Optimal Pile Caps Design Considering Soil Structures Interaction

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Abstract. The design of pile caps is typically performed by analyzing the cap individually based on the superstructure loads, the analysis of the piles receiving these loads, and the soil profile. This study aims to present a formulation for the optimal design of pile caps, considering soil-structure interaction. To solve the optimization problem, the Particle Swarm Optimization (PSO) algorithm was associated to the cap and piles design. As an objective function, CO_2 emissions and the final costs of the pile caps were considered individually, based on the materials used in their construction. The constraints of the problem include the requirements prescribed by Brazilian standards and the criteria for determining the optimal dimensions of the piles. The results indicate that the solution obtained from an environmental perspective is not the same as the solution from an economic perspective.

Keywords: Pile Cap, Optimization, Cost, CO_2 Emissions, Particle Swarm Optimization

1 Introduction

The pursuit of excellence in engineering is a frequent and much-discussed topic. In the field of civil construction, the design of structures should meet the ultimate limit state (ULS) and the serviceability limit state (SLS), always seeking the lowest possible cost. However, current studies by Silva et al. [1], Erlacher et al. [2], Fiorotti et al. [3], Santoro and Kripka [4], among others, shows that the CO_2 emissions from materials such as concrete, steel, and wood, used in civil constructions tends to become decisive in this process.

Cost analysis is essential to ensure that the design is not only technically and safely adequate, but also feasible and aligned with the client's conditions. For this reason, it was the main determining factor in the design and construction process for many years. However, the reduction of CO_2 emissions has become increasingly relevant due to growing environmental concerns, as observed in the study conducted by the GFN [5], which presents the Earth's biocapacity about the carbon footprint. In this regard, civil construction occupies a very important position, since approximately one-third of CO_2 emissions come from this sector. Additionally, the GNR [6] estimates that concrete is the second most used material worldwide, playing a significant role in environmental discussions.

Based on these two issues, the concept of optimization becomes an interesting tool in the quest to reconcile cost reduction and the decrease of carbon emissions in civil construction. This process applied to the design of structural elements proposes the best possible dimensions and material characteristics to obtain a final product that maximally meets these requirements.

In this context, the foundation pile caps are important structural elements, fundamental for the safety and material point of view. Once defined by the ABNT NBR 6118 [7] as structures of volume used to transmit loads to the piles, the amount of materials needed in just one element may be considerable and can present a significant comparative index when analyzed more carefully. That is; by studying the variation of certain design parameters of these pile caps, it is possible to obtain cost data, as well as CO_2 emissions, aiming to select those characteristics that best meet an optimal structural design.

Studies involving the optimization of pile caps can be found in the works of Alvez and Thomaz [8] and Turini et al. [9], which addressed cost optimization but did not consider the geotechnical design of the pile, and Rodrigues et al. [10], who analyzed the CO_2 emissions related to the materials used in the caps but not their total costs.

Thus, this study proposes to analyze the optimization problem of a three-pile cap, calculated using the Blévot and Frémy method [11], considering the environmental impacts as well as their respective costs. For the analysis of the soil-structure interaction of the piles, the methodologies of Aoki and Veloso [12] and Dequourt and Quarena [13] were considered for determining the load-bearing capacity of the pile. To solve the optimization problem, the Particle Swarm Optimization (PSO), proposed by Kennedy and Eberhart [14] and modified by Shi and Eberhart [15], was used to obtain the optimal results via Matlab R2020a [16].

2 Optimization Problem Formulation

The present study aims to analyze the optimization of a 3-pile cap as shown in Fig. 1.

