Integrating metaheuristic and machine learning for optimization of a full-scale transmission line tower

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Abstract. In long transmission lines (TLs), the design of a transmission line tower (TLT) can be replicated multiple times, a contrast to most structures that feature a unique design. Various optimization methods have been developed to reduce the overall mass of these structures. Historically, most research has relied on metaheuristic algorithms to identify optimal solutions. More recently, machine learning (ML) techniques have begun to be integrated with metaheuristics to speed up the optimization process, although ML has primarily been applied to smaller TLTs, in academic settings, or to simplify structural analysis, potentially compromising accuracy. This paper introduces a new approach that combines the metaheuristic Backtracking Search Algorithm (BSA), known for its efficacy in similar real-world TLT optimization challenges, with Kriging-based Efficient Global Optimization (EGO). This methodology starts by optimizing the size using BSA and subsequently employs EGO to refine the shape. This dual-step optimization, comprised of a metaheuristic (BSA) and an adaptive ML model (Kriging), significantly reduces the mass of the TLT with only a few hundred additional objective function evaluations (OFEs). In comparison, simultaneous size and shape optimization using BSA alone requires over one hundred thousand additional OFEs to achieve comparable results, showing the potential of the proposed approach in dealing with expensive engineering design optimization problems.

Keywords: Transmission line towers, Optimization, Kriging

1 Introduction

In a country with the size of Brazil, the design of a transmission line tower (TLT) may be replicated times within a single transmission line (TL), unlike most structures that feature a unique designs. Therefore, the appeal of employing structural optimization procedures arises, bringing performance and cost-effectiveness to the transmission system.

Initial investigations into TLT optimization centered primarily around academic research [1–9]. Despite the advancements, a direct industrial application requires additional considerations with regard to constructive feasibility and the design performance during prototype testing.

Recently, some works have addressed the problem by including industrial aspects [10–12]. Although, similar to previous studies, symmetry is imposed to all faces.

de Souza et al. [13], Tort et al. [14], de Souza et al. [15], Couceiro et al. [16], Khodzhaiev and Reuter [17] introduced different procedures for optimization of TLTs, adopting metaheuristic algorithms as the optimizer. Taheri et al. [18] employed a neural network and a metaheuristic algorithm to mitigate the computational costs associated with topology optimization of TLTs. Hosseini et al. [19] optimized two full-scale TLTs, comparing a metaheuristic, the Biogeography-Based Optimization (BBO), approach with a combined metaheuristic and machine learning (ML) technique, BBO with ANFIS, an adaptive fuzzy neural inference system, where the latter was used to mitigate the cost associated with the structural analysis done by the MSTOWER software. The proposed approach required 90% less time while obtaining a lighter structure for the CIGRÉ [20] tower when compared to de Souza et al. [13], even adopting fewer design variables. While the surrogate can reduce computation time, it also approximate the structural behavior, meaning that the optimal solution may not be feasible. Nguyen and Vu [21] performed a size optimization of a 160-bar tower, where a machine learning classification technique was combined with the Differential Evolution metaheuristic. The former was responsible for the elimination of unfeasible solutions, saving around 40% of the time. Deichmann et al. [22] extended de Souza et al. [13] methodology to a full-scale TLT family, leading up to a 7% reduction of the entire family. Alves et al. [23] utilized de Souza et al.

[13] approach for size, shape and topology optimization of guyed towers, achieving up to 8% of reduction.

In summary, metaheuristic algorithms are the standard in the literature for TLT optimization. The utilization of those algorithms to explore vast search spaces for an optimal solution is a time-consuming process, due to the elevated number of objective function evaluations (OFEs). This approach cannot be translated to more complex problems, such as a nonlinear analysis of full-scale tower families with multiple load case scenarios. In recent times, ML techniques, in combination with metaheuristics, have been applied [4, 9, 19, 21] to accelerate the optimization process. When ML is used to replace structural analysis [4, 19], the finite element method (FEM) solution may not be represented correctly, especially when dealing with more complex problems, such as nonlinear analysis. Another limitation encountered is the optimization of only small-scale towers [4, 21] or those centered around academic research [4, 9].

That being so, this paper introduces a procedure for the size and shape optimization of transmission line towers, encompassing all industrial aspects. The structural optimization is performed through a Kriging-based Efficient Global Optimization (EGO) framework, where the Kriging Krige [24], a ML technique, is applied to reduce computational cost without replacing the FEM analysis. The proposed methodology finds the optimal shape variables using a Kriging surrogate model, starting from a size optimization result performed by a metaheuristic called the Backtracking Search Algorithm (BSA) [25], chosen for its performance in similar real-world TLT optimization problems [13, 15, 22, 23]. While varying the shape, the size variables may need adjustment to meet optimization constraints. This is achieved through a few iterations by changing the profiles of specific groups. The methodology will be discussed further, in Section 2.

