Multi-objective structural optimization of ground structures considering dynamic and stability behaviors and automatic member grouping

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Abstract. This paper aims to formulate the many-objective structural optimization problem to find the optimal design of ground-structure systems. The objective functions are the weight of the topologically optimized ground structure (to be minimized), the compliance (to be minimized), the different number of discrete cross-sectional areas (to be minimized), the first natural frequency of vibration (to be maximized), and the first load factor concerning the global stability of the structure (to be maximized). It is worthwhile to emphasize that minimizing the number of distinct cross-sectional areas of the optimized ground structure makes finding the best member grouping possible. As a result, the smaller the number of different groups, the greater the weight of the structure, and the greater the number of groups, the smaller the structure's weight. Also, minimizing compliance, maximizing the first natural frequency of vibration, and maximizing the critical load factor directly impact the structure's mass. The constraints are defined as the maximum allowable stress in the bars and the number of crossings in the final topology, which shouldn't be greater than zero. One DE-based multi-objective algorithm (MMIPDE) is adopted to solve the proposed optimization problem. Benchmark problems are analyzed, and Pareto fronts are obtained, showing the non-dominated solutions. Multi-criteria decision-making (MCDM) is adopted to extract solutions from the Pareto front according to the preferences of the decision-maker (DM) used in the ground-structure method. The complete data for each extracted non-dominated solution, including its optimized topology, is provided.

Keywords: Structural optimization, ground-structures, multi-objective, differential evolution

1 Introduction

Multi-objective structural problems are common in many real-world situations. Before the emergence of Multi-Objective Structural Optimization (MOSO), treating a multi-objective optimization problem as multiple single-objective optimization problems was usual. Recently, numerous multi-objective optimization metaheuristic algorithms have been developed, and since then, this approach has been vastly used. As the most known algorithms, one can refer to NSGA-II by Deb et al. [1], GDE3 by Kukkonen and Lampinen [2], and so on. A complete survey on the state of the art of Multi-objective Evolutionary Algorithms (MOEA's) can be found in Zhou et al. [3].

The ground-structure optimization method developed by Dorn et al. [4] essentially consists of modeling a continuous domain with a finite but large enough number of bars that represents it properly. In a structural optimization problem, if subjected to a load and under linear and elastic conditions, this structure can optimize its weight by removing unnecessary bars from regions that are unnecessary. Many works have been developed in this subject lately, such as in Zegard and Paulino [5] (GRAND2d), Zegard and Paulino [6] (GRAND3d), Hagishita and Ohsaki [7], and many others.

This work proposes a many-objective structural optimization of known ground structures in the literature using the Multi-objective Meta-heuristic with Iterative Parameter Distribution Estimation (MM-IPDE). It has shown good performance for multi-objective structural optimization problems in recent works. Five conflicting objective functions (weight, number of distinct cross-section areas, compliance, first natural frequency, and the first linear critical load) are analyzed. Finally, once the Pareto Front (PF) is obtained, choosing one final solution can be done graphically by the Decision Maker (DM), but some extraction methods can make this task easy according to the DM's preferences. The extraction of solutions from Pareto in this work is made via the Multicriteria Tournament Decision Method (MCDM) described in Parreiras and Vasconcelos [8]. The final extracted topologies are shown, and the results are discussed.

This paper is organized as follows: The formulation of the MOSOP discussed in this paper is presented in Section 2. Section 3 discusses the numerical experiments analyzed in this paper. Finally, Section 4 reports the conclusions and future work.

2 The multi-objective structural optimization problem

The multi-objective structural optimization problem of this study can be defined as follows:

$$\begin{array}{ll} \min & (W(\mathbf{x}), ncs(\mathbf{x}), \mathrm{C}(\mathbf{x}), -f_1(\mathbf{x}), -\lambda_1(\mathbf{x})) \\ \text{s.t.} & stresses \quad and \quad bar \quad crossing \quad constraints \end{array} \tag{1}$$

where $\mathbf{x}$ is the vector containing the areas of the structure, $W(\mathbf{x})$ is the weight, $ncs(\mathbf{x})$ is the number of different cross-section areas, $\mathrm{C}(\mathbf{x})$ is the compliance, $f_1(\mathbf{x})$ is the first natural frequency and $\lambda_1(\mathbf{x})$ is the first critical load factor of the structure. This paper will optimize two ground structures under the constraints of stresses and crossing bars through the Finite Element Method (FEM) as the model simulator for the optimization algorithm, where the properties of the structures are obtained as follows:

- The weight of the structure is obtained by:

$$W(\mathbf{x}) = \sum_{i=1}^{N} \rho A_i L_i \tag{2}$$

where ρ is the specific mass of the material, A_i and L_i are the cross-sectional areas and the length of the i-th bar of the structure, respectively. The number of bars of the structure is denoted by N.

