Modified genetic algorithm for efficient optimization: application to prestressed concrete structural elements

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Abstract. Prestressing has become increasingly attractive as it allows for larger spans, greater architectural flexibility, agility in execution and durability. All of these advantages are capable of making prestressed concrete structures more economical solutions specially when associated with an efficient scaffolding system and formwork. The designer can appropriately define the dimensions of the beams and slabs, as well as parameters such as the quantity and layout of tendons by trial based on experience. However, numerical optimization techniques are tools recognized as appropriate for the search for a structural solution, where design parameters can be determined such that the solution minimizes, for example, cost, which takes into account the cost of materials, the volume of concrete, the weight of tendons, rebar reinforcement, and labor productivity, at the same time constraint functions related to, e.g., serviceability and ultimate conditions are verified. Given the discrete nature of some variables, Genetic Algorithms (GAs) have been widely used. GAs have control parameters and operators that are, in general, problem dependent, requiring calibrations in each case. The objective of this work is to review some GA models investigating alternative crossover operators, proposing a modified one thus aiming at the efficiency of the algorithm for prestressed elements, using a prestressed beam as a case study.

Keywords: optimization, prestressed beams, genetic algorithms.

1 Introduction

Prestressed concrete is made up of concrete, prestressed tendons and reinforcement bars. The prestressing mechanism consists of pulling the tendons using hydraulic jack, until the force determined in the project is reached. At the end of the process, the tendons are anchored and tend to return to their relaxed state, causing internal tensions to arise in the concrete structures, thus, the concrete will be working in its best state, compression. Therefore, prestressing allows the structures to reach greater spans and be slender.

For prestressed concrete structures, unlike reinforced concrete, serviceability limit state (SLS) come to the fore, given that service checks are decisive in the design of structural elements. Thus, based on the designer's experience, an initial estimate of the dimensions and number of tendons is made (which can be calculated based on the load to be balanced), and then it is checked whether all service limits are satisfied, if not are, the section is resized and verified again. Therefore, it is sized by the SLS, related to user comfort and durability and, subsequently, the ultimate limit states (ULS) are verified, which are related to collapse and structural ruin. It is also in the ULS that it is checked whether rebars should be added. Therefore, optimization can then be applied to prestressed structures design, making the structural design process more effective and efficient, as there will be no need for resizing.

Since traditional nonlinear programming techniques are ineffective for discrete variables and may converge to local optima, genetic algorithms (GAs), based on Charles Darwin's evolutionary theory, become an excellent alternative for solving such problems, given that GAs can find the global optimal solution with high probability

[1]. Some variants of GAs, developed for problems with real variables, have been proposed in the literature, but there are still few applications to problems of prestressed concrete structure design. There is, therefore, a demand for verifications of these variants applied to these types of structural problems. It is worth pointing out that the evolutionary algorithm operators can affect its exploitation aspect, where better solutions are constructed based on the current knowledge of the search space, or exploration aspect, where new regions of the search space are investigated [2].

Thus, the objective of this work is to apply alternative crossover operators and propose a modified one for real encoded GAs (for discrete and continuous search space) to the design of a prestressed beam, optimizing its cost, where the design variables are the width and depth of cross section and the amount of unbonded prestressing tendons. All analyzes are implemented in the GA of the BIOS program - developed at the Computational Mechanics and Visualization Laboratory (LMCV) of Universidade Federal do Ceará (UFC) – developed with C++ programming language.

2 Analysis

The following formulations are developed considering a simply supported beam and limit state verifications are applied at the central section.

2.1 Equivalent prestressing load

To modelling the effect the prestressing tendon on a concrete element, the Load Balancing Method, proposed by Lin [3], can be used, which consists of replacing the loads coming from the tendon with a uniformly distributed vertical load, assuming a parabolic shape for the tendon. The equivalent load formula is

$$q_e = \frac{8.P.e_p}{L^2} \tag{1}$$

where q_e is the equivalent prestress load, P is the constant prestressing force along the tendon, e_p is the distance from the tendon to the center of gravity of the section (CG) and L is the length of the span.

