length and angle of inclination are calculated relative to the global coordinate system. Based on this data, each element's stiffness matrix is determined in local and global coordinate systems. The local stiffness matrices are then used to assemble the global stiffness matrix for the ground structure. By applying the direct stiffness method, the equation $\{\boldsymbol{F}\} = [\boldsymbol{K}]\{\boldsymbol{u}\}$ is formulated. This allows the calculation of the free degrees of freedom displacements and, consequently, the stresses within each element.

In this investigation, the PDS method is tailored to optimize the design of plane trusses. The user defines initial parameters, such as the number of elements to select, and the convergence criterion C_n, which indicates the number of selection stages needed with the same selected elements to complete the optimization process. For instance, if C_5 is adopted, the algorithm requires five stages with the same selected elements to determine the optimized structure. The performance criterion, which in this study is the absolute stress, is also defined by the user and employed for ranking the elements. The user needs to set a selection tolerance ST, allowing elements with absolute stress values close to the last selected element to be included in the selection based on relative stress differences. A penalty factor PF is also necessary to penalize the contribution of removed elements in the global stiffness matrix assembly to avoid remeshing, which is computationally more expensive.

Another important aspect of this method is defining the number of individuals removed (NR_i^{step}) at each stage of the selection process. For the i-th selection stage, NR_i^{step} is initially proportional to the final number of individuals removed (NR), which, in the case of continuous problems, is related to the desired final volume for the structure. Thus, $NR_i^{step} = NR/i$, which must be an integer to access a vector by its index. When this does not occur, NR_i^{step} must be adjusted by redistributing the decimal part among the remaining stages, as illustrated in the third stage of the building floor problem.

Regarding ESO, as described in section 3, Querin et al. (2017) briefly shows the step-by-step process for implementing the method:

Step 1: Determine the initial domain of the structure, boundary conditions, and the physical and geometric properties of the elements;

Step 2: Discretize the structure using a finite element mesh;

Step 3: Perform the finite element analysis of the structure;

Step 4: Remove elements that satisfy the equation (1);

Step 5: Increase the rejection rate according to equation (2);

Step 6: Repeat steps 3 to 4 until a desired optimized solution is obtained.

Regarding step 2, two-dimensional truss elements in a ground structure domain are used instead of finite elements in the continuous domain.

4.3 Analysis of the optimized structures

After finding the optimized structures, an analysis of the structures for each method is performed. This involves examining the deformed configuration, stress distribution, processing time, and total length of elements (i.e., the sum of the lengths of all elements). This analysis aims to ensure that the optimized truss meets performance criteria under loading conditions while reducing material cost by minimizing the total length of elements. Processing time is also crucial as it affects computational efficiency and the feasibility of the optimization process.

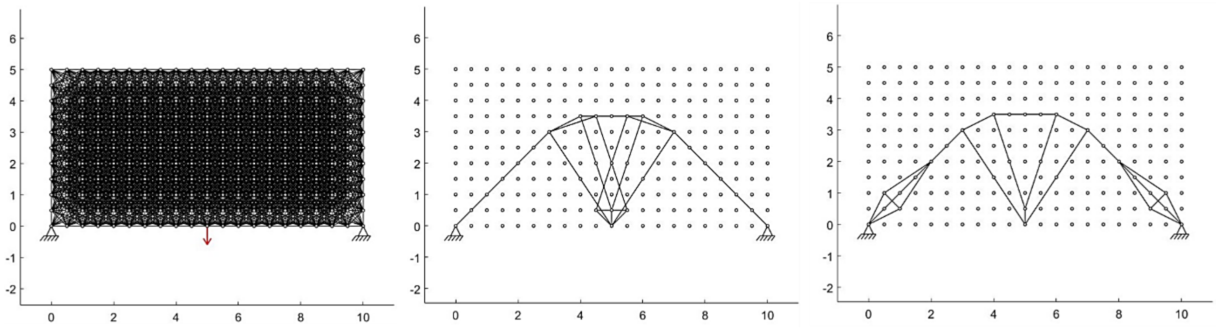
5 Results and Discussions

The results of the optimization process based on the PDS and ESO methods are presented below. Two types of structures are analyzed: Michell structure and Cantilever beam. For all analyses, the adopted ground structure has connectivity level 3, and all bars have the same cross-sectional area ($A=0.01\ m^2$) and longitudinal modulus of elasticity ($E=70\ GPa$). The input parameters for the PDS are convergence criterion C_7, selection tolerance $ST = 10^{-6}$, and penalty factor $PF = 10^{-6}$. The computational environment in terms of programming language and machine can be defined as Matlab R2020a (64-bits) for Windows 10, and processor of 5th Gen Intel(R) Core(TM) i5-5200U 2.20 GHz, RAM 8.0 GB DDR3L.

