



# Multiobjective Optimization of prestressed composite steel and concrete beam via Multi-Objective Particle Swarm Optimization (MOPSO)

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**Abstract.** The use of prestressing in steel and composite steel and concrete beams is still underexplored in the design of structures with large spans due to the lack of specific normative codes for this type of structural element. Studies involving optimization techniques for these types of structures are scarce in the literature, thus opening up a vast field of research to be explored. This study aims to propose the formulation of a multi-objective optimization problem for composite steel and concrete beams with external prestressing. The final cost of the beams, CO<sub>2</sub> emissions from the materials used in their fabrication, and the maximization of the live load that the beam can support are considered minimization objectives. Constraints include the requirements for composite steel and concrete elements from NBR 8800:2008 and the prestressing provisions from NBR 6118:2021. The Multi-Objective Particle Swarm Optimization implemented in the Matlab package will be used to solve the optimization problem. Examples demonstrating the solution's effectiveness were compared with examples from the literature, and Pareto Frontiers shows that for the same load, different solutions are found, and the best solutions were obtained when using concrete with compressive strength over 45 MPa.

**Keywords:** Multiobjective Optimization, Prestressed Steel and Concrete Composite Beam, Cost, CO<sub>2</sub> emission and maximum load

## 1 Introduction

The use of prestressing in steel and composite steel-concrete beams is still not widely explored in the design of large-span structures due to the lack of specific normative codes for this type of structural element. Among the recent studies on the optimization of prestressed steel beams, the works of Netto et al. [1] (2023) and Fiorotti et al. [2] stand out. In the study by Netto et al. [1] (2023), the costs of prestressed steel beams with straight cables were analyzed and compared with problems from the literature, and the authors concluded that the application of optimization techniques could reduce the final cost of the beam by up to 23%. Fiorotti et al. [2] analyzed the costs and CO<sub>2</sub> emissions of steel beams with straight and polygonal cables and concluded that for these beams, the solution with straight cables was not different from the solution for polygonal cables. However, the studies by Netto et al. [1] and Fiorotti et al. [2] analyzed either the cost or CO<sub>2</sub> emissions separately, thus motivating the proposed study in this work.

Among the algorithms developed for multi-objective optimization, the MOPSO (Multi-Objective Particle Swarm Optimization) algorithm, initially proposed by Coello [3], is an extension of the PSO (Particle Swarm Optimization) algorithm to solve multi-objective optimization problems, dealing with multiple objectives simultaneously. Multi-objective optimization studies applying MOPSO can be found, for example, in the works

of Baei and Terzic [4] (Dampers in Seismic), Xu et al. [5] (Tubes and Shells), and Malashin et al. [6] (Neural Network in Polymer), among others.

Despite several studies in the literature on multi-objective optimization, whether applying MOPSO or another optimization algorithm, no studies were found in the literature on the multi-objective formulation of prestressed steel and concrete composite beams focusing on minimizing CO<sub>2</sub> emissions and cost while considering the beam's load capacity.

Given the absence of works addressing the multi-objective optimization of prestressed composite steel-concrete beams, this work aims to present the formulation of the multi-objective optimization problem for composite steel-concrete beams with external prestressing and straight cables. The minimization functions will consider the final cost of the beams and the CO<sub>2</sub> emissions from the materials used in their manufacture, in addition to maximizing the useful load the beam can support. To solve the problem, the MOPSO algorithm was implemented in MATLAB 2020[7].

## 2 Optimization Problem Formulation

For the definition of the multi-objective problem, the design variables presented in Figures 1(a) and 1(b) were defined, showing the longitudinal section and the beam cross-section, respectively, for monosymmetric welded steel profiles with steel deck slabs.

