Multi-objective truss structural optimization considering dynamic and stability behaviors and automatic member grouping

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Abstract. This paper aims to address the challenge of multi-objective structural optimization in the search for the most efficient configuration of members in truss structures. Conflicting objectives, such as the structure's weight, the number of discrete cross-sectional areas, the first natural vibration frequency, and the first load factor relative to overall structural stability, must be optimized simultaneously, resulting in a Pareto front (PF) that provides non-dominated solutions. NSGA-III is the multi-objective evolutionary algorithm adopted to solve the proposed optimization problems. Non-dominated solutions are extracted from the PF using a multi-criteria decision-making (MCDM). New competitive configurations of structural members can be discovered, offering attractive alternatives to decision-makers in manufacturing, cutting, transportation, checking, and welding.

Keywords: Multi-objective structural optimization, Automatic member grouping, Global stability of structures, Dynamic behavior of structures, NSGA-III.

1 Introduction

The literature extensively discusses multi-objective structural optimization problems (MOSOPs) with two objectives. For trusses, most MOSOPs are frequently formulated to minimize both the weight and the maximum nodal displacement. Incorporating the natural frequencies of vibration into the formulations of MOSOPs aims to keep structural configurations away from frequencies that can cause problems, such as resonance, which can lead to structural collapse. Additionally, considering load factors related to the structure's global stability ensures that loads are accounted for in the design, maintaining structural integrity even if higher-than-expected loads occur.

Beyond dynamic and global stability considerations, designing structures like trusses and frames often involves leveraging the advantages of grouping members to achieve the designer's objectives. These benefits can include improvements in architecture, manufacturing, transportation, assembly, and final inspection. However, determining the optimal member grouping can be complex, requiring extensive design experience and leading to a costly and time-consuming trial-and-error process. Even after optimization, the ideal member grouping for the final design is not always guaranteed.

Several approaches in the literature are designed to address optimal member groupings of bars. Key works in this context include those by Grierson and Cameron [1], Biedermann and Grierson [2, 3], Biedermann [4], Galante [5], Shea et al. [6], Barbosa and Lemonge [7, 8], Herencia and Haftka [9], Herencia et al. [10], Liu et al. [11], Angelo et al. [12], Carvalho et al. [13], Azad et al. [14], Woudenberg and Meer [15], and Turay et al. [16]. More details on these references can be found in a recent work by Carvalho et al. [17]. A method for addressing the challenge of determining the optimal grouping of bars is to formulate it as a MOSOP, with conflicting objectives to minimize the structure's weight and the different number of cross-sections or profiles used in the optimized design. Therefore, the main objective of this paper is to use the formulation and solution of MOSOPs to face the challenge of determining the ideal grouping of members of a truss structure, even if groupings based on symmetry, architectural and aesthetic aspects, and the designer's experience are possible in the conceptual design of the structure. In this context, this paper proposes a formulation for a MOSOP that simultaneously minimizes the structure's total weight and the distinct number of cross-sectional areas assigned to structural members, maximizes the first natural frequency of vibration, and the first load factor concerning global stability.

The experiment evaluated in this paper is the benchmark 10-bar truss, and the algorithm adopted to solve the MOSOP is the third non-dominated sorting genetic algorithm (NSGA-III). The non-dominated solutions are collected in a Pareto front (PF), and a multi-criteria decision-maker (MCDM) is adopted to extract solutions from this PF according to the decision-maker preferences.

This paper is organized as follows: The formulation of the MOSOP discussed in this paper is presented in Section 2. Section 3 briefly describes the evolutionary algorithm used in this paper to solve the proposed MOSOP. Section 4 descibes the computational experiment analyzed in this paper. Section 5 presents the results of the analyzed numerical experiment. Finally, Section 6 reports the conclusions and future work.

