Evaluation of Optimization Algorithms for Calibration of Numerical Models of Engineering Structures

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Abstract. In civil engineering, the finite element method (FEM) is employed to simulate the behavior of various structures. To address associated uncertainties, numerical model calibration based on experimentally obtained modal data is used, allowing for the update of difficult-to-obtain properties to better match the model's dynamic response to that of the real structure. This study evaluates two metaheuristic algorithms, Particle Swarm Optimization (PSO) and Genetic Algorithm (GA), for model calibration. A simply supported beam was modeled in ANSYS to extract pseudo-experimental modal data, including ten natural frequencies and corresponding mode shapes, with uncertainties represented by five design variables. The search spaces were defined based on literature values, and initial populations were generated using Latin Hypercube Sampling (LHS). MATLAB managed the optimization process, interfacing with ANSYS to iteratively adjust model parameters and minimize deviations in natural frequencies and Modal Assurance Criterion (MAC) values between numerical and reference models. Results show satisfactory performance for both algorithms, with PSO achieving better values for the same computational effort. Optimal settings for PSO were determined by balancing accuracy and computational efficiency. These findings highlight the effectiveness of calibration methods within the problem's constraints and PSO's superior capability in consistently handling calibration.

Keywords: optimization algorithm, numerical modeling, calibration, modal analysis, finite elements.

1 Introduction

Civil structures are intricately connected to societal life globally, and their design and functionalities both influence and are influenced by the societal organization in many aspects. Therefore, the collapse of a structure or its inability to serve its intended purpose has a significant impact on economic activities, daily social routines, and public safety. Despite the importance of maintaining structural integrity, assessments often rely on subjective visual inspections, which can be inaccurate and may overlook critical issues, as noted by Aktan et al. [1] and Sanayei et al [2]. Furthermore, the deterioration of civil structures occurs due to numerous factors, most of them presenting high levels of uncertainty, such as cyclic traffic loads, steel corrosion, material aging, environmental conditions, and extreme events, making it harder to define the structural health in terms of age and usage alone, as Chen [3] presented. Given these challenges, continuous in-service monitoring of the structure is an essential process for reliably assessing the structural health and safety. The development of finite element (FE) models is crucial for this analysis, as they provide essential baseline information to be later compared with measured values and aid in detecting structural damage [3].

However, these models often suffer from inaccuracies due to factors such as simplified modeling assumptions or uncertainties in material and geometric properties, which can lead to incorrect conclusions, as discussed by Ribeiro et al [4]. Thus, proper calibration is essential for accurately representing the structure. Model updating methods are employed to adjust the FE model based on measured modal data and can be divided into two groups:

direct and indirect [5] [6]. Direct methods update the mass and stiffness matrices of the structure to reproduce the measured vibration modal data, while indirect methods use parameter sensitivity to update the analytical model, minimizing discrepancies between measured and simulated data by optimizing a set of parameters [3].

The effectiveness of the calibration process depends on three key factors: the objective function and constraints, the structural updating parameters, and the optimization techniques [7]. While the first factor can be defined as a numerical function that assesses the discrepancies between measured modal data and the corresponding FE model results, the second factor strongly depends on the experimenter's physical understanding of the structure at a local level, such as the behavior of the connections between structural elements and their impact on the modal response [8]. Finally, the selection of an optimization algorithm plays a critical role. While a manual parameter tuning is possible, in practical cases it becomes unfeasible and inefficient. Some global optimization algorithms, such as the genetic algorithm and the simulated annealing, have shown success in minimizing the objective function. Despite their applicability, these algorithms face a significant barrier: the demand for high computational efforts, especially when the number of updating parameters is large and when significant differences between the measured data and the original model exist. In addition to the aforementioned metaheuristic algorithms, gradient-based methods, such as Levenberg-Marquardt [9], can also be used. These algorithms offer rapid convergence when there is a clear relationship between the parameters and the objective function, although they are less effective for escaping local minima.