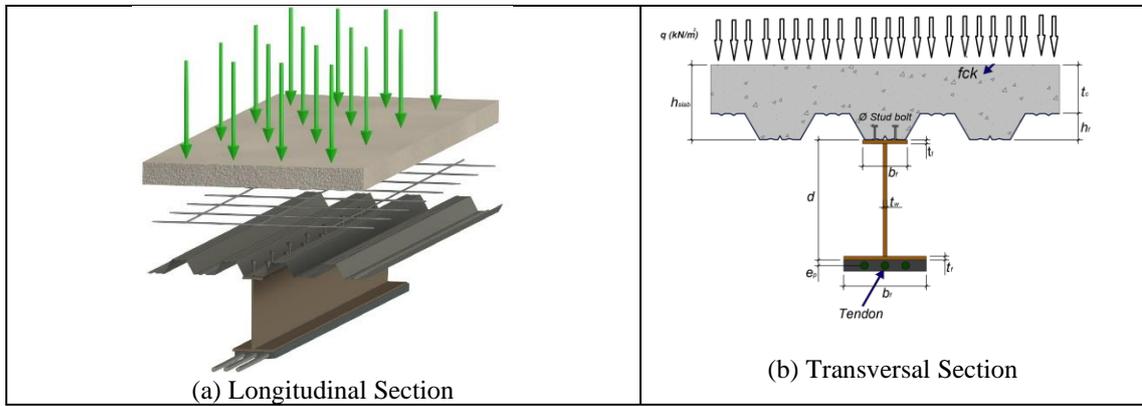


Figure 1 – Design Variables

Each variable had the following range in the Table 1.

Table 1 – Upper and lower bounds of design variables

Profile height ( $d$ ): $50 \text{ mm} \leq d \leq 1100 \text{ mm}$ ;	Lower flange thickness ( $t_f^{bot}$ ): $4 \text{ mm} \leq t_f^{bot} \leq 30 \text{ mm}$
Upper flange ( $b_f^{top}$ ): $50 \text{ mm} \leq b_f^{top} \leq 700 \text{ mm}$ ;	Web thickness ( $t_w$ ): $4 \text{ mm} \leq t_w \leq 30 \text{ mm}$
Lower flange ( $b_f^{bot}$ ): $50 \text{ mm} \leq b_f^{bot} \leq 700 \text{ mm}$	Slab concrete thickness ( $t_c$ ) and steel deck formwork height ( $h_f$ ) were selected from the options in the Metform steel formwork catalog [34]
Upper flange thickness ( $t_f^{top}$ ): $4 \text{ mm} \leq t_f^{top} \leq 30 \text{ mm}$	Slab concrete compressive strength ( $f_{ck}$ ): $20 \text{ MPa} \leq f_{ck} \leq 50 \text{ MPa}$ , with a step of 5 MPa.
Lower flange thickness ( $t_f^{bot}$ ): $4 \text{ mm} \leq t_f^{bot} \leq 30 \text{ mm}$	Number of tendons: $0 \leq No. Tendons \leq 20$
Web thickness ( $t_w$ ): $4 \text{ mm} \leq t_w \leq 30 \text{ mm}$	Live Load: $3 \text{ kN/m}^2 \leq q \leq 8 \text{ kN/m}^2$

### 3.1. Objective Functions

The multiobjective problem is defined with three objective functions: CO<sub>2</sub> emission minimization, Cost minimization, and maximization of the load capacity of the beam present in Eqs. (1) to (3).

$$\begin{aligned} \text{Min } CO_2 = & CO_{2,S} V_S \rho_S + CO_{2,TR} V_{TR} \rho_S + CO_{2,Slab} \\ & + CO_{2,Studbolt} + L_W CO_{2,W} \end{aligned} \quad (1)$$

$$\text{Min } COST = \text{Cost } V_S \rho_S + COST_{(tendons)} + COST_{(slab)} + COST_{(conectors)} \quad (2)$$

$$Max Load = q \tag{3}$$

The parts of Eq. (1) and (2) represent the CO<sub>2</sub> emissions and cost of each beam component. The first term refers to the steel profile, where  $CO_{2,S}$  and  $Cost$  is the CO<sub>2</sub> emission in kgCO<sub>2</sub> per kilogram and cost of steel profile,  $V_s$  is the volume of steel profile given by area  $A_s$  and beam span  $L$ , and  $\rho_s$  is the specific mass of steel. The second term represents the CO<sub>2</sub> emission and cost of the tendons, where  $CO_{2,TR}$  and  $COST_{(tendons)}$  is the emission in kgCO<sub>2</sub> per kilogram of tendon and tendon cost respectively,  $V_{TR}$  is the volume of the tendon given by the product of area  $A_{TR}$  and length of the tendons  $L$ ,  $CO_{2,CS}$  and  $COST_{(conc.slab)}$  is the CO<sub>2</sub> emission and the cost of the composite slab consisting of the steel deck formwork emission and cost, reinforcement emission and cost, and concrete emission and cost,  $CO_{2,Studbolt}$  is the stud bolt emission, and  $L_W CO_{2,W}$  is the weld emission of the profile manufacturing.