- the compliance (or deformation energy) of the structure is obtained by:

$$\mathrm{C}(\mathbf{x}) = \{F\}^\top \{\delta\} \tag{3}$$

where $\{F\}$ is the nodal force vector, and δ are the nodal displacements obtained by the equilibrium equation for a discrete system of tridimensional truss bars (3 degrees of freedom per node):

$$[K]\{\delta\} = \{F\} \tag{4}$$

where $[K]$ is the elastic stiffness matrix of structure. When displacements are obtained, the internal forces can be calculated for the bars and, hence, the stresses.

- The natural frequencies of vibration are obtained by evaluating the eigenvalues of the matrix

$$\left([K] - f_{m_f}^2 [M]\right) \phi_{m_f} = 0 \tag{5}$$

where $[M]$ is the mass matrix (consistent formulation) and ϕ_{m_f} is the m_f-th eigenvector corresponding to the m_f-th eigenvalue (mode of vibration).

- The critical load factors are obtained by evaluating the eigenvalues of the matrix

$$\left([K] - \lambda_{c_f}^2 [K_g]\right) \phi_{c_f} = 0 \tag{6}$$

where $[K_g]$ is the geometric stiffness matrix (consistent formulation) and ϕ_{c_f} is the c_f-th eigenvector corresponding to the c_f-th eigenvalue (instability mode).

3 Numerical experiments

In this section, the numerical experiments are described. They are the L-shape (Mela [9]) and a bi-supported cantilever beam (Kanno [10]). Both experiments were performed into 20 independent runs, and the respective number of function evaluations (nfe) per run is indicated in each description. Each bar has its crossing bar count to handle these constraints. For every candidate solution, the number of crossings for each independent bar is evaluated and treated by the constraint technique if it's greater than 0. During the optimization process, removing bars might lead to collinear bars; hence, a Reduced Order Model (ROM - Sanders et al. [11]) is employed to merge all col linear bars. At the end of the optimization process, the Pareto set is made by obtaining the non-dominated individuals of all feasible ones from each run, and the solution is extracted via MCDM. In this work, two scenarios of weights (w_i) are established for the objective functions to extract the individual from the Pareto set:

- Scenario 1 (Sc 1): $w_1 = w_2 = w_3 = w_4 = w_5 = 20\%$
- Scenario 2 (Sc 2): $w_1 = 90\%$, $w_2 = w_3 = w_4 = w_5 = 2.5\%$

where the indices 1, 2, 3, 4, and 5, refer, respectively, to the objective functions $W(\mathbf{x})$, $ncs(\mathbf{x})$, $C(\mathbf{x})$, $f_1(\mathbf{x})$ and $\lambda_1(\mathbf{x})$. Each set will lead to a solution extracted from the Pareto set, fitting these preference weights defined by the Decision-Maker.

3.1 The L-Shape domain

This experiment is the L-Shape domain shown in Figure 1. It has 21 nodes and 86 members, where the vertical load is applied at the right side middle node corresponding to 100 kN. The search space of the areas is composed of the Square Hollow Sections (SHS) profiles from Table 6 of the study by Mela [9]. The Young modulus is equal to 210 GPa, the maximum stress in each bar considered is the steel yield strength (fy), which equals approximately 450 MPa, and the specific mass is equal to 7850 kg/m^3. This experiment was performed with a population size of 200 individuals throughout 1000 generations, leading to 200000 nfe.