The EGO approach is compared with the metaheuristic-only procedure for a 230 kV self-supporting TLT, showing that, for a much smaller computational budget, better results are achieved. This procedure can be extended to a TLT family and the results show the potential for the application where the objective function is much more costly.

2 Formulation of the size and shape optimization

The optimization objective is to reduce the total mass of the tower (W), including the redundant members, while adhering to the ASCE [26] constraints and the ABNT NBR 8850 [27] angle between adjacent elements constraint. Mathematically, it can be written as:

$$
\begin{aligned}
&\textbf{find}\;\; \mathbf{x} = \{a_1, \ldots, a_m,\, \xi_1, \ldots, \xi_q\}\\
&\textbf{that minimizes}\;\; W(\mathbf{x}) = \sum_{i=1}^{m} \rho_i l_i(\mathbf{x}) a_i(\mathbf{x}) \qquad (1)\\
&\textbf{subjected to axial force constraints:}\;\; g_i(\mathbf{x}) = |S_{d,i}(\mathbf{x})| - R_{d,i} \le 0,\; (i = 1, 2, \ldots, m)\\
&\textbf{slenderness ratio constraints:}\;\; g_{i+m}(\mathbf{x}) = \lambda_i(\mathbf{x}) - \bar{\lambda_i} \le 0,\; (i = 1, 2, \ldots, m)\\
&\textbf{cross sectional area constraints:}\;\; g_{i+2m}(\mathbf{x}) = \frac{\omega_{f,i}(\mathbf{x})}{t_i(\mathbf{x})} - (\omega_f/t)_{max} \le 0,\; (i = 1, 2, \ldots, m)\\
&\textbf{minimum angle constraints:}\;\; g_{i+3m}(\mathbf{x}) = 13^\circ - \alpha_i(\mathbf{x}) \le 0, (i = 1, 2, \ldots, n_\alpha)\\
&\textbf{minimum width constraints:}\;\; g_{i+3m+n_\alpha} = w_{i+1}(\mathbf{x}) - w_i(\mathbf{x}) \le 0, (i = 1, 2, \ldots, n_l - 1)\\
&\textbf{and minimum thickness constraints:}\;\; g_{i+3m+n_\alpha+n_l-1} = t_{i+1}(\mathbf{x}) - t_i(\mathbf{x}) \le 0, (i = 1, 2, \ldots, n_l - 1)
\end{aligned}
$$

in which W is determined by the FE model, including the redundant members. A supplementary value of 20% of W is added to consider the mass of bolts, plates, and galvanization. The parameters ρ_i, l_i, $S_{d,i}$, $R_{d,i}$, λ_i, $\bar{\lambda_i}$, t_i, $\omega_{f,i}$, and $(\omega_f/t)max$ represent the material density, length, axial force, ASCE [26] design capacity, slenderness ratio, maximum slenderness ratio, thickness, flat width, and allowable ratio of flat width to thickness, respectively, for the i^{th} tower member. The angle α (in degrees) between concurrent bars cannot exceed 13 degrees, as required by ABNT NBR 8850 [27], so all $n\alpha$ angles between concurrent bars are checked. The width w and thickness t of a leg member group profile must be equal to or less than those of the leg member group situated directly below. Hence, n_l is the number of leg member groups, with the first at the bottom of the tower and the n_l at the top of the tower head.

The vector $\mathbf{x}$ contains the size and shape design variables: $\mathbf{x} = [\mathbf{a}, \boldsymbol{\xi}]$. The profiles of the structural members' groups, selected from a catalog of angle sections provided by the manufacturer, are discrete size design variables defined by $\mathbf{a} = a_1, \ldots, a_m$. The shape variables, $\boldsymbol{\xi} = \xi_1, \ldots, \xi_q$, also discrete, are rounded to centimeters for practical purposes and modify the coordinates of some predefined nodes.

The structural elements adhere to the design recommendations outlined in ASCE [26], which encompass verification for tension, compression, and connections. The internal forces envelope for all conceivable combinations is obtained by an in-house routine of a linear elastic analysis. The slenderness ratio λ_i limits are: 150 for leg members, 200 for compressed members, and 250 for members subjected to tension. The maximum allowed flat width to thickness ratio $(w_f/t)_{max}$ is 25. For handling constraints, a death penalty strategy is adopted. The basic idea is to add the penalty factor, P_t, as done in Alves et al. [23].