2 The many-objective structural optimization problem

The formulation of the MOSOP discussed in this paper is written as:

$$\begin{aligned} \min \quad W(\mathbf{x}) \quad \text{and} \quad \min \quad \text{ncs} \quad \text{and} \quad \max \quad & f_1(\mathbf{x}) \quad \text{and} \quad \max \quad \lambda_1(\mathbf{x}) \\ \text{s.t.} \quad & \sigma_i(\mathbf{x}) \leq \overline{\sigma} \\ & \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U, \end{aligned} \tag{1}$$

where $W(\mathbf{x})$ is the weight of the structure, written as:

$$W(\mathbf{x}) = \sum_{i=1}^{N} \rho A_i L_i \tag{2}$$

where ρ is the specific mass of the material, A_i is the cross-sectional areas and L_i is the length of the *i*-th bar of the structure. The number of bars of the structure is denoted by *N*. $f_1(\mathbf{x})$ is the first natural frequency of vibration, $\lambda_1(\mathbf{x})$ is the smallest load factor concerning the maximum elastic critical load able to be applied to the structure, and $\sigma_i(\mathbf{x})$ is the axial stress at the i-th bar. The search space of the design variables is defined by the lower $\mathbf{x}^L$ and upper $\mathbf{x}^U$ bounds.

3 The adopted evolutionary algorithm

The algorithm utilized is the third non-dominated sorting genetic algorithm (NSGA-III), first introduced by [18, 19]. The algorithm has proven its efficacy in solving many-objective optimization problems. NSGA-III (Non-dominated Sorting Genetic Algorithm III) is an evolutionary algorithm designed for solving multi-objective optimization problems, particularly those involving many objectives (more than three). The main characteristics of NSGA-III are: 1) NSGA-III extends the capabilities of NSGA-II to handle many-objective optimization problems efficiently. It aims to find solutions that represent a good approximation of the PF; 2) uses predefined reference points to guide the selection process. These reference points help maintain diversity among the solutions and ensure a well-distributed PF; 3) The algorithm assigns individuals to reference points based on their proximity. During the environmental selection process, individuals are chosen to ensure a good spread of solutions along the PF; 4) incorporates a niching strategy that assigns solutions to reference points and maintains a diverse population. This prevents solutions from convergently affecting a small region of the PF; 5) Like NSGA-II, NSGA-III uses a non-dominated sorting approach to rank solutions based on Pareto Dominance. Solutions are grouped into different fronts, with the non-dominated solutions being the first front; 6) While NSGA-II used crowding distance for maintaining diversity, NSGA-III relies more on the distribution of solutions concerning reference points. However, crowding distance can still be used as a secondary criterion; 7) NSGA-III ensures elitism by preserving the best solutions from the current population to the next generation. This helps maintain high-quality solutions over successive generations; 8) uses genetic operators like crossover and mutation to generate new solutions. These operators help explore the search space and create diversity in the population; 9) Is designed to be scalable and perform well with many objectives. It addresses previous algorithms' challenges when dealing with many-objective optimization problems, and 10) it has been shown to perform well in terms of convergence and diversity. It is widely used in various fields, including engineering, economics, and logistics, for solving complex multi-objective optimization problems. Overall, NSGA-III is a robust and efficient algorithm for tackling many-objective optimization problems, offering improved diversity and convergence properties compared to earlier algorithms like NSGA-II.

4 Computational experiment

The computational experiment analyzed in this section is the traditional 10-bar truss, illustrated in Figure 1 and first introduced by [20]. The sizing design variables are the cross-sectional areas of the bars and they must be chosen from a discrete set of 42 options (in^2): 1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00, 16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, 33.50. The material has specific mass $\rho = 0.1$ lb/in^3 and Young's modulus E = 10^4 ksi. Vertical downward loads of 100 kips are applied at nodes 2 and 4. The stress in each bar is limited to $\pm$ 25 ksi. The experiment was evaluated in 20 independent runs with a population size of 100 candidate vectors and 2000 generations.

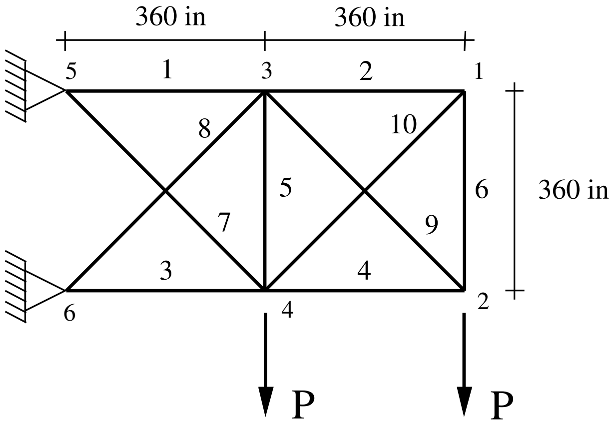


Figure 1. The 10-bar truss.