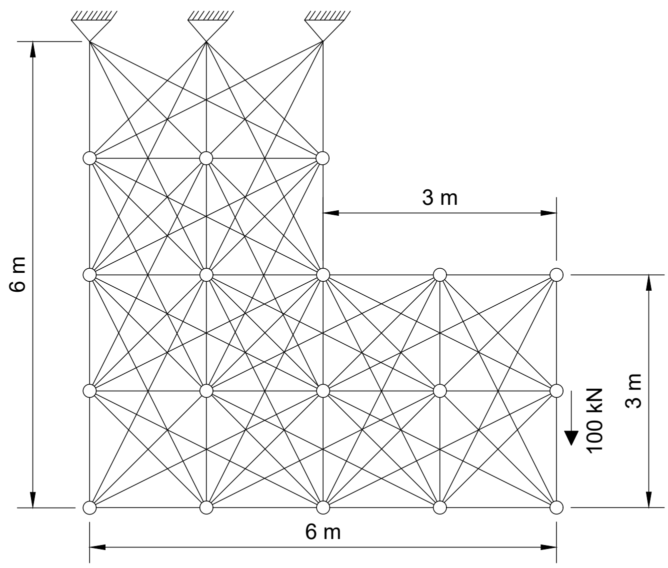


Figure 1. The L-shape domain.

The parallel coordinates of the objective functions for all non-dominated solutions are shown in Fig. 2. Each line of a random color corresponds to a different solution in the final non-dominated population. The extracted solutions via MCDM for the two scenarios of extraction combinations are shown in Figs. 3 and 4.

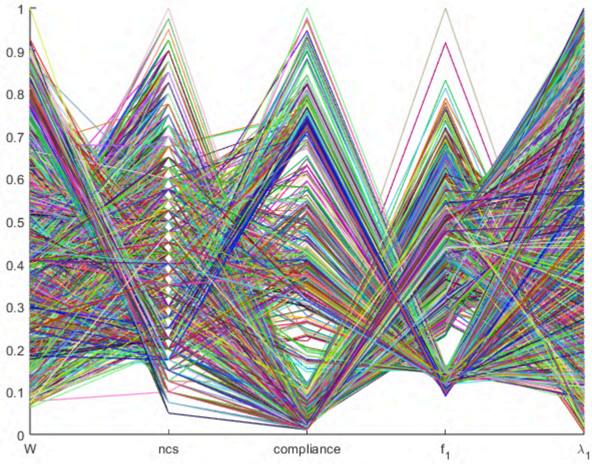
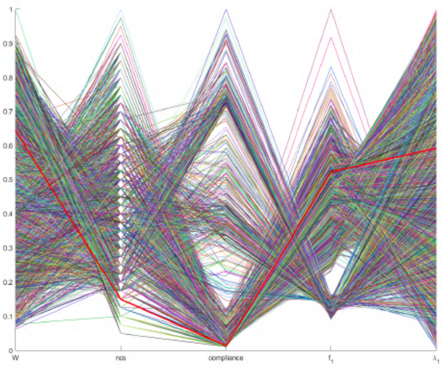
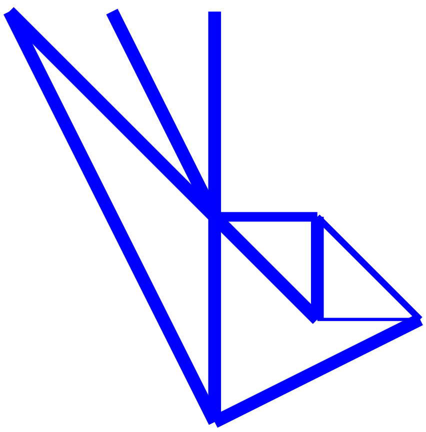


Figure 2. Parallel coordinates for L-shape truss experiment.

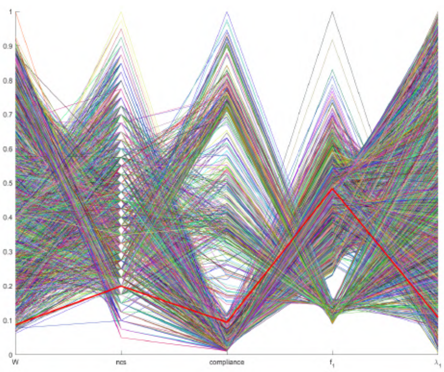


(a)

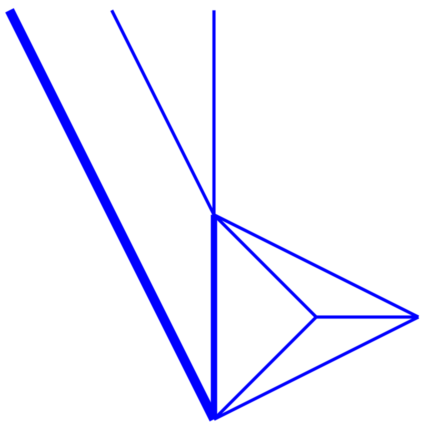


(b)

Figure 3. Extracted solution of L-shape truss for Sc 1, indicated by a red thick line in a) and its corresponding topology in b), with objective functions: W(**x**) = 616.62 kg, ncs(**x**) = 6(4), C(**x**) = 13.52 Nm, f_1(**x**) = 34.04 Hz and λ_1(**x**) = 38848.34.



