Multi-objective structural optimization of a space truss considering geometric nonlinearity and global stability aspects

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Abstract. The literature has broadly discussed developing and solving multi-objective structural optimization problems (MOSOPs) with two objectives. The conflicting objective functions commonly addressed are minimizing the structure's weight and the maximum displacement. In this paper, the two objective functions considered are minimizing the weight and maximizing the first critical load factor related to the structure's global stability. The constraints are related to the maximum stresses in the bars, the maximum allowed nodal displacements, and the minimum value determined for the first natural frequency of vibration. The analyzed structure is a 25-bar truss. When defining the displacements and deformed configurations of the structure, a geometrically nonlinear analysis is applied using the cylindrical arc-length method. This analysis allows the designer to obtain more accurate values regarding the objective functions and constraints. Three evolutionary algorithms are applied to solve the proposed MOSOP, comparing their performances and providing solutions. The Pareto front obtained in the proposed problem is presented, as it is possible to observe, for example, how the growth of the truss' weight causes increases in the first critical load factor. Finally, optimized solutions are extracted from the Pareto front according to the decision-maker's preferences.

Keywords: Multi-objective structural optimization, Global stability, Geometrically nonlinear analysis.

1 Introduction

In a real-world structural optimization problem, a designer or decision-maker (DM) wants to find a structural configuration that satisfies the requirements imposed by a standard or a usually recommended practice. Most of these problems aim to minimize the structure's weight and, consequently, the material consumption and execution costs. The literature also widely discusses problems with two or more conflicting objective functions, known as multi-objective structural optimization problems (MOSOPs).

This paper proposes and solves a MOSOP with two objective functions. In addition to minimizing the weight, the other objective is maximizing the first critical load factor related to global stability, aiming to ensure the structure's integrity according to the applied load. The constraints are the allowable stresses on the bars, the maximum nodal displacements, and the minimum value defined for the first natural frequency of vibration.

The structure under study is a 25-bar space truss, with the cross-sectional area of these bars as the sizing design variables. When defining the displacements and the deformed configurations, a geometrically nonlinear analysis is applied using the cylindrical arc-length method. This analysis allows the decision-maker to obtain more realistic and accurate values regarding the objective functions and constraints.

Three differential evolution algorithms are applied to solve the MOSOP, and the Pareto fronts obtained are presented. Afterward, a multi-criteria decision-making (MCM) is applied to extract desired solutions from the Pareto fronts, according to the DM preferences. Therefore, this paper's main objective is to apply the proposed objective functions and constraints in formulating a structural optimization problem, considering the geometric nonlinearity of the analyzed truss, and extracting solutions according to the designer's criteria.

This paper is organized as follows: Section 2 describes the aspects of the multi-objective structural optimization applied in the paper. Section 3 discusses the application of the geometrically nonlinear analysis. The

MOSOP's formulation is presented in Section 4. Section 5 presents and analyzes the Pareto front obtained and the non-dominated solutions extracted from it. Finally, this research's conclusions and future works are reported in Section 6.

2 Multi-objective structural optimization

This section theoretically presents important concepts covered in this paper, such as the multi-objective structural optimization and the differential evolution algorithms adopted to solve the proposed problem. It also presents the structural importance of the objective functions applied in the MOSOP and the method applied to extract the desired solutions.

2.1 Definition and objective functions

By definition, MOSOPs present two or more conflicting objective functions to be minimized or maximized simultaneously. As mentioned above, the MOSOP solved in this paper has two objective functions: minimizing the structure's weight and maximizing its first critical load factor regarding global stability.

As usual in the literature, the first objective function is to minimize the weight, aiming to reduce the material consumption in the structure's design for economic, environmental, and structural reasons. As commonly considered in the literature (e.g, in [1], [2] and [3]), the total weight of the truss is approximated by the sum of each bar's weight, disregarding the masses of the connections and the non-structural masses acting on some of the nodes.

The global stability of structures indicates their sensitivity to second-order effects, that is, those generated by their displacements. Therefore, verifying the global stability concerning the Euler buckling loads is an important requirement in the design of a structure, aiming to guarantee safety in terms of the ultimate limit state of stability. The critical load factor indicates the ratio between the estimated critical load at which the structure becomes unstable and the load effectively applied. In this sense, maximizing the first critical load factor aims to find a solution that allows the application of the highest possible load to the structure without causing instability.