5 Results

The PF, in parallel coordinates using normalized objective functions, including dominated and non-dominated of all independent runs, is presented in Figure 2.

Figures from 3(a) to 3(f) provide pair-in-pair combinations of the PFs concerning the four objective functions. In these figures, the complete set of non-dominated solutions concerning all runs is represented in blue, whereas the non-dominated solutions presented in the unified PF are presented in red.

Table 2 shows extracted non-dominated solutions using three scenarios simulating the preferences of an artificial DM, in which the weights are pointed to w_1 ($W(\mathbf{x})$), w_2 ($W(\text{ncs})$), w_3 ($f_1(\mathbf{x})$), and w_4 ($\lambda_1(\mathbf{x})$). For Scenario 1 (Sc_1), the following weights w_i were set: w_1 =0.25, w_2 =0.25, w_3 =0.25, and w_4 =0.25. For Scenario 2 (Sc_2): w_1 =0.5, w_2 =0.2, w_3 =0.15, and w_4 =0.15, and, finally, for Scenario 3 (Sc_3), w_1 =0.9, w_2 =0.033, w_3 =0.033, and w_4 =0.034.

After obtaining all the solutions and non-dominated solutions, it can be difficult for the decision-maker (DM) to choose the desired non-dominated solutions. Multi-criteria decision-making helps by considering weights (w_i) defined by the DM to the objective functions of the problem. With this strategy, a solution is extracted from the PFs. The complete details on how the adopted MCDM works can be found, for instance, in [21, 22].

Table 1 provides the design variables dv, the weights $W(\mathbf{x})$, the number of distinct cross-sectional areas ncs, the first natural frequency $f_1(\mathbf{x})$, and the first load factor $\lambda_1(\mathbf{x})$ for the extreme non-dominated solutions of the PFs presented in Figure 3(a)–(f), in which nde^L and nde^U are the non-dominated extreme solutions at the lower bound and the non-dominated solutions at the upper bound of the PFs, respectively.

Figure 4(a), (b), and (c) depicts the unified PFs (presenting only non-dominated solutions among all independent runs) and the extracted non-dominated solutions using the adopted MCDM. The members of the 10-bar truss are presented in the same figure 4(d), (e), and (f) – in different colors for each distinct cross-sectional area – concerning the extracted non-dominated solutions.

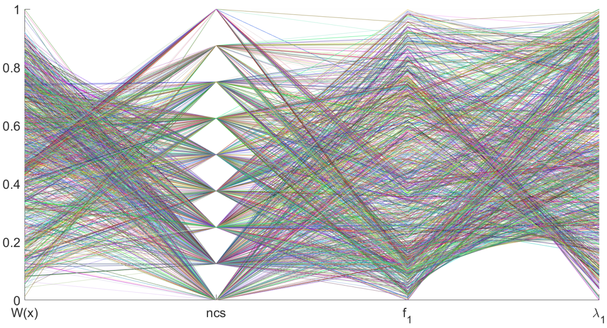
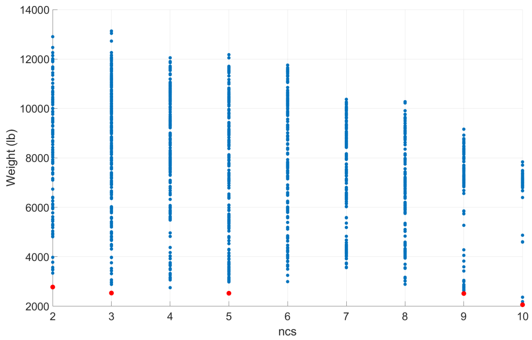
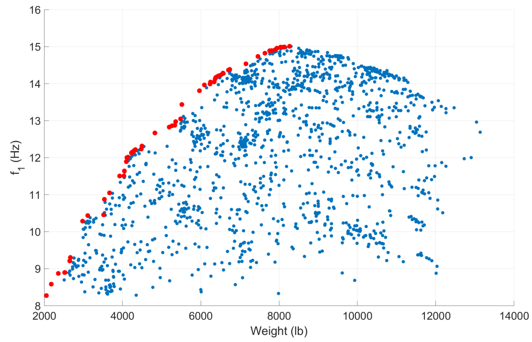


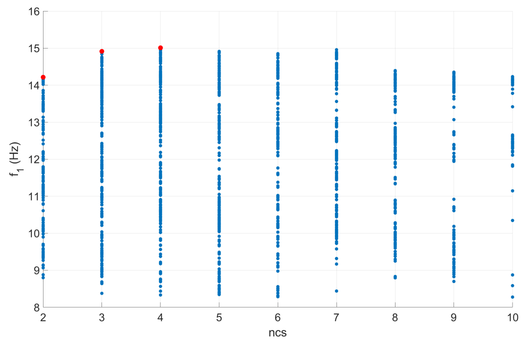
Figure 2. The PF, in parallel coordinates, including dominated and non-dominated of all independent runs.