Discrete variables, along with the nonconvexity and nonlinearity of the objective function, necessitate the use of an appropriate optimization algorithm to find an optimum within a reasonable amount of time. With that in mind, the Backtracking Search Algorithm [25] is applied due to its performance in previous studies for similar real-world structure optimization problems [13, 15, 22, 23, 28]. The BSA is used for the size optimization that precedes the shape optimization performed by the EGO procedure. Additionally, the BSA will be used for a simultaneous size and shape optimization to compare with the results achieved by the proposed approach.

The objective function is divided into five stages:

(1) The tower model is constructed using the information from the tower design and the design vector $\mathbf{x}$;
(2) Wind forces are calculated according to IEC 60826 [29], combined with the point loads (specified for the wind load cases in the loading tree) and the self-weight, which receives an additional 20% in all load cases to account for bolts, plates, and galvanization;
(3) The FEM analysis is conducted;
(4) Bolts and redundant members are sized; and
(5) The penalization scheme is applied.

3 Proposed approach for shape optimization

The basic idea of the proposed procedure for size and shape optimization is to start with a tower design that has been optimized for size only (with a fixed shape set a priori by the designer), and then improve its shape using the EGO algorithm. In this work, the size only optimization is performed with the BSA. The overview of the proposed method is illustrated in Figure 1. The procedure depicted in the flowchart is executed for each point of the EGO. The update of the size variables $\mathbf{a}$ is also illustrated in Figure 1. The procedure is performed until it outputs $W(\mathbf{x})$ or until five iterations are executed. The former case is achieved when $\mathbf{a}^*$ was feasible in the last iteration and the $\mathbf{a}^*_{new}$ barely changed the mass of the structure. In the latter case, it means that the procedure was not able to find size variables that would generate an unpenalized solution. This can occur if the selected set of shape variables requires profiles that are heavier than those available in the optimization. On average, the procedure takes about 2.3 iterations to converge to a viable result, that is, 130% more expensive than an objective function evaluation described in the previous section.

The reason for the procedure to break at 5 iterations, is that it rarely takes more than 3 iterations to adjust the size variables. Otherwise, the process would be stuck in an infinite loop. And the same can happen if the size variables are already feasible. If only a few groups change ($\mathbf{a}^*_{new} \approx \mathbf{a}^*$) and the mass of the structure practically does not decrease, the tower that $\mathbf{a}^*_{new}$ brings in this case is usually penalized. This also creates an infinite loop.

After evaluating all points, the vector $\boldsymbol{\xi}^*$, with the adjusted $\mathbf{a}^*$, that minimized W the most, is the result of the EGO.

The EGO framework includes the following steps [30]:

(1) Initial sampling plan creation;
(2) Kriging metamodel generation;
(3) Inclusion of a new infill point to the sampling plan, followed by a return to step (2).

The initial sampling plan, composed of n_s points, is generated using the optimal Latin Hypercube sampling (LHS) scheme as described in Forrester et al. [31]. Following this, the objective function value W for each point is assessed.

Steps (2) and (3) are repeated until a stopping criterion is achieved. In the second step of EGO, a predictive model is developed based on the current sampling plan using Kriging [32].

To estimate the infill points in Step (3), the Expected Improvement (EI) criterion [33] is applied. The fundamental idea of the EGO infill criteria is to utilize the model uncertainty, as indicated by Kriging interpolation, to guide the optimization search. This study uses the expected improvement infill criterion to measure the anticipated improvement at specific points within the domain. In each EGO iteration, the point that maximizes expected improvement is added to the sampling plan, and a new surrogate model is constructed. This iterative process continues until a stopping criterion, such as the maximum number of OFEs dictated by the computational budget, is satisfied.

In short, the complete procedure can be summarized as follows:

(1) BSA size optimization to obtain $\mathbf{a}^*$;

(2) Execute EGO procedure until maximum computational budget is achieved:

  (a) Generate initial sampling plan with the LHS;

  (b) Evaluate the objective function for the initial sampling plan;

  (c) Begin the optimization:

    (i) Train the Kriging metamodel using the Genetic Algorithm (GA) [34] from MATLAB;

    (ii) Generate new infill point $\boldsymbol{\xi}$ using the GA again to minimized the EI, inputting the sampling plan and the trained metamodel;

    (iii) Evaluate objective function with new $\boldsymbol{\xi}$, as in Fig. 1;

    (iv) Add the new $\boldsymbol{\xi}$ to the sampling plan and go back to step (i.) if computational budget was not reached.