2.2 Algorithms and multi-criteria decision-making

The differential evolution algorithm (DE), introduced by Storn and Price [4], is based on the generation and evolution of a population of candidate solutions with continuous variables. Currently, it is considered one of the most popular meta-heuristics for solving optimization problems. The DE-based multi-objective structural optimization algorithms (MOEAs) used to solve the MOSOP formulated in this paper are the success history–based adaptive multi-objective differential evolution (SHAMODE) and its variation using whale optimization (SHAMODE-WO), both proposed by Panagant *et al.* [5]. In addition, the multi-objective meta-heuristic with iterative parameter distribution estimation (MM-IPDE), proposed by Wansasueb *et al.* [6], is also applied.

The non-dominated solutions provided by the MOEAs are presented through Pareto fronts. In this study, the multi-criteria tournament decision (MTD) method, proposed by Parreiras and Vasconcelos [7], is employed to extract the desired solutions from the Pareto front obtained in the MOSOP. Derived from the MCDM framework, this method ranks the best and worst solutions based on the values of the objective function and the weights (w_i) assigned to them by the decision-maker, according to the level of importance the DM attributes to each objective of the problem.

3 Geometrically nonlinear analysis

The primary objective of nonlinear analysis is to search for the equilibrium configuration of structures, which are under the action of external loads [8]. The geometrically nonlinear analysis contemplates the effects caused by the deformation and displacements of the structures.

As highlighted by Bonet [9], several methods combining load and displacement increments have been proposed to deal with nonlinear problems, but they have been replaced by arc-length methods, that compel the iterative solution to follow a certain route toward the equilibrium path. Thus, the family of techniques known as arc-length consists of controlling the length of the vector that connects a known point of the equilibrium trajectory to the desired unknown point, that is, the length of the arc for the trajectory to be determined.

In this paper, the cylindrical arc-length technique, proposed by Crisfield [10], is used to solve the geometrically nonlinear problems in the analyzed truss, providing more realistic and exact values for the nodal displacements. The variable tangent stiffness matrix is calculated in each iteration according to the Updated Lagrangian formulation, in which the equations are defined based on the equilibrium configuration of the previous step. This process was accomplished through the program "NUMA-TF (Numerical Analysis of Trusses and Frames)". The program (originally called "NLframe2D") was developed by Rangel in [11]. The calculated displacements are analyzed in the constraints of the proposed MOSOP. The parameters of the arc-length procedure used in the program's simulation are presented in Table 1.

Table 1. Numerical parameters for nonlinear analysis in the program "NUMA-TF".

Parameter	Value
Increment of load ratio in the predicted solution of first step ($\Delta\lambda^0$)	0.01
Limit value of load ratio to stop analysis	1
Maximum number of steps to stop analysis	5000
Maximum number of iterations in each step (i_{max})	100
Desired number of iterations in each step	2
Tolerance to assume that equilibrium has been reached (tol)	10^{-6}

4 Formulation of the MOSOPs and computational experiments

The 25-bar space truss in Figure 1 is the structure to be analyzed. The loading data and the division of design variables (cross-sectional areas of the bars) in 8 groups are detailed by Rajeev and Krishnamoorthy in [12]. The variables are discrete and chosen from the 30 options in the set S_{in^2} = 0.1, 0.2, 0.3, ...2.6, 2.8, 3.0, 3.2 and 3.4 in^2, that is, from S_{cm^2} = 0.6452, 1.2903, 1.9355, ...16.7742, 18.0645, 19.3548, 20.6451 and 21.9354 cm^2. The aluminum bars have a specific mass of 2700 kg/m^3 and Young's modulus of 68.95 GPa. A non-structural mass of 45 kg is applied at each free node of the truss.