In this paper, the application of optimization algorithms towards the calibration of a finite element model is explored by using an example of a simply supported beam. Different metaheuristic algorithms are tested, due to their capability of explore the solution space widely, avoiding local minima, and flexibility to handle diverse optimization challenges, and key algorithm parameters are discussed.

2 Methodology

To evaluate the performance of the iterative calibration method based on optimization algorithms, a numerical model of a simply supported concrete beam with a rectangular cross-section of 0.5 m width, 1.0 m height, and 10 m length was proposed, as shown in Fig. 1. The beam is considered to be on flexible supports, represented by springs with stiffnesses k_1 and k_2.

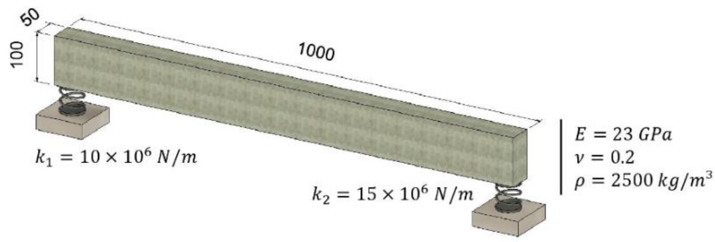
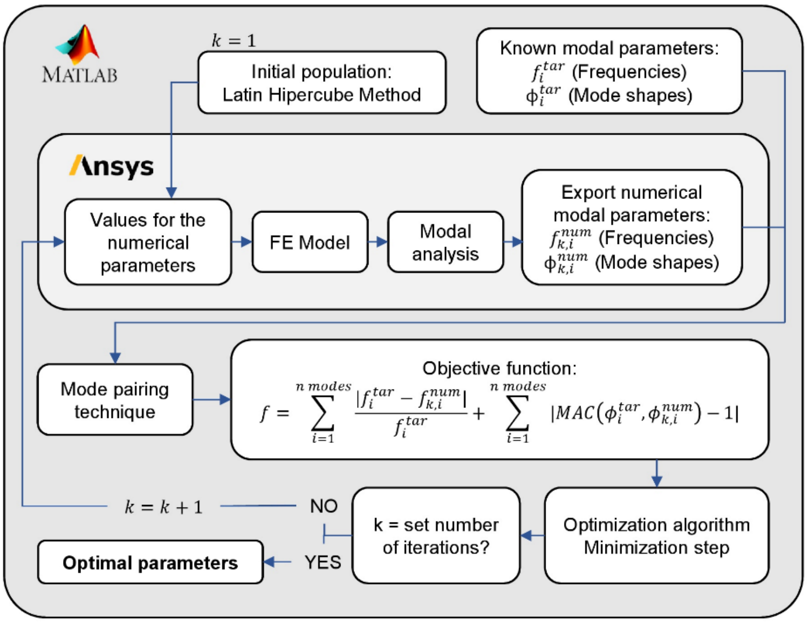


Figure 1. Schematic representation of the structure, dimensions in centimeters

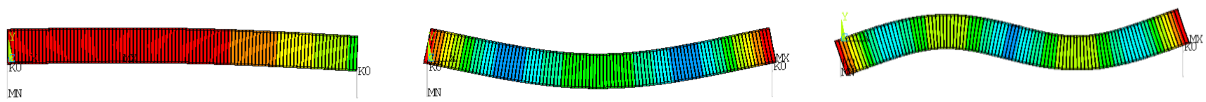
The process involved the use of two software packages: ANSYS Mechanical APDL 2020 R1 [10], which is used for finite element modeling and extracting modal parameters, and MATLAB R2020a [11], which receives the model data, manages the optimization algorithm, and executes ANSYS with adjusted parameters at each iteration, as illustrated in the flowchart in Fig. 2.