To determine the potential contribution to global warming in kgCO<sub>2</sub>, the weld emission value of 3.3 kgCO<sub>2</sub>-Equiv./m was adopted, adjusted by the ratio between the actual cross-section of the weld chord ( $A_{wl}$ ) and the reference cross-sectional area of the weld chord for the 20 mm plate, which is 158 mm<sup>2</sup> (Sproesser et al., 2015).

For emissions related to concrete, Santoro and Kripka [8] carried out a study considering the analysis of the beam cycle of materials from cradle to the gate for all materials that make up the concretes adopted in this study, and the material costs were used the values proposed by SINAP [9] (2023) and a local supplier in Espírito Santo. Furthermore, a transport radius of 100 km was considered for emissions due to concrete transport. Table 1 presents the CO<sub>2</sub> emissions for each material considered in Equation (18).

Table 1 – Units Costs and Emissions

Component	Strength	CO <sub>2</sub> Emission (kgCO <sub>2</sub> /m <sup>3</sup> )	Source	Cost (R\$)	Source	
Concrete	20 MPa	129.85	Santoro e Kripka [8]	463.14	SINAPI [9]	
	25 MPa	142.71		474.87		
	30 MPa	153.68		491.01		
	35 MPa	163.25		504.22		
	40 MPa	171.73		518.15		
	45 MPa	189.6		532.09		
Steeldeck formwork	280 MPa	2.638kg/kg	Worldsteel Association [10]	90.5/m <sup>2</sup>	Local Supplier(2024)	
				esp.0.8mm		107.64 m <sup>2</sup>
				esp.0.95mm		141.64 m <sup>2</sup>
				esp.1.25mm		99.00 m <sup>2</sup>
Steel Profile	AGR52	1.116kg/kg		117.00 m <sup>2</sup>		
				esp.0.95mm		154.00 m <sup>2</sup>
Stud Bolt		1.924kg/kg		12.0/kg		
Tendons	CP190	1.924kg/kg		11.40/unit		
Reinforcement Mesh	CA60	1.924kg/kg		15.60/kg	SINAPI [9]	

The constraints proposed by Fiorotti [11] for the different stages of execution of the prestressed composite beam were adopted for the problem as follows: Stage 1 – prestressed steel beam subjected to its own weight; Stage 2 – prestressed steel beam subjected to its own weight plus the weight of the slab and construction loads; Stage 3 – analysis of the prestressed composite steel and concrete section with live load; Stage 4 – verification of the serviceability limit state at infinite time. The MOPSO implemented on the MATLAB platform was used to determine the optimal solution to the problem.