2.2 Serviceability limit state verification (SLS)

To verify the serviceability limit state of crack formation, the frequent load combination (FLC) must be applied, in accordance with item 8.2.5 of NBR 6118 [4], taking as tension inferior ($\sigma_{i,lim(flc)}$) and compression superior ($\sigma_{s,lim(flc)}$) limits values:

$$\begin{cases} \sigma_{i,lim(flc)} = 0.7 \cdot \alpha \cdot f_{ctm} \\ \sigma_{s,lim(flc)} = -\,0.6 \cdot (f_{ck}) \end{cases} \tag{2}$$

where, for concretes with $fck \leq 50$ MPa, $f_{ctm} = 0.3 \cdot (\sqrt[3]{f_{ckj}})$ and the coefficient α takes on different values depending on the section of the concrete element, being 1.5 for rectangular sections.

Then, for the lower part ($\sigma_{i(flc)}$) and top of the section most requested ($\sigma_{s(flc)}$), the calculation of the stress in this combination are

$$\begin{cases} \sigma_{i(flc)} = -\left(\frac{P_\infty}{A_c}\right) - \left(\frac{M_p}{w}\right) + \left(\frac{M_g}{w}\right) + \psi_1 \cdot \left(\frac{M_q}{w}\right) \\ \sigma_{s(flc)} = -\left(\frac{P_\infty}{A_c}\right) + \left(\frac{M_p}{w}\right) - \left(\frac{M_g}{w}\right) - \psi_1 \cdot \left(\frac{M_q}{w}\right) \end{cases} \tag{3}$$

where: P_∞ and M_p are, respectively, the prestressing force and the resulting isostatic moment due to prestressing considering the losses in infinite time; M_g is the moment due to permanent loading and self-weight; M_q is the moment due to live loads; ψ_1 is the load factor of the accidental load to FLC; A_c is the area and w is the resistance modulus of the cross section.

For the decompression limit state, the quasi-permanent combination (QPC) must be applied and, according to NBR 6118 [4], the entire section analyzed in this state must be completely compressed. The acting stress on the lower ($\sigma_{i(qpc)}$) and upper ($\sigma_{s(qpc)}$) surfaces of the section are given by:

$$\begin{cases} \sigma_{i(qpc)} = -\left(\frac{P_\infty}{A_c}\right) - \left(\frac{M_p}{w}\right) + \left(\frac{M_g}{w}\right) + \psi_2 \cdot \left(\frac{M_q}{w}\right) \\ \sigma_{s(qpc)} = -\left(\frac{P_\infty}{A_c}\right) + \left(\frac{M_p}{w}\right) - \left(\frac{M_g}{w}\right) - \psi_2 \cdot \left(\frac{M_q}{w}\right) \end{cases} \tag{4}$$

where ψ_2 is the load factor of the accidental load to QPC.

The compressive stress limit in concrete, both in the lower and upper fibers of the section, in this combination is given by:

$$\sigma_{i,lim(qpc)} = \sigma_{s,lim(qpc)} = -\,0.45 \cdot (f_{ck}) \tag{5}$$

For the limit state of excessive deformations, ABNT NBR 6118 [4] establishes the limit displacements referring to the deflections in the structures, with sensory acceptability due to visual limitation being considered in the relevant work due to the visual limitation, which the limit displacement is L/250, with L being the span of the structural element.

In prestressed concrete structural elements, it is sufficient to consider the integral stiffness of the cross-section to calculate the immediate deflection [4]. To consider deformation over time, simply multiply the permanent portion of the immediate deflection by (1+ φ), where φ is the creep coefficient obtained in ABNT NBR 6118 [4].

2.3 Ultimate limit state (ULS) verification

ABNT NBR 6118 [4] specifies stress limits during prestressing, immediately transferring loads, which are presented below.

- The maximum compressive stress during prestressing in the concrete section, obtained through factored loads, must not exceed 70% characteristic resistance (f_{ckj}) predicted at the fictitious age *j* (in days), clearly specified in the project. Eq.6 explains the procedure for calculating the limit reference and eq.7 presents the calculation of the tension at the bottom of the section.

$$\sigma_{i,lim(act)} = -\,0.7 \cdot (f_{ckj}) \tag{6}$$

$$\sigma_{i(act)} = -\gamma_p \cdot \left(\frac{P_0}{A_c}\right) - \gamma_p \cdot \left(\frac{M_{p0}}{w}\right) + \gamma_f \cdot \left(\frac{M_{g0}}{w}\right) \tag{7}$$

where: P_0 e M_{p0} respectively the prestressing force and the resulting isostatic moment due to prestressing considering the immediate losses; M_{g0} the moment due to the self-weight of the part; γ_p (1.1 if the prestressing is unfavorable and 0.9 otherwise) and γ_f (1.0) are the load weights for post-tension prestressing and self-weight, respectively.