(3) Output $\boldsymbol{\xi}$, that minimized W the most, with the adjusted $\mathbf{a}^*$.

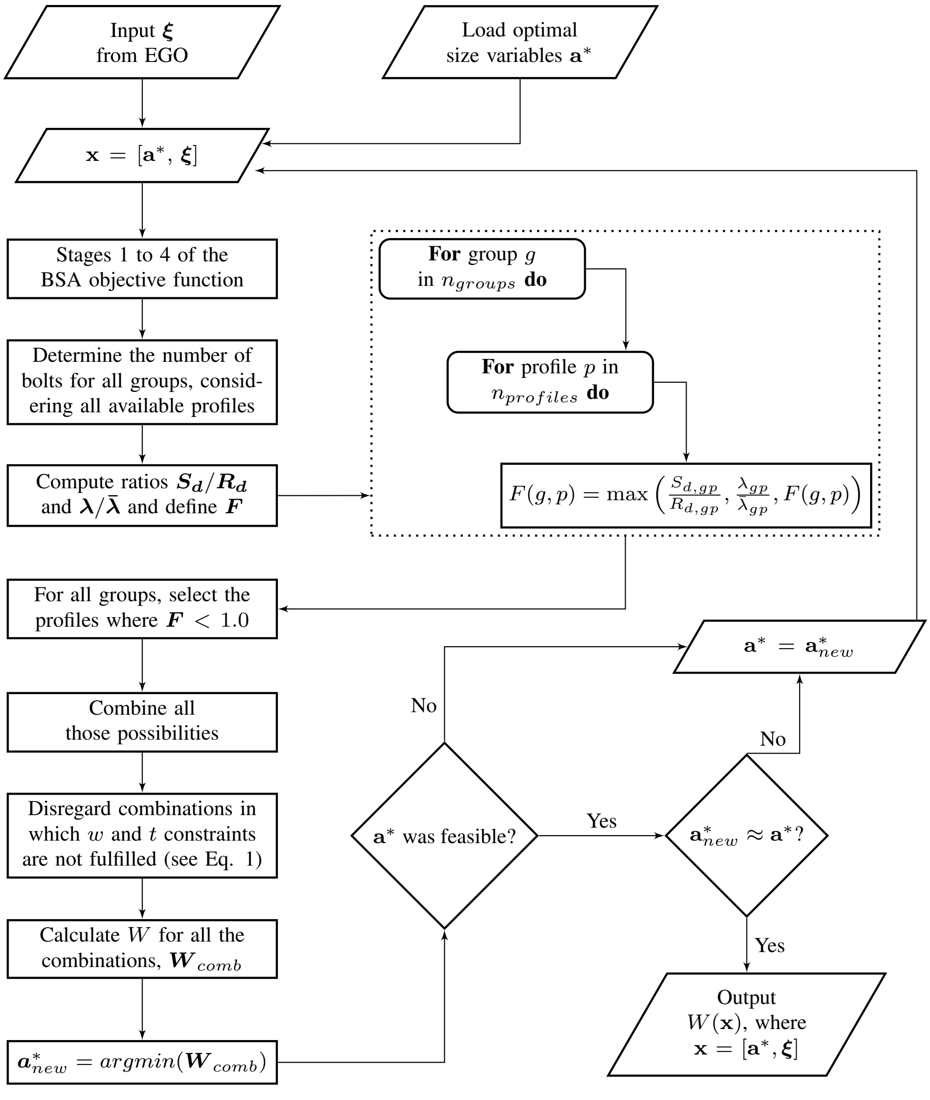

Figure 1. Flowchart of the proposed approach for size and shape optimization of TLTs

4 Numerical example

A 230 kV single circuit self-supporting TLT is optimized here. There are a total of 67 size variables, considering 17 different angle profiles of the metric series to the primary members, the first seventeen profiles from Table 3 of Deichmann et al. [22]. For the redundant members sizing, 5 profiles are available. In the optimization, 6 shape variables are considered, the same as the variations 1.1, 1.2, and 3.1 to 3.4 of the optimization performed by Deichmann et al. [22]. These variables change horizontally the tower base, the tower head and the base of the tower head, all acting on the transverse and longitudinal faces. The members feature ASTM A572 g50 steel and the bolts 12 mm ASTM A394 Type 0. The load hypothesis involve 11 different load cases, where the wind load is determined according to IEC 60826 [29], with a dynamic pressure of 408.9 N/m^2 and terrain roughness category B. A coefficient of 0.36 is applied to the reduced wind cases. There are three wind hypotheses, one rupture scenario for each conductor and one for the ground wire, three construction hypotheses and one anti-cascade. During the optimization process, the projected area of the tower elements is automatically updated. This approach guarantees that the final wind force aligns with the optimal geometry ascertained through the optimization.