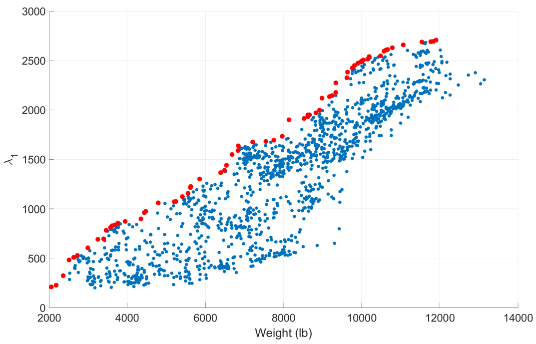
(a) $W(\mathbf{x}) \times$ ncs



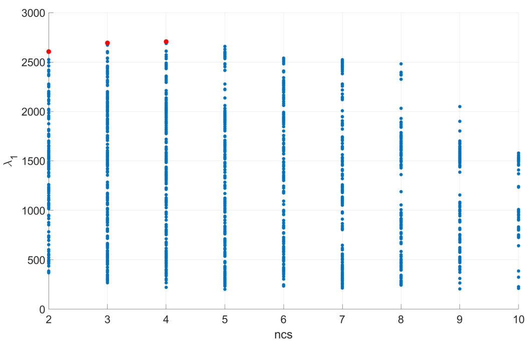
(b) $W(\mathbf{x}) \times f_1(\mathbf{x})$



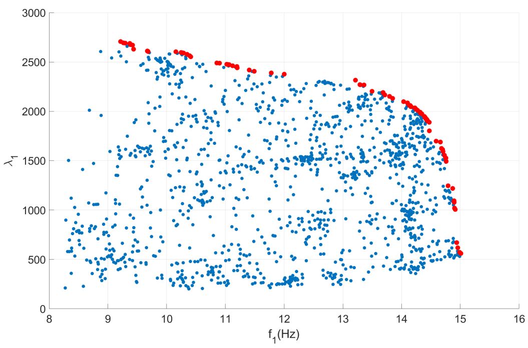
(c) ncs $\times f_1(\mathbf{x})$



(d) $W(\mathbf{x}) \times \lambda_1(\mathbf{x})$



(e) ncs $\times \lambda_1(\mathbf{x})$



(f) $\lambda_1(\mathbf{x}) \times f_1(\mathbf{x})$

Figure 3. Non-dominated solutions for the 10-bar truss.

Table 1. Extreme non-dominated solutions of the PFs presented in Figure 3(a)–(f).