5.1 Michell structure

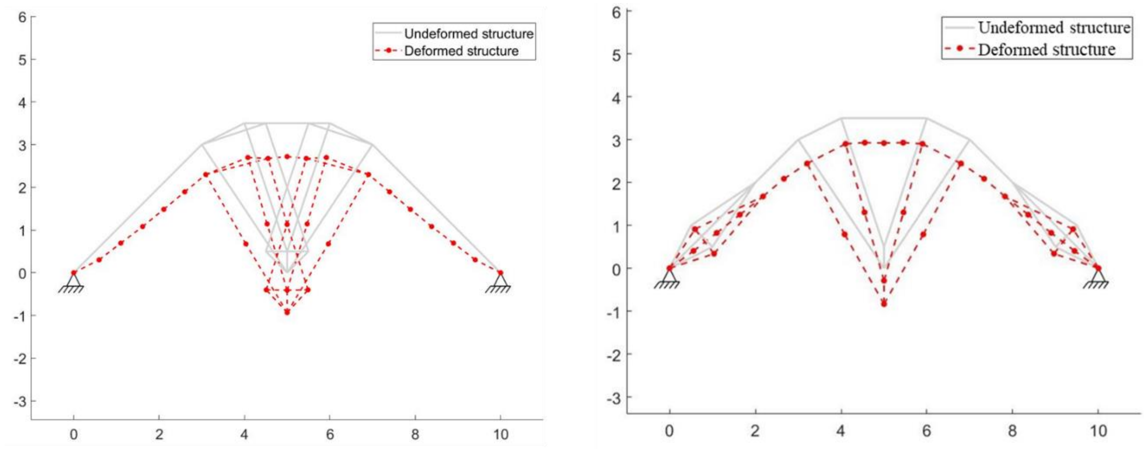
A classic example of topological structural optimization for trusses, developed by the Australian engineer A.G.M. Michell, involves minimizing their self-weight (Michell, 1904). The initial design domain consists of a grid of 21 × 11 points, with $10\ m \times 5\ m$ dimensions, as shown in Figure 4, with an applied load $P = 1000\ kN$. The optimized structures obtained by the PDS and ESO methods are shown in Figure 4-b) and Figure 4-c), respectively.

The optimized structure obtained by PDS consists of 37 elements and requires 560 selection stages to converge. For ESO, the initial rejection rate is $RR_0 = 0.1\%$, and the evolution rate is $ER = 0.17\%$. The final rejection rate is $RR = 10\%$ to reach the optimized structure. Figure 5 shows the deformations, and Figure 6 shows the stress distributions of the optimized structures for each method.



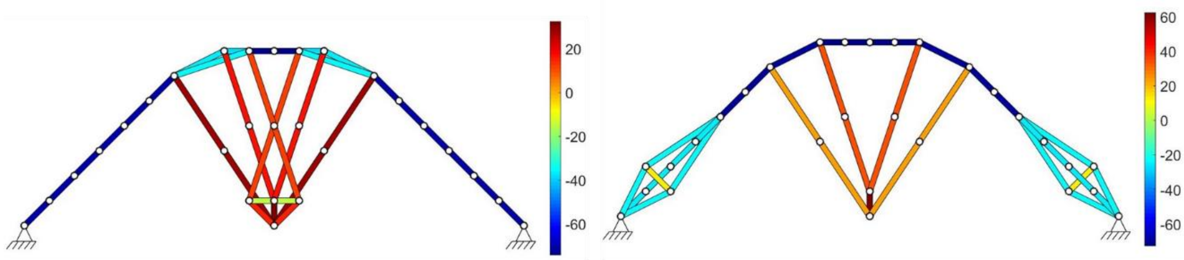
a) b) c)

Figure 3. Michell type optimized structure: a) Initial domain; b) PDS; c) ESO.



a) b)

Figure 4. Deformation of the Michell type optimized structure: a) PDS; b) ESO.