(a)



(b)

Figure 4. Extracted solution of L-shape truss for Sc 2, indicated by a red thick line in a) and its corresponding topology in b), with objective functions: W(**x**) = 83.93 kg, ncs(**x**) = 8(3), C(**x**) = 95.12 Nm, f_1(**x**) = 31.56 Hz and λ_1(**x**) = 7156.43.

3.2 The bi-supported cantilever domain (bscd)

This experiment is the ground structure shown in Figure 5. The grid comprises 5 x 5 nodes leading to 200 bars as design variables. The continuous search space for the areas is bounded by 1.0 and 20.0 cm^2. The Young modulus equals 210 GPa, the maximum stress in each bar considered is the steel yield strength (fy), which equals approximately 450 MPa, and the specific mass equals 7850 kg/m^3. This experiment was performed with a population of 250 individuals throughout 500 generations (125000 nfe).

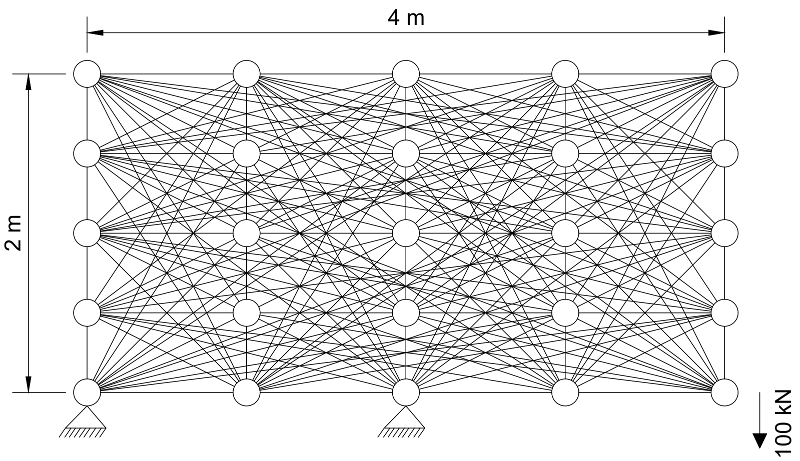


Figure 5. The bi-supported cantilever domain.

The parallel coordinates of the objective functions for all non-dominated solutions are shown in Fig. 6. The extracted solutions via MCDM for the two scenario extraction combinations are shown in Figs. 7 and 8.