- The maximum tensile stress of the concrete, shown in eq.8, must not exceed the tension strength f_{ctm} corresponding to the value f_{ckj} specified.

$$\sigma_{s,lim(act)} = 1.2 \cdot (f_{ctm}) \tag{8}$$

To calculate the tension in the upper part of the section:

$$\sigma_{s(act)} = -\gamma_p \cdot \left(\frac{P_0}{A_c}\right) + \gamma_p \cdot \left(\frac{M_{p0}}{w}\right) - \gamma_f \cdot \left(\frac{M_{g0}}{w}\right) \tag{9}$$

2.4 Calculation of rebar reinforcement

The rebar reinforcement is evaluated by means of the equilibrium equation between the requesting moment M_{Sd} and resisting moment M_{Rd}.. The requesting bending moment is defined as

$$M_{Sd} = \gamma_g \cdot M_g + \gamma_q \cdot M_q \tag{10}$$

Where M_g is the resulting moment due to the permanent load and self-weight, M_q is the resultant moment due to the accidental load, γ_g and γ_q are the load factors with values of 1.4 for both for normal combination.

The requesting moment is then equated with the resisting moment. Then for an initial case, in which only the tendon reinforcement was defined, that is, there is only concrete and tendons, according to the formulations developed by Ferreira [5], the positive rebar reinforcement (A_s) can be calculated subtracting from the required prestressing reinforcement (A_{pn}) the effective prestressing reinforcement (A_p), previously defined, doing

$$A_s = (A_{pn} - A_p) \cdot \left(\frac{\sigma_{pd}}{f_{yd}}\right) \tag{11}$$

where f_{yd} represents the calculation resistance to yielding of passive reinforcement steel, being equal to 435 MPa for CA-50 steel and σ_{pd} is the total tension in the active reinforcement, which is the sum of the stress due to prestressing and the increase in stress in the reinforcement tendons calculated by ABNT NBR 6118 [4]. The minimum reinforcement of the section is also calculated, adopting the largest value between the steel area calculated by eq. 11 and the minimum steel area.

As for the negative rebar (A'_s), if necessary, do:

$$A'_s = \frac{M_{Rd2}}{[f_{yd} \cdot (d_p - d')]} \tag{12}$$

Where M_{Rd2} is the resistant moment referring to the compressed region, d' represents the distance from the CG of the rebars reinforcement in the compressed region to the most distant upper point of the section and d_p the distance from the CG of the prestressing reinforcement to the most distant upper point of the section.

3 Optimization Model

3.1 Formulation of the optimization model

The optimization model of the related work aims to minimize the total unit cost of materials for the production of a prestressed concrete beam with unbonded tendons, with the objective function given by:

$$C_{TOT} = C_c . V_c + C_p . m_p + C_s . m_s \tag{13}$$

where C_{TOT} (R$) is the total cost materials, C_c (R$/m³) is the concrete cost, V_c (m³) is the concrete volume, C_p (R$/kg) is the tendons cost, m_p (kg) is the tendons mass, C_s (R$/kg) is the rebar cost and m_s (kg) is its mass.

The design variables are the section width (b_w) and the section depth (h), which are continuous variables, and the number of tendons (n), which is a discrete variable. 9 design constraints are considered due to the restrictions during prestressing, decompression limit state, crack formation limit state and excessive deformation limit state, and are summarized in Tab. 1. Note that the compression is considered negative and tension positive in these expressions. Moreover, the side constraints are satisfied implicitly by the GA algorithm used.