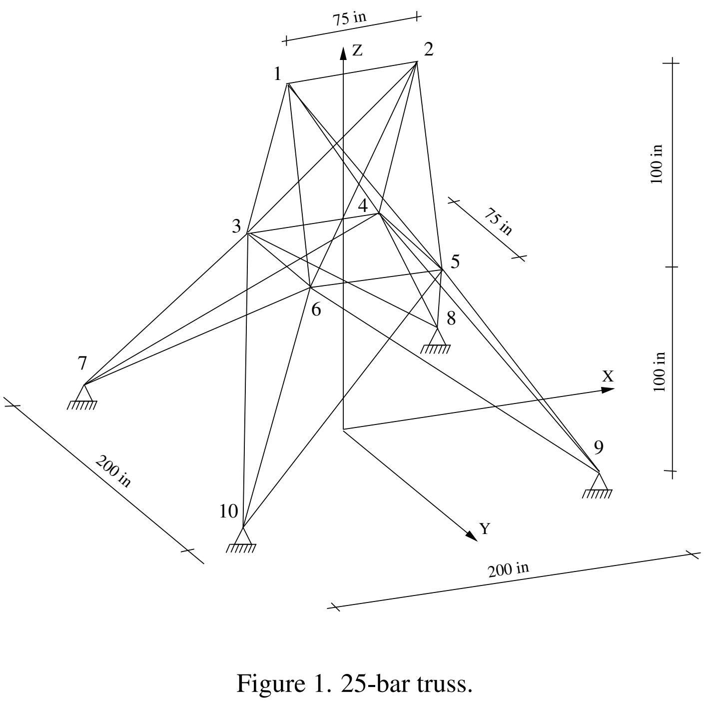


Figure 1. 25-bar truss.

The MOSOP was developed to evaluate the truss's weight and global stability. Regarding the constraints under study, the stress in each bar must not exceed 275.80 MPa in tension or compression, the maximum displacement of nodes 1 and 2 is 0.89 cm in all directions, and the first natural frequency of vibration must be greater than or equal to 10 Hz. The experiment was evaluated through 30 independent runs of the algorithms, each run with 100 generations of 20 individuals (2000 nfe).

Since $W(\mathbf{x})$ is the weight of the truss, $f_1(\mathbf{x})$ is the first natural frequency of vibration, $\lambda_1(\mathbf{x})$ is the first critical load factor, $\sigma_i(\mathbf{x})$ is the stress in the i^{th} bar, $u_j(\mathbf{x})$ is the displacement of the j^{th} node, $\mathbf{x}$ represents the design variables (cross-sectional areas of the bars), and $\mathbf{x}^L$ and $\mathbf{x}^U$ indicate their lower and upper boundaries, this optimization problem can be written as follows:

MOSOP 1:

$$\begin{aligned} \min \quad W(\mathbf{x}) \quad \text{and} \quad \max \quad \lambda_1(\mathbf{x}), \\ \text{s.t.} \qquad |\sigma_i(\mathbf{x})| \leq 275.80\ MPa \\ |u_j(\mathbf{x})| \leq 0.89\ cm \\ f_1(\mathbf{x}) \geq 10\ Hz \\ \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U \end{aligned} \tag{1}$$

5 Pareto fronts and extracted solutions

After formulating MOSOP 1, the optimization problem was solved through the three DE-algorithms presented in Section 2.2. Figure 2 presents the non-dominated solutions obtained in the resolution of MOSOP 1 by each meta-heuristic. According to the DM's preferences, two solutions were extracted from the Pareto front using the MTD method. The first scenario (sc_1) adopts equal importance/weights to the objective functions, with w_i = 0.5 for both. The second scenario (sc_2) indicates w_1 = 0.75 for the minimization of the structure's weight and w_2 = 0.25 for the maximization of $\lambda_1(\mathbf{x})$, favoring the extraction of lighter structural solutions. MTD(sc_1) is highlighted in green, and MTD(sc_2) in cyan, as shown in Figure 2.