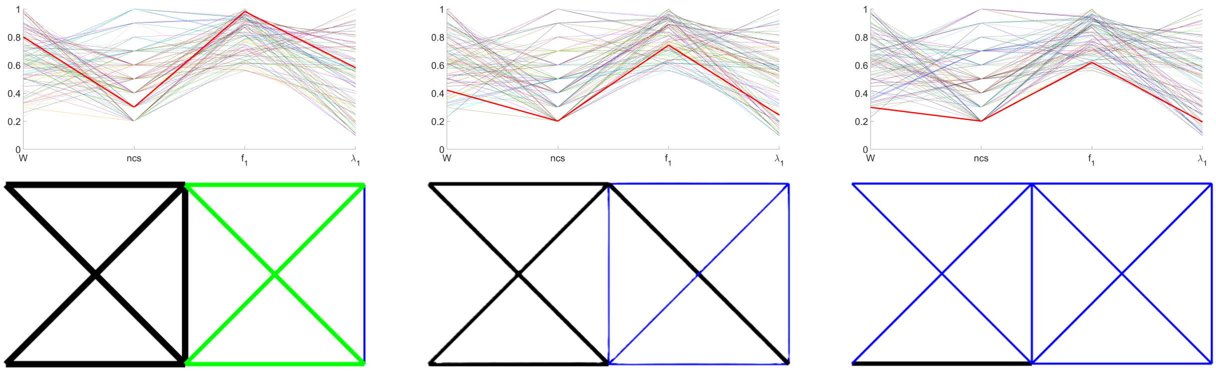
dv	$W(\mathbf{x}) \times$ ncs		$W(\mathbf{x}) \times f_1(\mathbf{x})$		$W(\mathbf{x}) \times \lambda_1(\mathbf{x})$		ncs $\times f_1(\mathbf{x})$		ncs $\times \lambda_1(\mathbf{x})$		$f_1(\mathbf{x}) \times \lambda_1(\mathbf{x})$	
	nde^L	nde^U	nde^L	nde^U	nde^L	nde^U	nde^L	nde^U	nde^L	nde^U	nde^L	nde^U
A_1	11.50	5.74	7.97	33.50	5.74	33.50	33.50	33.50	33.50	33.50	33.50	33.50
A_2	3.63	2.13	7.97	13.90	2.13	33.50	33.50	13.90	33.50	33.50	33.50	13.90
A_3	11.50	13.50	14.20	33.50	13.50	4.97	33.50	33.50	5.12	4.97	4.97	33.50
A_4	3.63	1.99	7.97	11.50	1.99	16.90	4.80	11.50	5.12	16.90	16.90	11.50
A_5	11.50	1.80	7.97	1.62	1.80	33.50	4.80	1.62	33.50	33.50	33.50	1.62
A_6	3.63	3.38	7.97	1.62	3.38	18.80	4.80	1.62	33.50	18.80	18.80	1.62
A_7	3.63	11.50	7.97	33.50	11.50	33.50	33.50	33.50	33.50	33.50	33.50	33.50
A_8	11.50	2.38	7.97	33.50	2.38	33.50	33.50	33.50	33.50	33.50	33.50	33.50
A_9	3.63	2.88	7.97	13.90	2.88	33.50	4.80	13.90	33.50	33.50	33.50	13.90
A_{10}	3.63	3.47	7.97	13.90	3.47	33.50	33.50	13.90	33.50	33.50	33.50	13.90
$W(\mathbf{x})$ (lb)	2772.960	2056.647	2056.647	8266.509	2056.647	11900.027	9494.002	8266.509	12010.507	11900.027	11900.027	8266.509
ncs	2	10	2	4	10	4	2	4	2	4	4	4
$f_1(\mathbf{x})$ (Hz)	9.131	8.273	9.266	15.010	8.273	9.215	14.215	15.010	8.876	9.215	9.215	15.010
$\lambda_1(\mathbf{x})$	462.418	209.959	521.015	559.934	209.959	2707.264	1195.072	559.934	2606.338	2707.264	2707.264	559.934

Table 2. Extracted solutions

dv	Sc_1	Sc_2	Sc_3
A_1	33.50	15.50	7.97
A_2	14.20	7.97	7.97
A_3	33.50	15.50	14.20
A_4	14.20	7.97	7.97
A_5	33.50	7.97	7.97
A_6	2.38	7.97	7.97
A_7	33.50	15.50	7.97
A_8	33.50	15.50	7.97
A_9	14.20	15.50	7.97
A_{10}	14.20	7.97	7.97
$W(\mathbf{x})$ (lb)	9579.626	5035.037	3567.588
ncs	3	2	2
$f_1(\mathbf{x})$ (Hz)	14.700	11.105	9.266
$\lambda_1(\mathbf{x})$	1559.240	652.121	521.015

6 Conclusions

This paper proposed and solved a multi-objective structural optimization problem considering four objective functions: the total weight of the structure to be minimized, the number of different cross-sections of the bars to be minimized, and the first natural frequency of vibration and the first critical load factor to be maximized. It is important to emphasize that minimizing weight conflicts with minimizing the number of different cross-sections of the bars and maximizing the first natural frequency of vibration and the first critical load factor. The results obtained by NSGA-III were non-dominated solutions represented in a Pareto front using parallel coordinates. Non-dominated solutions were presented from both the ends of the PF and those extracted according to the decision maker's preferences. Future work will address other structures in the context of the MOSOP formulation presented in this paper.



(a) (b) (c)

Figure 4. Extracted non-dominated solutions from MCDM for the 10-bar truss.

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