Figure 2. Flowchart of the calibration methodology based on optimization algorithms



2.1 Structural Modelling

The structure was modeled in ANSYS using the BEAM188 element, a 2-node element with 6 degrees of freedom per node (three translations and three rotations). The mesh was discretized into 100 elements along the length, and the adopted mechanical properties were as follows: Young's modulus as 23 GPa, Poisson's ratio as 0.2, and the density of the concrete was taken as 2500 kg/m³. Elastic supports were defined using COMBIN14 spring elements, with stiffness set to 10×10^6 N/m for the first support and 15×10^6 N/m for the second support. Regarding boundary conditions, out-of-plane displacement was restricted for all nodes, vertical displacement was limited with the spring at both hinged supports and horizontal displacement was prevented at one of the supports. Rotation was allowed at both ends. For the modal analysis, the determination of ten natural frequencies and their corresponding vibration modes was requested, using the Lanczos method to solve the eigenvalue and eigenvector problem. The modal data of the structure is then exported and considered as the target values for the calibration to be obtained with an optimization procedure (in real structures, these would be obtained experimentally). Three of the vibration modes of the numerical model are presented in Fig. 3.



Vibration mode 1 (f = 6,27 Hz) Vibration mode 3 (f = 33,4 Hz) Vibration mode 4 (f = 80,2 Hz)

Figure 3. Vibration modes of the target model.

2.2 Design Variables

The input parameters of the initial model were used as design variables, namely the modulus of elasticity (E), Poisson's ratio (ν), density (ρ), and the stiffness coefficients of both supports (k_1 and k_2). The geometric characteristics (cross-sectional dimensions and beam length) were considered fixed. The search range limits for the material parameters were defined based on the literature, given that the structure is made of concrete, while the range of support stiffness was kept broad due to significant uncertainty. The parameters and ranges are indicated in Tab. 1:

Table 1. Design variables and search range

Parameter	Lower Bound	Upper Bound	Unit
E	20	30	GPa
ν	0.2	0.5	-
ρ	2400	2600	kg/m³
k_1	10×10^5	10×10^7	N/m
k_2	10×10^5	10×10^7	N/m

2.3 Objective Function

The function to be minimized by the optimization techniques employed is presented in Eq. (1) and comprises a term related to the natural frequencies and another to the MAC values, proposed by Allemang and Brown [12] and shown in Eq. (2):

$$f = \sum_{i=1}^{n\,modes} \frac{|f_i^{tar} - f_i^{num}|}{f_i^{tar}} + \sum_{i=1}^{n\,modes} |MAC(\phi_i^{tar}, \phi_i^{num}) - 1| \tag{1}$$

$$MAC(\phi_i^{tar}, \phi_i^{num}) = \frac{\left|\phi_i^{tar^T} \phi_i^{num}\right|^2}{(\phi_i^{tar^T} \phi_i^{tar})(\phi_i^{num^T} \phi_i^{num})} \tag{2}$$

Where the terms f_i^{tar} and ϕ_i^{tar} represent pseudo-experimental vibration data, specifically the natural frequency and the displacement vector (determined from ten equally spaced points along the beam) corresponding to vibration mode *i*, both of which were extracted from the initially established target model. Conversely, f_i^{num} and ϕ_i^{num} refer to the values from the numerical model to be analyzed at each iteration of the optimization problem.

2.4 Solution Algorithms

To solve the proposed problem, a piece of code was developed on the MATLAB platform to serve as the interface between the structural analysis program used in this work, which was ANSYS software, and the algorithms employed to obtain the optimal solution. For this study, two metaheuristic algorithms were used to verify the optimal solution of the problem: the Genetic Algorithm proposed by Holland [13] and the Particle Swarm Optimization (PSO) algorithm proposed by Kennedy and Eberhart [14] and modified by Shi and Eberhart [15]. The algorithms employed in this study are available within the MATLAB toolbox; however, custom implementations developed by the authors were utilized.

3 Results and Discussion

3.1 Optimization

The optimization step aimed to calibrate the parameters to minimize the differences between the dynamic responses of the numerical model and the pseudo-experimental data from the target (known) model. For the structure under analysis, 5 design variables were updated using 10 frequencies and 10 associated vibration modes. The initial population was generated using the Latin Hypercube Sampling (LHS) method to ensure a more uniform and representative sampling of the search space compared to purely random sampling methods.