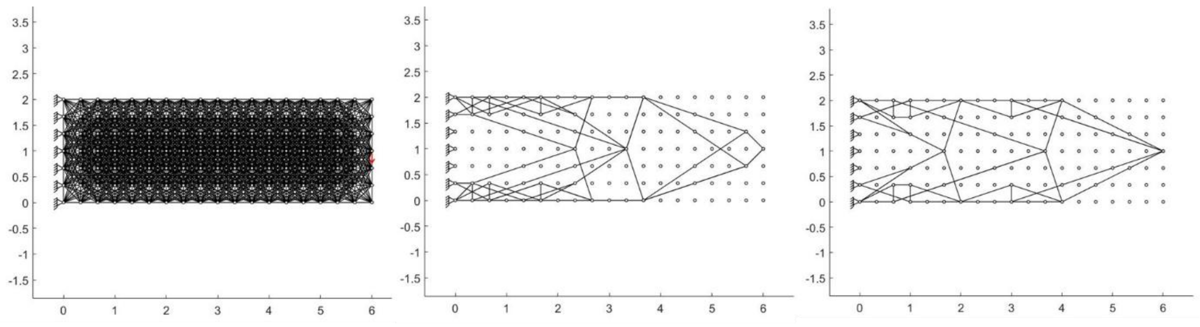
a) b)

Figure 5. Stress distribution in the elements of the Michell type optimized structure: a) PDS; b) ESO.

5.2 Cantilever beam

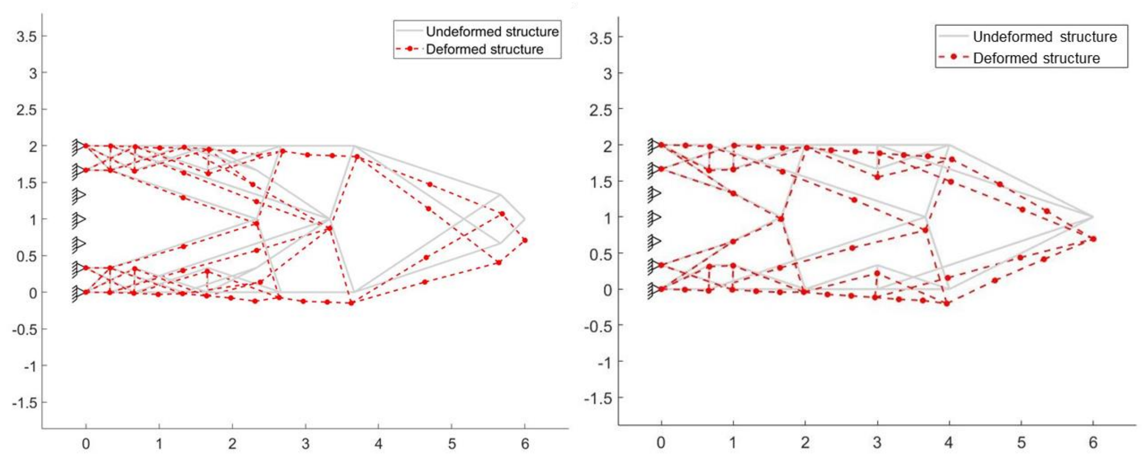
The cantilever beam is a standard design structure. The initial design domain is rectangular, with all nodes restricted at the left board and the load applied at the middle of the right board. The initial design domain consists of a grid of 19 × 7 points, with *6 m × 2 m* dimensions, as shown in Figure 7-a), and the applied load has an intensity of *P = 800 kN*. The optimized structures by PDS and ESO methods are shown in Figure 7-b) and Figure 7-c), respectively.

The optimized structure obtained by PDS has 74 elements and requires 271 selection stages to converge. For the structure obtained by ESO, the input parameters are $RR_0 = 0.1\%$ and $ER = 0.1\%$, and the rejection ratio evolves to $RR = 16.70\%$. Figure 8 shows the deformation, while Figure 9 shows the stress distribution in the elements of the optimized structures obtained by each method.



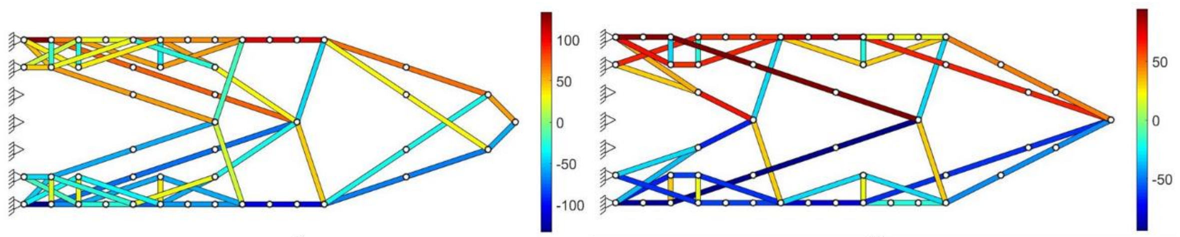
a) b) c)

Figure 6. Cantilever beam type optimized structure: a) Initial domain; b) PDS; c) ESO.



a) b)

Figure 7. Deformation of the Cantilever beam type optimized structure: a) PDS; b) ESO.



a) b)

Figure 8.Stress distribution in the elements of the Cantilever type beam optimized structure: a) PDS; b) ESO.