The population of BSA is taken as 32 and its individuals are run in parallel. The initial sampling plan used to build the Kriging surrogate is also parallelized. Thus, since a 16 thread CPU is used, 32 population is adopted for the BSA and 32 n_s points are chosen for the EGO procedure. Running in parallel, considering 16 workers, means that the 32 towers of each cycle are processed in parallel, costing the same as only 2 towers processed in series. While the infill points of EGO cannot run in parallel, the load cases are parallelized in this step. Since there are 11 load cases, the cost of an infill point is divided by 11.

A total of 20 independent runs are carried out for each of the optimization types. For the EGO, the initial size variables adopted are taken from a BSA size run, at a point where the metaheuristic found a set of variables that resulted in a feasible solution. The worst of all the feasible solutions of the 20 runs was taken as the initial starting point of the EGO. This corresponds to a tower with a mass of 7460.7 kg, obtained at the 226th iteration. On average, the BSA found a viable solution after 218 iterations, with a standard deviation of 57, a minimum of 139 and a maximum of 337.

On average, 2.3 iterations are performed in the proposed framework, in order to adjust the size variables so that they produce a feasible structure, while the BSA method does not need this part. But the structural analysis for all load cases, the most expensive part, is parallelized during the evaluation of the infill points. Thus, the cost of the EGO, considering 160 points, being 32 for the initial sampling plan and 128 infill ones, is, on average, equal to running 31 ($[32/16 + 128/11] \times 2.3$) towers in parallel, i.e., around 16 BSA cycles (with 32 of population). A fair comparison of the proposed procedure with the simultaneous size and shape optimization using BSA would be considering around 350 iterations for the latter, plus the cost of the Kriging computations, in a worst case scenario for the EGO, i.e., 337 BSA cycles for a viable size optimization solution, plus 16 which is the calculated EGO cost. The cost of Kriging for this problem, which has a low-cost objective function, around 50 times less expensive than the one presented by Alves et al. [23], is about 80% of the EGO procedure. Hence, the final cost becomes $337 + 16/0.20 = 417$ cycles.

Table 1 presents the results obtained with size optimization and simultaneous size and shape runs with the BSA and the EGO runs for the 20 independent runs. The optimization with BSA stops at 3,000 iterations (96,000 OFEs), which is where the 20 size runs become very close to each other and the size and shape BSA optimization results close to the EGO.

Results show that, to achieve similar results, the cost with a metaheuristic approach is up to 6 times more than with the proposed method. Also, if considering the same cost, the presented framework reduces the weight of the tower by 20.5% and 17.7% in relation to a size and to a size and shape optimization, respectively.

The proposed procedure presented herein can be expanded to a family of TLTs. Also, a small portion of the cost is due to the Kriging calculations, which is dissolved when considering more expensive objective functions, such as more loading case scenarios, common in the industry, and nonlinear structural analysis, employed for guyed towers [23].

Table 1. Comparison between the proposed approach (size + EGO) and the size and shape optimization using BSA (statistics for 20 independent runs and units in kg)

	Size opt		Size opt + EGO	Size and shape (BSA)	
OFEs	16,000	96,000	$\lesssim$ 16,000	16,000	96,000
Minimum mass	5927.6	5236.1	4711.7	5727.2	4723.9
Average	6109.8	5240.6	4804.3	5998.8	4791.8
Standard deviation	125.3	4.5	44.8	159.4	41.6
Median	6104.1	5239.1	4805.7	5954	4804.1

5 Conclusions

The presented approach to the size and shape optimization of a 230 kV single circuit self-supporting transmission line tower involves performing a quick size optimization with a metaheuristic, until a feasible solution is found, and then starting a shape optimization with the Kriging-based Efficient Global Optimization framework, with an iterative adjustment of the size variables.

Compared to a metaheuristic only solution, the proposed procedure was able to achieve similar results six times faster. When considering the same cost, the presented framework reduced the weight of the tower by 20.5% and 17.7% in relation to a size and to a size and shape optimization, respectively, with the metaheuristic Backtracking Search Algorithm.

This highlights the potential of the method for objective functions that are more complex, that represent common situations in the industry, such as the ones that involve nonlinear analysis, more load cases scenarios and also for a family of towers.

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