Initially, a configuration step was performed by varying the number of iterations and population size in independent runs to evaluate the relationship between computational cost and the minimization of the objective function (presented in Eq. (1)), conducting five runs for each configuration and recording the averages found for both optimization algorithms. The stopping criterion employed in the analyses was based on the number of iterations, to evaluate the accuracy of each run for a given number of evaluations performed.

For the PSO, whose results are presented in Tab.2, the values of the personal and social acceleration constants were both set to 2.05, as recommended by [16], in order to balance the individual exploration of particles with cooperative group search. A damping ratio for the inertia weight of 0.99 was applied, which gradually decreases the inertia coefficient over the iterations to promote greater exploration in the initial phase and refined convergence, thus avoiding local minima. For the GA, indicated in Tab. 3, a crossover rate of 90%, a mutation rate of 5% and an elitism rate of 5% were established, which was considered to adjust the initial population adopted in the GA compared to PSO.

Table 2. Optimization results using PSO Algorithm

Parameter	Target Value	30 iterations, population of 70		50 iterations, population of 50		50 iterations, population of 70		70 iterations, population of 70	
		Updated Value	Error (%)	Updated Value	Error (%)	Updated Value	Error (%)	Updated Value	Error (%)
E	23.00 GPa	22.75	-1.09%	23.02	0.09%	23.10	0.43%	23.20	0.87%
ν	0.20	0.20	-0.96%	0.20	0.01%	0.20	0.00%	0.20	0.00%
ρ	2500 kg/m³	2474.86	-1.01%	2503.97	0.16%	2509.67	0.39%	2520.60	0.82%
k_1	1.00E+07 N/m	9.90E+06	-1.03%	1.00E+07	0.17%	1.00E+07	0.38%	1.01E+07	0.82%
k_2	1.50E+07 N/m	1.48E+07	-1.04%	1.50E+07	0.15%	1.51E+07	0.39%	1.51E+07	0.82%
Individuals Evaluated		2100		2500		3500		4900	
Cost Function Value		2.24E-03		7.27E-05		1.32E-05		3.73E-06	

Table 3. Optimization results using Genetic Algorithm

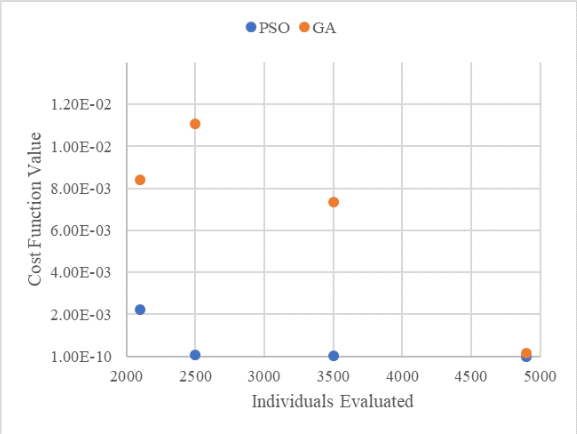
Parameter	Target Value	30 iterations, population of 70*		50 iterations, population of 50*		50 iterations, population of 70*		70 iterations, population of 70*	
		Updated Value	Error (%)	Updated Value	Error (%)	Updated Value	Error (%)	Updated Value	Error (%)
E	23.00 GPa	23.43	1.88%	23.40	1.74%	23.30	1.30%	23.70	3.04%
ν	0.20	0.21	2.60%	0.22	11.61%	0.20	-0.29%	0.20	-0.33%
ρ	2500 kg/m³	2543.28	1.73%	2524.72	0.99%	2531.68	1.27%	2574.70	2.99%
k_1	1.00E+07 N/m	1.02E+07	1.87%	1.01E+07	0.57%	1.02E+07	2.09%	1.01E+07	1.32%
k_2	1.50E+07 N/m	1.53E+07	1.92%	1.52E+07	1.01%	1.51E+07	0.97%	1.60E+07	6.57%
Individuals Evaluated		2100		2500		3500		4900	
Cost Function Value		8.41E-03		1.11E-02		7.35E-03		1.64E-04	