Table 1. Model Constraints

Constraints	Function	Normalized constraints
Restrictions during prestressing		
Maximum tension	$\sigma_{s(act)} \leq \sigma_{s,lim\,(act)}$	$[\sigma_{s(act)} / \sigma_{s,lim(act)}] - 1 \leq 0$
Maximum compression	$\sigma_{i(act)} \geq \sigma_{i,lim\,(act)}$	$[\sigma_{i(act)} / \sigma_{i,lim(act)}] -1 \leq 0$
Decompression limit state		
Maximum tension on the lower fiber	$\sigma_{i(qpc)} \leq 0$	$\{[\sigma_{i(qpc)} + f_{ck}] / f_{ck}\}-1 \leq 0$
Maximum compression in the lower fiber	$\sigma_{i(qpc)} \geq \sigma_{i,lim\,(qpc)}$	$[\sigma_{i(qpc)} / \sigma_{i,lim\,(qpc)}] - 1 \leq 0$
Maximum tension on the upper fiber	$\sigma_{s(qpc)} \leq 0$	$\{[\sigma_{s(qpc)} + f_{ck}] / f_{ck}\}-1 \leq 0$
Maximum compression in the upper fiber	$\sigma_{s(qpc)} \geq \sigma_{s,lim\,(qpc)}$	$[\sigma_{s(qpc)} / \sigma_{s,lim\,(qpc)}] - 1 \leq 0$
Crack formation limit state		
Maximum tension in frequent combination	$\sigma_{i(flc)} \leq \sigma_{i,lim\,(flc)}$	$[\sigma_{i(flc)} / \sigma_{i,lim\,(flc)}] - 1 \leq 0$
Maximum compression in frequent combination	$\sigma_{s(flc)} \geq \sigma_{s,lim\,(flc)}$	$[\sigma_{s(flc)} / \sigma_{s,lim\,(flc)}] - 1 \leq 0$
Excessive deformation limit state		
Maximum vertical displacement	$\delta \leq \delta_{max}$	$[\delta / \delta_{max}] - 1 \leq 0$

3.2 Crossover operators variants in Genetic Algorithms

In Genetic Algorithms, a population of individuals (design variables) evolves through generations, improving its fitness. In this process, in each generation, a set of individuals are selected to process crossover, where current population is combined to find new solutions. Mutation operator is also applied. Since this work focus on the crossover operator, the details of the genetic algorithm used are omitted, but are available in the literature [6].

In the following, various crossover operators are discussed. The Arithmetic crossover consists of carrying out an arithmetic operation to obtain the new generation. Therefore, two individuals $\mathbf{x}^1$ e $\mathbf{x}^2$ are presented in the form:

$$\mathbf{x}^1 = [x_1^1, x_2^1, \dots, x_n^1] \; ; \; \mathbf{x}^2 = [x_1^2, x_2^2, \dots, x_n^2] \tag{14}$$

The result of the arithmetic crossover, that is, the vector of descendants $\mathbf{y} = [y_1, y_2, \dots, y_n]$, with α varying randomly from 0 to 1, is expressed as:

$$\mathbf{y} = \alpha \cdot \mathbf{x}^1 + (1 - \alpha) \cdot \mathbf{x}^2 \tag{15}$$

To generate two offspring from two parents, $\beta = (1 - \alpha)$ is used for the second descendent.

The geometric crossover, according to Katoch [7], being parents x_1 e x_2, with δ varying randomly from 0 to 1, the descendants y_1 e y_2 can be defined as:

$$y_1 = x_1^{\delta} \cdot x_2^{(1-\delta)} \; ; \; y_2 = x_2^{\delta} \cdot x_1^{(1-\delta)} \tag{16}$$

According to Yu [8], Simulated Binary Crossover (SBX) attempts to reproduce characteristics of the binary crossover in a real code, making the descendants of the single-point crossover have the same centroid as the parents (x_1 e x_2). Applying the centroid idea to offspring genes (y_1 e y_2):

$$y_1 = 0.5 \cdot (x_1 + x_2) + 0.5 \cdot \beta \cdot (x_1 - x_2) \; ; \; y_2 = 0.5 \cdot (x_1 + x_2) + 0.5 \cdot \beta \cdot (x_2 - x_1) \tag{17}$$

Wherein the propagation factor β is generated from a uniformly distributed random number u that varies from 0 to 1, with n being a control parameter previously defined by the user as shown in the equations below.