### 3 Numerical Results

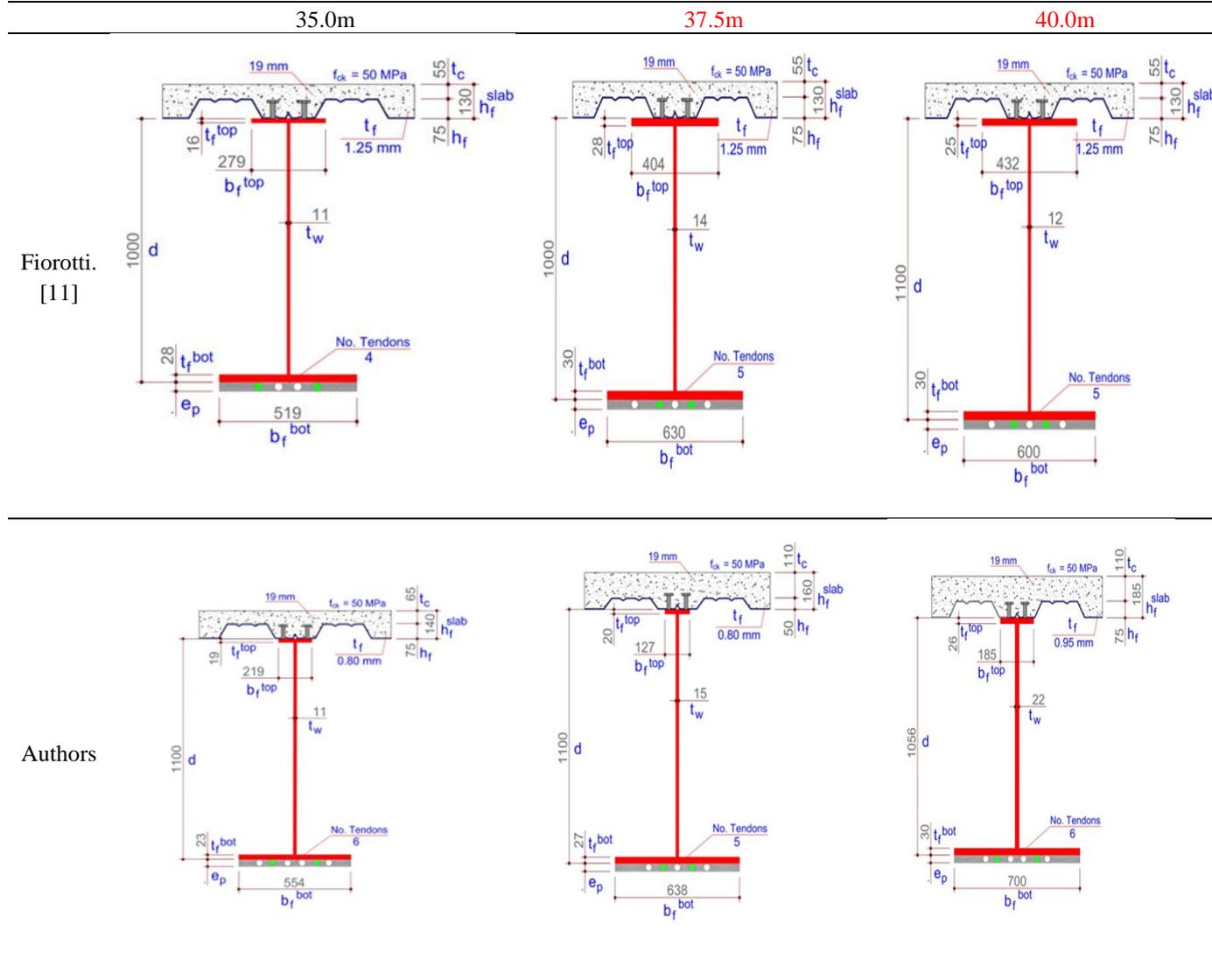
To demonstrate the application of the proposed formulation beams with spans of 35m, 37.5m, and 40m analyzed by Fiorotti [11] were examined. This study analyzed the CO<sub>2</sub> emissions of prestressed composite steel beams with doubly symmetric and monosymmetric profiles, with the monosymmetric profiles providing the best solutions. This analysis considered a beam spacing of 3m, a steel yield strength ( $f_y$ ) of 345MPa, and all design variables: composite slab thickness, concrete compressive strength, number of tendons, profile dimensions, and choice of steel deck slab. Besides the slab and beam self-weight, a dead load of 2kN/m<sup>2</sup> and a construction load of 1kN/m<sup>2</sup> were also considered. The tendons should be 50mm below the lower flange of the steel profile, and the steel CP190 should have a diameter of 15.2mm. For the multi-objective optimization, 10 rounds were performed

for each example, with an initial population of 300 individuals and 100 iterations for each round. Table 2 presents the CO<sub>2</sub> emissions results obtained by Fiorotti [11] using the SABO algorithm for a live load of 5kN/m<sup>2</sup> and the values found by applying MOPSO, and Table 3 presents the profile geometries presented by Fiorotti [11] and the geometries proposed in this work.

Table 2 – Comparison of results between the Authors and Fiorotti [11].

Span (m)	CO <sub>2</sub> Emission (kg) Fiorotti [11]	CO <sub>2</sub> Emission(kg) Authors	Cost (R\$) Authors	Ratio [%] CO <sub>2</sub> Authors/ CO <sub>2</sub> Fiorotti [11]
35.0	16080.7	17099.50	132203.76	6.3
37.5	19146.7	19044.69	154370.22	-0.5
40.0	22738.0	21821.20	178434.29	-4

Table 3 – Geometries to load of 5kN/m<sup>2</sup>.