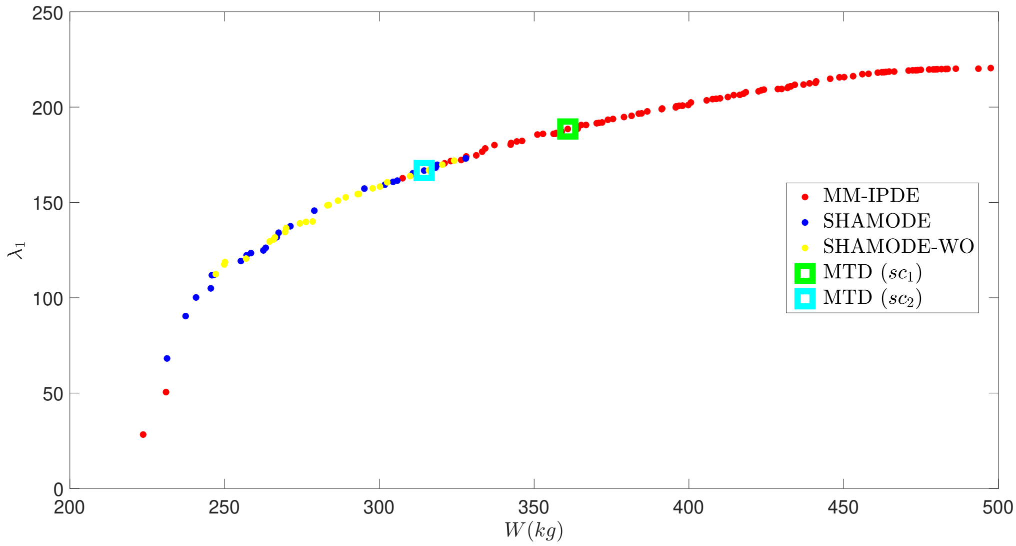


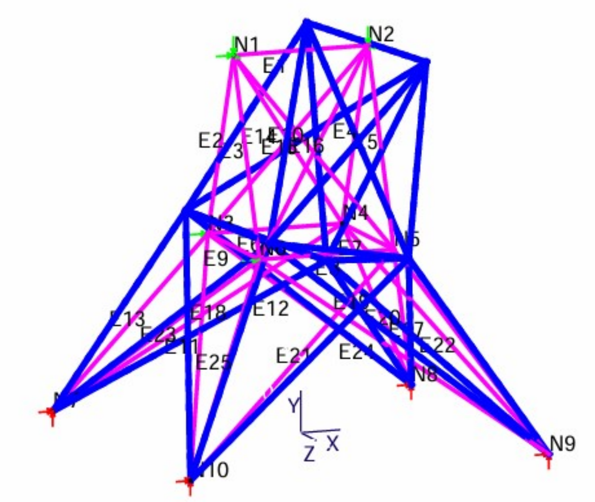
Figure 2. Solutions obtained for MOSOP 1.

Analyzing the non-dominated solutions obtained, the truss' weight varied from 223.68 kg to 497.51 kg. Regarding the first critical load factor, the minimum value observed is 28.25, while the maximum is 220.47. The structural configuration MTD(sc_1), taken from MM-IPDE, presents the values of $W(\mathbf{x})$ = 360.86 kg and $\lambda_1(\mathbf{x})$ = 188.53. As for the solution MTD(sc_2), which gives more importance to the minimization of the weight, it was extracted from SHAMODE, and the objective functions obtained were $W(\mathbf{x})$ = 314.47 kg and $\lambda_1(\mathbf{x})$ = 166.71, providing a lighter structure, but still presenting a high value for the load factor. Table 2 provides a compilation of the MTD solutions extracted with both scenarios, as well as the solution with the lowest weight and critical load factor (indicated as W^-) and the one that provided the highest values of $W(\mathbf{x})$ and $\lambda_1(\mathbf{x})$ (appointed as W^+).

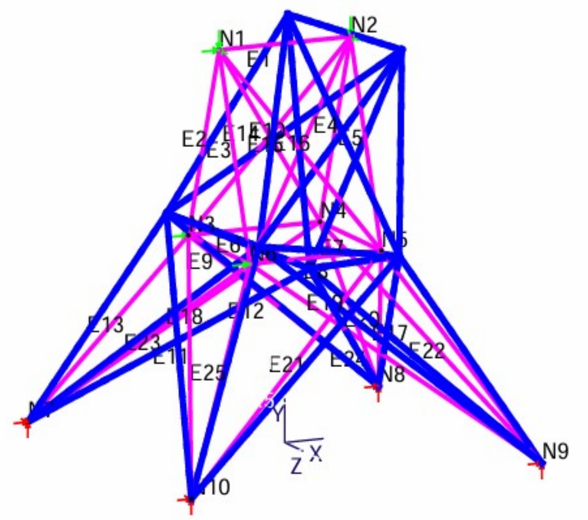
Table 2 also presents the cross-sectional areas of the bars, the DE-based algorithms that provided the solutions, and the values of the objective functions. This data provides indications to the designer of the bars for constructing the structure to obtain the desired configuration and characteristics. To show the structural behavior of the truss, Figure 3 (obtained through the software MASTAN2 [13]) illustrates the first instability mode of the extracted solutions MTD(sc_1) and MTD(sc_2).