$$\beta = \begin{cases} (2u)^{\frac{1}{n+1}} & \rightarrow u \leq 0{,}5 \\ 2 \cdot (1-u)^{-\frac{1}{n+1}} & \rightarrow u > 0{,}5 \end{cases} \tag{18}$$

Standard arithmetic crossing is a linear interpolation of two individuals, so it only produces offspring on the line connecting two parents. Blend Crossover (BLX) expands the arithmetic crossover range [7]. Thus, for genes x_1 and x_2, supposing that $x_1 < x_2$, the expansion is:

$$y_1 = rand\{[x_1 - \alpha \cdot (x_2 - x_1)], \; [x_2 + \alpha \cdot (x_2 - x_1)]\} \tag{19}$$

Where $rand(c_1,c_2)$ is a function to generate a uniformly distributed random number in the range (c_1,c_2), α is a user-defined parameter that controls the expansion of the linear interpolation. Therefore, it is normally written BLX-α, making the value of α clear [8]. The search interval for this method, as in the SBX, allows the exploration and exploration regions to be investigated at the same time.

In this work, a modified version of the Blend crossover is proposed (Mod Blend). Here, the expansion (c_1 *e* c_2) of x_1 and x_2 are weighted using the fitness function of the parents:

$$c_1 = x_1 - \left(\frac{f_1}{f_1+f_2}\right) \cdot \alpha \cdot (x_2 - x_1) \; ; \; c_2 = x_1 + \left(\frac{f_1}{f_1+f_2}\right) \cdot \alpha \cdot (x_2 - x_1) \tag{20}$$

where f_1 and f_2 are the fitness function of the parents x_1 and x_2, respectively. This modification aims to prioritize the chromosome of the best individual in the crossover process. A random number is then generated in a way similar to eq.19.

4 Numerical Results

The example to be analyzed was taken from Chust's book [9] (page 341). The structural element is a bi-supported beam (Fig.2) with width b_w (with a range of 0.14m to 0.7m) and depth h (with a range of 0.2m to 1.5m) to be determined by optimization, with f_{ck} = 40MPa and f_{ckj} = 20MPa, span of 14.75 m. The tendons are CP RB 190 unbonded with posterior prestressing with limits on the number of tendons between 1 and 30. The loadings are permanent load moment of 30.99 kN.m, accidental load moment of 1552 kN.m with weights of ψ_1 = 0.4 and ψ_2 = 0.3 and total requesting moment of 4225 kN.m. The coverings are 5 cm for rebars and 5.5 cm for tendons, totaling a maximum distance from the CG from the tendon to the bottom edge of 6 cm.

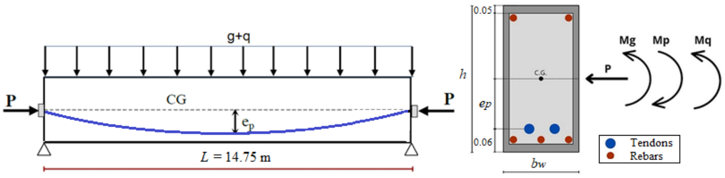

Figure 1. Prestressed beam supported at both ends

The optimization algorithm parameters used in all cases are: 20 optimizations, population size = 150, maximum number of generations = 100, migration rate = 10%, Fitness Proportional method for selection, crossover rate = 90%, penalty method "Deb 2000", constraint tolerance of 10^{-5}, mutation rate = 0%. The types of methods used are Nonlinear Analysis (Excel) and Genetic Algorithm with Arithmetic, Geometric, SBX, Blend crossover and Mod Blend. The cost data was taken in July 2024 from SEINFRA (Secretaria da Infraestrutura do Ceará): concrete=647.01 R$/m³, tendons=15.12 R$/kg, rebars=13.46 R$/kg. To study the SBX crossover method, the parameter u was varied. For the Blend crossover method and Mod Blend, the parameter α was varied within its tolerance.

The success rate can be used as a way to check the performance of the algorithm, being defined as the number of optimizations that achieved the lowest value for the problem divided by the total number of optimizations. Below in Tab.2 are the results of the optimizations for each method used and the comparison between the crossovers used is in Fig.2.