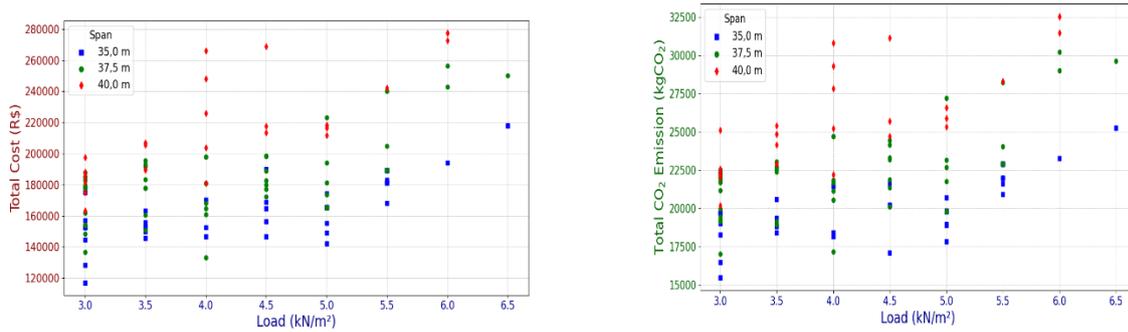


As shown in Table 2, although the results are different, the values for multi-objective optimization are close to those obtained by Fiorotti [11] when applied to multi-objective optimization, with the 37.5m span being the nearest. Despite these close values, it can be observed that the final geometry of the profiles and cables differed from those proposed by Fioroti [11], as presented in Table 3. For the three analyzed examples, the  $f_{ck}$  of the slab

was the same for spans of 35m and 40m, but with different steel deck formwork and concrete thickness. Figures 3(a), 3(b), and 3(c) show the Pareto Fronts for Load versus Emission, Load versus Cost, and Emission versus Cost.

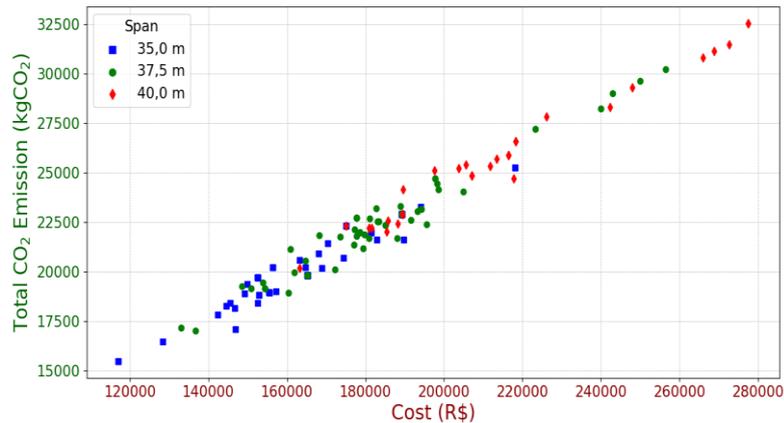
As can be observed in Figure 3, the lowest load obtained was 3kN/m<sup>2</sup>, and the highest load was 6.5kN/m<sup>2</sup> for the 35m and 37.5m spans and 6kN/m<sup>2</sup> for the 40m span. It can also be noted that the algorithm obtained more than one solution when analyzing costs and CO<sub>2</sub> emissions for different load conditions on the Pareto Frontiers. Figure 3(c) shows that the relationship between cost and CO<sub>2</sub> emission for the different solutions is nearly linear. The best solutions for each span are presented in Table 3, and Figure 4 presents a graph of the evolution of cost and CO<sub>2</sub> emission per beam meter.

Figure 3 –Pareto Fronts



(a) Load versus Cost

(b) Load versus CO<sub>2</sub> Emission



(c) Cost versus CO<sub>2</sub> Emission

Table 3 – Best Solutions Analysis

Span (m)	Load (kN/m <sup>2</sup> )	CO <sub>2</sub> Emission (kgCO <sub>2</sub> )	Cost (R\$)
35.0	6.5	25248.20	218111.75
37.5	6.5	29644.83	250060.35
40.0	6.0	31479.14	272730.44

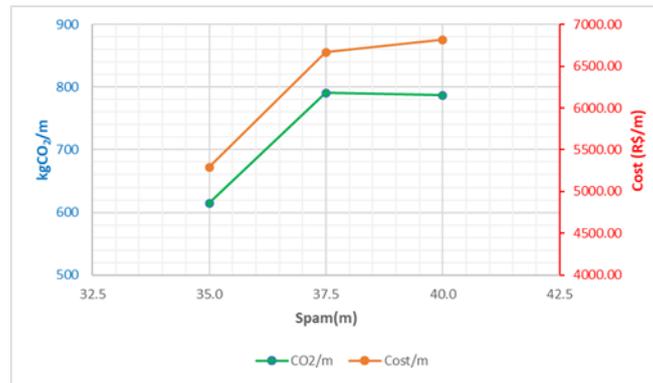


Figure 4 –Cost and CO<sub>2</sub> Evolution

As observed, the relationship between CO<sub>2</sub> emissions and beam costs follows a nearly linear trend as the span of the beam increases. Figure 5 presents the best solution for each span analyzed to the maximum load.