As shown in Table 1, in relation to the Michell type structure, the optimized structures of both methods presented the same number of elements. The displacement at the load application point is smaller in the optimized structure obtained by ESO, but the values of both structures are very close. The total lengths of the elements presented approximate values, with a lower value for the optimized structure obtained by PDS. Regarding processing time, the PDS optimization process takes much longer than the ESO algorithm.

In the case of the Cantilever Beam structure, there was a challenge in controlling the number of elements selected using the ESO method, resulting in 68 elements compared to the 74 elements in the structure obtained through the PDS. The displacement at the load application point showed similar values for the structures optimized by both methods, with the optimized structure by PDS displaying a slightly lower value. As a result of the smaller number of elements, the ESO-optimized structure has a shorter total length of the elements. The time needed to achieve the structure optimized by PDS is significantly longer than the time required for ESO optimization.

Table 1. Comparison of optimized structures obtained by PDS and ESO methods.

Example	Method	Nº of elements	Displacement at the load application point	Total length of the elements	Processing time
Michell structure	PDS	37	9.33 mm	38.66 m	25’28”
Michell structure	ESO	37	8.35 mm	39.85 m	7.11”
Cantilever beam	PDS	74	29.17 mm	53.08 m	2’51”
Cantilever beam	ESO	68	30.40 mm	47.26 m	7.84”

6 Conclusions

In this study, we conducted a comparative analysis of two heuristic evolutionary methods for the topological optimization of truss structures: Progressive Directional Selection (PDS) and Evolutionary Structural Optimization (ESO). The initial design domain was based on a ground structure approach, and the performance of these methods is evaluated in optimizing two-dimensional trusses, specifically the Michell structure and a cantilever beam. The PDS method iteratively removes less efficient elements to achieve an optimized structure with a specific number of selected elements. In contrast, the ESO method progressively eliminates elements with low stress levels based on a rejection ratio.

The results demonstrated that both the PDS and ESO methods effectively optimized the structural topology of the trusses, achieving configurations capable of efficiently supporting the applied loads. ESO is simpler to implement and requires significantly less processing time than PDS. However, the ESO method exhibited challenges in controlling the number of elements in the optimized structure, occasionally removing too many elements in a single step, with a slight increase in the rejection ratio.

Overall, the choice between PDS and ESO should be informed by the specific requirements of the optimization task. PDS may be the preferred method for applications where precise control over the final number of elements is critical despite its longer computation times. Future work should focus on improving the computation efficiency of PDS.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work and that all material included as part of the present paper is either the property (and authorship) of the authors or has the permission of the owners to be included here.

References

[1] C. Darwin. *On the Origin of Species by Means of Natural Selection, or the Preservation of Favoured Races in the Struggle for Life*. John Murray, 1859.
[2] D.C. Dennett. *Darwin's Dangerous Idea: Evolution and the Meanings of Life*. Simon & Schuster, 1995.
[3] L. C. L. Véras and M. A. A Cavalcante, “Topology optimization of continuum elastic structures through progressive directional selection method”. In. *Ibero-Latin American Congress On Computational Methods In Engineering (XLI CILAMCE)*, pp. 1-7, 2020.
[4] Y. M. Xie and G. P. Steven. *Evolutionary Structural Optimization*. Springer, 1997.
[5] X. Huang and Y. M. Xie. *Evolutionary topology optimization of continuum structures: methods and applications*. John Wiley & Sons, 2010.
[6] D. J. Munk, G. A. Vio and G. P. Steven, “Topology and shape optimization methods using evolutionary algorithms: a review”. *Structural and Multidisciplinary Optimization*, v. 52, pp. 613-631. 2015.
[7] Tuo. Zhao. *An Implementation of The Ground Structure Method Considering Buckling and Nodal Instabilities*. M.Sc. diss. University of Illinois at Urbana Champaign, 2014.

[8] T. Zegard and G. H. Paulino, "GRAND - Ground structure based topology optimization for arbitrary 2D domains using MATLAB. *Structural and Multidisciplinary Optimization*, v. 50, pp. 861–882, 2014.
[9] O. M. Querin, M. Victoria, C. Alonso, R. Ansola, P. Martí. *Topology design methods for structural optimization*. Academy Press, 2017.
[10] A. G. M. Michell. "The limits of economy of material in frame structures". *Philosophical Magazine*, v. 47, 1904,