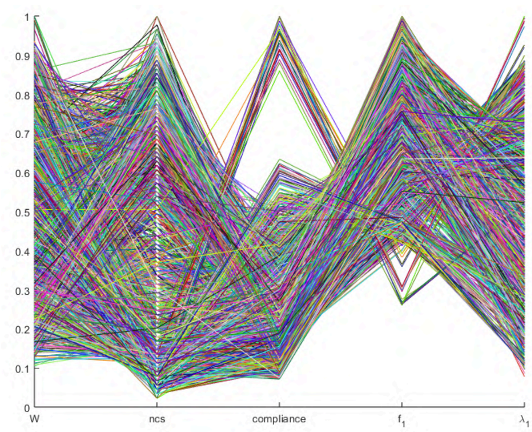
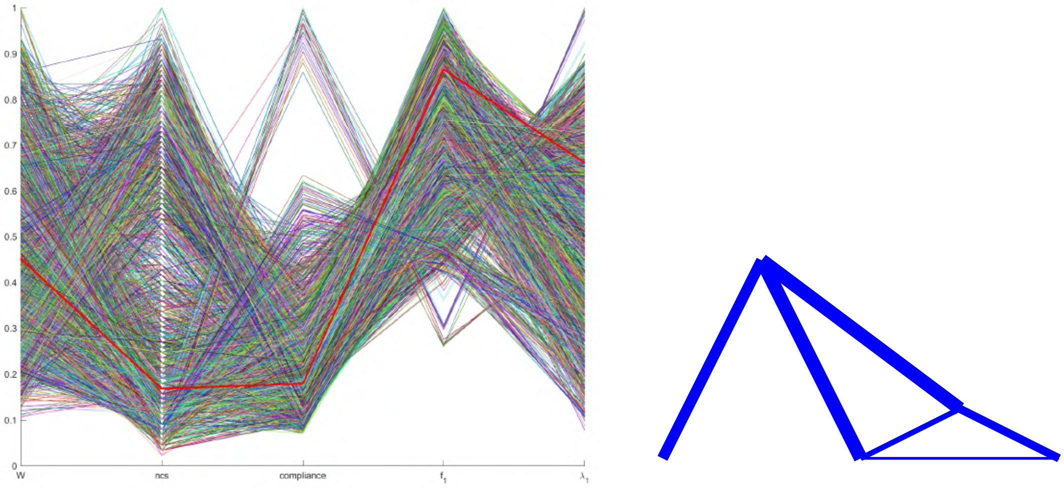
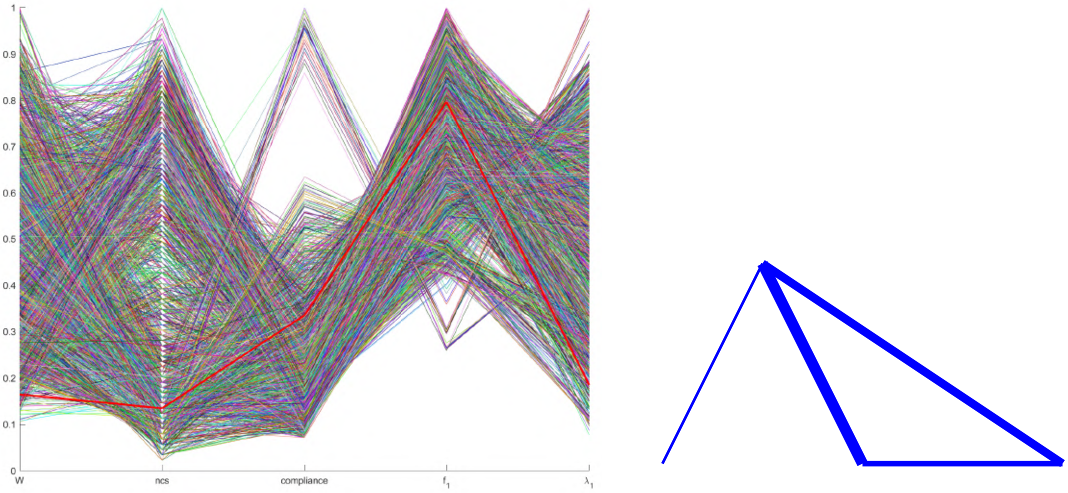


Figure 6. Parallel coordinates for bi-supported truss experiment.



(a) (b)

Figure 7. Extracted solution of bi-supported truss for Sc 1, indicated by a red thick line in a) and its corresponding topology in b), with objective functions: W(**x**) = 135.35 kg, ncs(**x**) = 15(5), C(**x**) = 1426.44 Nm, f_1(**x**) = 125.45 Hz and λ_1(**x**) = 543.21.



(a) (b)

Figure 8. Extracted solution of bi-supported truss for Sc 2, indicated by a red thick line in a) and its corresponding topology in b), with objective functions: W(**x**) = 49.27 kg, ncs(**x**) = 11(4), C(**x**) = 2663.01 Nm, f_1(**x**) = 115.41 Hz and λ_1(**x**) = 152.29.

4 Conclusions

This work proposes a methodology to solve ground-structure multi-objective optimization problems considering five objective functions and constraints of non-overlapping bars and maximum allowable stresses. This methodology showed coherent and intuitive results. The parallel coordinates indicated that the objective functions were conflicting, and the extracted topologies led to intuitive and expected structures. The set of weights for scenario 2 was chosen to obtain essentially light topologies (w_1 = 90%); hence, the extracted topologies have fewer remaining bars than the others. The parallel coordinates showed a good diversity of solutions, essentially occupying a good range for each design variable, indicating a good performance of the MM-IPDE in this type of problem. As future works, it is intended to solve these problems by adopting frame models, comparing them to the current trusses, and increasing the number of objective functions.

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