*with a 5% increase to account for individuals preserved due to elitism

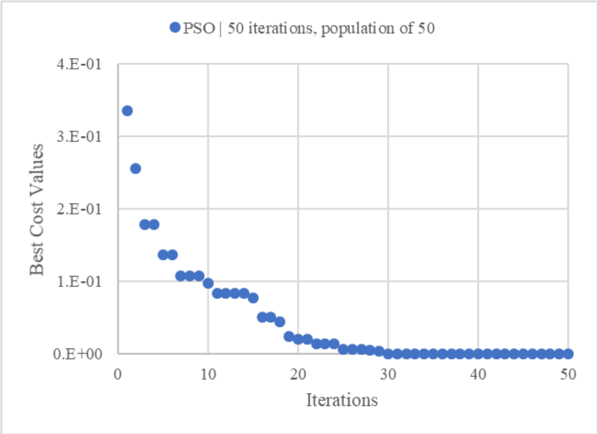
It is possible to verify that the PSO algorithm demonstrates better performance, consistently achieving lower cost function values for the same computational cost, as indicated in the tables above by the evaluated individuals, in addition to achieving parameter values that are closer to the target values. This can be attributed to the exploratory nature of PSO, which allows for a more effective global search within the solution space. The GA, although robust, may get trapped in local minima due to its reliance on crossover and mutation operators.

3.2 Discussion of Results

Evaluating the PSO results, indicated in Tab. 2, it is noticeable that using 50 iterations for a population of 50 individuals presents a satisfactory value for the cost function, significantly better than the case of 30 iterations and 70 individuals and close to the solution with 70 individuals and 70 iterations, but with practically half the computational cost. This trend is shown in the graph in Fig. 4 (a), indicating that increasing the number of individuals beyond 2500 does not result in significant performance improvement, which was confirmed in consecutive runs with the same parameters. Figure 4 (b) shows the convergence graph of PSO for the case with 2500 individuals.



(a)



(b)

Figure 4. (a) Cost function value versus computational cost; (b) PSO Convergence History

Therefore, to solve the problem under study, the PSO algorithm was chosen, with a run of 50 iterations and a population of 50 individuals. Five more independent runs were performed under these conditions, and the results shown in Tab. 4 represent the values obtained for the first six vibration modes for the average solution. Table 5 presents the final values for the calibrated parameters.

Table 4. Resulting modal parameters

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
f^{tar} (Hz)	6.26920	12.08592	33.35528	80.17519	145.79792	223.66127
f^{num} (Hz)	6.26906	12.08573	33.35534	80.17555	145.79849	223.66191
Error (%)	-0.00223%	-0.00157%	0.00018%	0.00045%	0.00039%	0.00029%
MAC	0.99999973	0.99999986	0.99999972	0.99999982	0.99999984	0.99999981

Table 5. Resulting calibrated model parameters

Parameter	E (GPa)	ν -	ρ (kg/m³)	k_1 (N/m)	k_2 (N/m)
Target Value	23.00	0.20	2500.00	1.00E+07	1.50E+07
Updated Value	23.32	0.20	2534.40	1.01E+07	1.52E+07
Error (%)	1.38%	0.02%	1.38%	1.37%	1.38%

As shown in Tab. 4 and 5, the discrepancies between the frequencies of the calibrated and reference models are negligible, and the largest difference in the calibrated parameters values was less than 1.4%. These results demonstrate that the updated numerical model achieved properties very close to those of the target model.