Table 2. Design variables and objective functions for the MTD and the extreme solutions.

A$_i$(cm^2)	sc_1	sc_2	W^-	W^+
1	4.5161	8.3871	0.6452	21.9354
2	19.3548	14.8387	9.0322	21.9354
3	13.5484	12.2580	21.9354	21.9354
4	3.8710	10.9677	0.6452	21.9354
5	5.1613	3.2258	7.0968	21.9354
6	15.4838	9.0322	5.8064	21.9354
7	21.9354	20.6451	1.9355	21.9354
8	14.8387	16.1290	21.9354	21.9354
W(kg)	360.86	314.47	223.68	497.51
λ_1	188.53	166.71	28.25	220.47
Origin	MM-IPDE	SHAMODE	MM-IPDE	MM-IPDE



(a) 1st instability mode of MTD(sc_1) (λ_1 = 188.53).



(b) 1st instability mode of MTD(sc_2) (λ_1 = 166.71).

Figure 3. First instability mode of the MTD solutions. The original configuration of the 25-bar truss is represented in pink, and the deformed shape is colored in blue.

Regarding the algorithms, MM-IPDE was the one that provided more non-dominated solutions (65.31% of the total amount of the Pareto front, including the lightest W^- and heaviest W^+ solutions), followed by SHAMODE (17.68%) and SHAMODE-WO (17.01%). Using the Hypervolume (HV) [14] as the performance indicator, the three meta-heuristics presented good efficiency in solving MOSOP 1. Observing the relative HVs of the algorithms (HV_n, normalized by the HV of the total Pareto front, with all non-dominated solutions), the HV-metric indicated SHAMODE as the best performer (HV_n = 0.8700), and then MM-IPDE (HV_n = 0.8503) and SHAMODE-WO (HV_n = 0.8434).

6 Conclusions

Solving MOSOPs with the objective functions of minimizing the weight and maximizing the first critical load factor is very useful for obtaining lighter and more economical structural solutions while maintaining safe indicators of global stability. The constraints related to the stresses in the bars, the nodal displacements, and the frequency of vibration are also important to guarantee the structure's safety and integrity and avoid the resonance effect. The geometric nonlinearity results in obtaining more realistic and reliable values in the definition of displacements and deformed configurations of structures despite the higher computational cost.

The solutions obtained for the proposed MOSOP clearly show that minimizing the weight is a conflicting objective concerning maximizing the first critical load factor. In other words, $\lambda_1(\mathbf{x})$ grows while increasing the weight and stiffness of the structure and decreases as the truss becomes lighter. It can also be seen that the $\lambda_1(\mathbf{x})$ values obtained in the solutions are very high, indicating that the analyzed 25-bar truss does not present imminent risks related to its global stability, given the load applied to it in the problem.

Regarding the solutions extracted through the MTD method, the structural configuration of MTD(sc_2) is more favorable than the one obtained in MTD(sc_1), since MTD(sc_2) provides a lighter truss while preserving a high and safe value for the first critical load factor. Concerning the meta-heuristics applied to solve MOSOP 1, all three algorithms were efficient and satisfactory in solving the proposed problem, with MM-IPDE being the one to provide more non-dominated solutions. At the same time, SHAMODE presents the best hypervolume, with a calculated HV_n slightly higher than the others.

Future work is expected to propose new MOSOPs that consider the inclusion of more objective functions (many-objective optimization problems) and to apply these problems in large-scale and more complex trusses, including shallow domes with intense nonlinear behavior. Finally, machine learning algorithms are expected to be applied to minimize the high computational costs required to evaluate the objective functions and constraints.

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