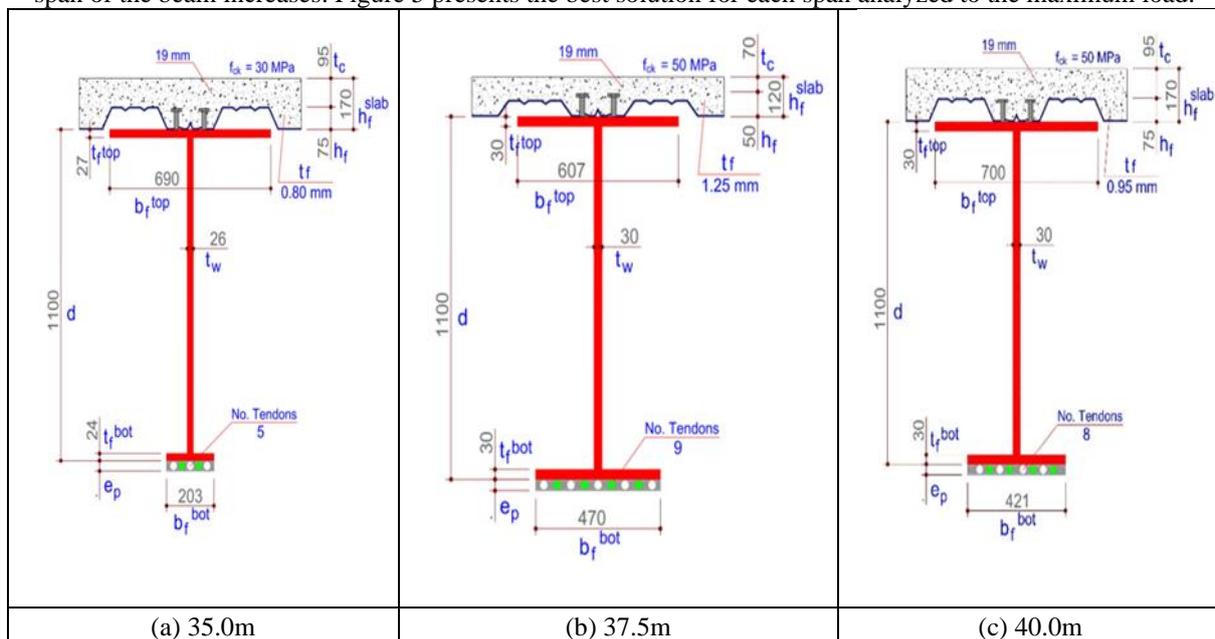


Figure 5 –Final geometries

As observed, different steel deck slabs were chosen for each span analyzed. However, it's important to note that concrete with compressive strength over 45MPa was used for the slab with similar solutions to the load of 5kN/m<sup>2</sup>. Despite these concretes emitting a higher amount of CO<sub>2</sub>, their use compensates for the final strength of the composite section.

## 4 Conclusions

The present study aimed to present a multi-objective formulation for prestressed steel and concrete composite beams, applying the MOPSO to obtain optimal solutions. According to the results, the algorithm was efficient in searching for solutions for different load stages and according to the results of the comparative analysis with the single-objective optimization algorithm presented by Fiorotti [11]. Although the results are similar, the geometries obtained for the MOPSO solutions differed from those proposed in Fiorotti's study [11].

The results indicated that for the same load situation, more than one solution was presented in the Pareto Fronts, thus allowing the design engineer to choose the best solution from the cost and environmental points of view. It can also be observed that for the analyzed spans, the maximum loads of the beams were 6.5kN/m<sup>2</sup> for the spans of 35m and 37.5m, and 6kN/m<sup>2</sup> for the span of 40m, with composite slabs with concrete above 45 MPa. These results highlight the importance of using higher-strength concrete in prestressed composite elements.

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