4 Conclusions

The methodology employed proved to be effective for the calibration of numerical models using metaheuristic algorithms. The PSO algorithm demonstrated superior performance compared to the GA in terms of accuracy and computational cost. Future research could explore the application of this methodology to different types of structures and boundary conditions, as well as consider other optimization techniques for comparison, such as the application of a hybrid genetic optimization strategy with a Levenberg-Marquardt local search, as presented in [17]. Also, the effects of noise in vibration measurement data as well as a limited number of points for determination of the vibration modes should be addressed.

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References

[1] A. E. Aktan, D. N. Farhey, A. J. Helmicki, D. L. Brown, V. J. Hunt, K. Lee and A. Levi, “Structural identification for condition assessment: Experimental arts”. *Journal of Structural Engineering*, vol.123, 1997.
[2] M. Sanayei, A. Khaloo, M. Gul and F. N. Catbas, “Automated finite element model updating of a scale bridge model using measured static and modal test data”. *Engineering Structures*, vol. 102, pp. 66-79, 2015.
[3] H. Chen, *Structural health monitoring of large civil engineering structures*. Wiley Blackwell, 2018.
[4] D. Ribeiro, C. Bragança, C. Costa, P. Jorge, R. Silva, A. Arêde, R. Calçada, “Calibration of the numerical model of a track section over a railway bridge based on dynamic test”. *Structures*, vol.34, pp. 4121-4141, 2021.
[5] J. S. Theyssen, E. Aggestam, S. Zhu, J. C.O. Nielsen, A. Pieringer, W. Kropp, W. Zhai, “Calibration and validation of the dynamic response of two slab track models using data from a full-scale test rig”. *Engineering Structures*, vol.234, 2021.
[6] X. Chen, P. Omenzetter, S. Beskhyroun, “Calibration of the Finite Element Model of a Twelve-Span Prestressed Concrete Bridge Using Ambient Vibration Data”. *EWSHM - 7th European Workshop on Structural Health Monitoring*, 2014.
[7] J.M.W. Brownjohn, P. Xia, H. Hao, Y. Xia, “Civil structure condition assessment by FE model updating:: methodology and case studies”. *Finite Elements in Analysis and Desing*, vol.37, pp. 761-775, 2001.
[8] M. Palmonella, M. I. Friswell, J. E. Mottershead, A. W. Lees, “Finite element models of spot welds in structural dynamics: review and updating*”. Computer and Structures*, vol 83, pp. 648-661, 2005.
[9] D. W. Marquardt, “An Algorithm for Least-Squares Estimation of Nonlinear Parameters. *SIAM J. Appl. Math*., 11, pp. 431–441, 1963.
[10] Ansys® Mechanical APDL, Release 2020 R1, ANSYS, Inc.; 2020.
[11] MATLAB version: 9.8.0 (R2020a), Natick, Massachusetts: The MathWorks Inc.; 2020.
[12] J. H. Holland, *Adaptation in Natural and Artificial Systems*. University of Michigan Press, pp. 232, 1975.
[13] R. J. Allemang and D. L. Brown, “A Correlation Coefficient for Modal Vector Analysis”. In: *Proceedings of the 1st International Modal Analysis Conference (IMAC 1982)*, pp. 110–116, 1982.
[14] J. Kennedy and R. Eberhart, “Particle swarm optimization”. *Proceedings of ICNN’95 - International Conference on Neural Networks*, pp. 1942-1948, 1995.
[15] Y. Shi and R. Eberhart, “A modified particle swarm optimizer”. *1998 IEEE International Conference on Evolutionary Computation Proceedings. IEEE World Congress on Computational Intelligence (Cat. No.98TH8360),* pp. 96-73, 1998.
[16] R. C. Eberhart and J. Kennedy. “A new optimizer using particle swarm theory”. *In Proceedings of the sixth international symposium on micro machine and human science*, vol 1, pp. 39–43, 1995.
[17] S. Kokot and Z. Zembaty, “Damage reconstruction of 3D frames using genetic algorithms with Levenberg–Marquardt local search”. *Soil Dyn Earthq Eng*, 29 (2), pp. 311-323, 2009.