Table 2. Results for method

Reference / Method	b_w (m)	h (m)	Tendons	Objtictive function value (R$)	Success Rate
Chust [9]	0.7000	1.5000	22	17365.3	-
Nonlinear Analysis (Excel)	0.6196	1.4976	21	15838.1	-
Arithmetic Crossover (GA)	0.6641	1.4389	22	16662.7	5%
Geometric Crossover (GA)	0.6956	1.4179	23	16757.3	5%
SBX - n=0.5 (GA)	0.6065	1.4927	21	15786.1	90%
Blend Crossover - α=0.9 (GA)	0.6065	1.4927	21	15786.1	90%
Mod Blend - α=0.9 (GA)	0.6065	1.4927	21	15786.1	95%

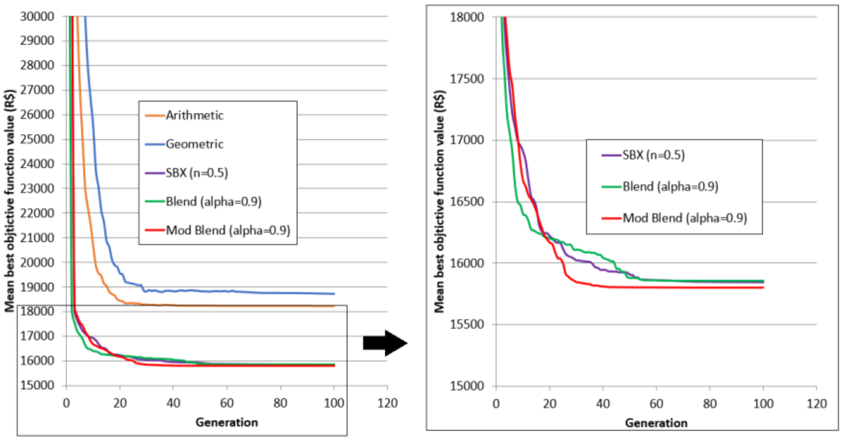

Figure 2. Comparison between the crossover operators

From the values found for the objective function, the most efficient methods were Mod Blend Crossover (α=0.9) with a success rate of 95%, Blend Crossover (α=0.9) and SBX (n=0.5) with a success rate of 90%, all with a reduction of cost of 9% in relation to the initial value obtained by Chust [9]. Furthermore, it can be seen from Fig.2 that the SBX and Blend crossovers converges more quickly, reaching lower values for the cost function in relation to the Arithmetic and Geometric crossovers, assuming therefore that the global minimum may have been reached. It can also be observed that the variable that presented the greatest reduction with the optimization was the width of the section (b_w), considering that the depth (h) and number of tendons contribute more significantly to the resistant moment of the prestressed concrete section.

Regarding the SBX parameter n, it is clear that for n<1 the success rate is between 75% and 90%, however, for values greater than 1 the success rate begins to decrease. As for Blend and Mod Blend, for α from 0.8 to 1 the success rate normally remains between 80% and 95%. The processing time of each optimization were around 3 seconds for each method. The number of generations until convergence is 32 for Arithmetic, 35 for Geometric, 79 for SBX, 49 for Blend, and for Mod Blend around 53 generations were necessary.

All restrictions were satisfied for all methods, the most decisive in optimization (close to zero) being the maximum tension during prestressing $\sigma_{s(act)}$, the maximum tension in the lower fiber in the quasi-permanent combination $\sigma_{i(qpc)}$. Furthermore, the variable h reached values very close to the limit of the established range, indicating that the greater the depth of the beam, the more resistant the section.

5 Conclusions

From the results found for the optimizations carried out, it was found that it was possible to reduce the cost of a prestressed beam by up to 9%, mainly reducing the width of the section, which in cases of transfer beams, for example, this is a significant reduction, considering that this type of beam is expensive for a project, in addition to the fact that the reductions in section dimensions are favorable for architecture.

Regarding the crossover operators analyzed, the most efficient operators were BLX, Blend and Mod Blend crossovers, since their success rates were higher, which can be attributed to the investigation space, which is formed by exploration and exploitation regions.

In short, using BLX, Blend and Mod Blend crossovers operators in optimization, it was possible to calculate the dimensions of prestressed element so that all restrictions were satisfied, with a high success rate and in a reduced time compared to resizing in a conventional structural modeling program.

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