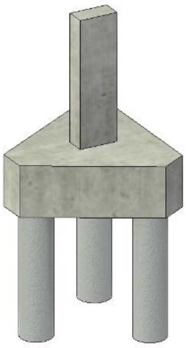


Figure 1. 3-Pile cap triangular model

For this purpose, two functions were analyzed in the optimization of the pile caps, which seek to individually reduce costs and CO_2 emissions

$$Min\ C = (V_b) \cdot C_{cb} + N_e \cdot \pi \frac{{d_{pile}}^2}{4} \cdot L_{pile} \cdot C_{cpile} + A_f \cdot C_f + A_s \cdot \gamma_a \cdot C_a, \tag{1}$$

$$Min\ E = (V_b) \cdot E_{cb} + N_e \cdot \pi \frac{{d_{pile}}^2}{4} \cdot L_{pile} \cdot E_{cpile} + A_f \cdot E_f + A_s \cdot \gamma_a \cdot E_a. \tag{2}$$

Eq. (1) refers to the minimization of total costs, where C_{cb} and C_{cpile} are the total costs of the concrete of the cap and piles, respectively, V_b is the volume of the cap, N_e is the number of piles, d_{pile} and L_{pile} are the diameter and length of the piles, C_c is the cost of concrete per m³ based on its compressive strength (f_{ck}), A_f and C_f are the area and cost of the formwork in m², and A_s, γ_a and C_a are the area, specific weight, and cost of steel per kg. The Eq. (2) is related with the minimization of emissions, where E_{cb} and E_{cpile} are the total CO_2 emissions from the concrete of the cap and the pile, respectively, and, E_c, E_f and E_a are the CO_2 emissions per m³ of concrete, per m² of formwork, and per kg of steel, respectively.

For the optimization of the pile caps using the method proposed by Blévot and Frémy [11], the following constraints must be respected, according to the guidelines of ABNT NBR 6118 [7].

$$h - \frac{A - a_p}{3} \leq 0 \tag{3}$$

$$h - \frac{B - b_p}{3} \leq 0 \tag{4}$$

$$\frac{R_{e,max}}{R_{e,lim}} - 1 \leq 0 \tag{5}$$

$$45^{o} \leq \theta \leq 55^{o} \tag{6}$$

$$\frac{\sigma_{column}}{\sigma_{column,lim}} - 1 \leq 0 \tag{7}$$

$$\frac{\sigma_{pile}}{\sigma_{pile,lim}} - 1 \leq 0 \tag{8}$$

$$\frac{e_x}{e_{x,min}} - 1 \leq 0 \tag{9}$$

$$\frac{e_y}{e_{y,min}} - 1 \leq 0 \tag{10}$$

$$A_s - \frac{R_{sd}}{f_{yd}} = 0 \tag{11}$$

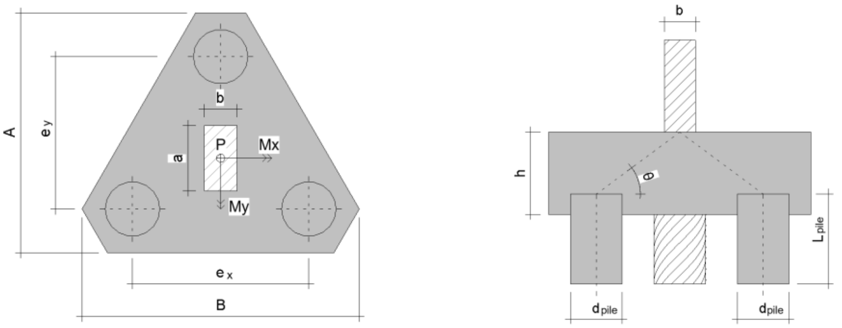
$$20 \leq f_{ck} \leq 50 \tag{12}$$

$$2 \leq N_e \leq 6 \tag{13}$$

$$1 \leq L_{pile} \leq L_{max} \tag{14}$$

$$d_{min} \leq d_{pile} \leq d_{max} \tag{15}$$

From Eq. (3) to Eq. (15), the design variables are presented in Fig. 2, where h is the total height of the cap, A and B are the larger and smaller dimensions of the cap, a_p and b_p are the larger and smaller dimensions of the column, $R_{e,max}$ is the maximum normal load transmitted to the piles, $R_{e,lim}$ is the pile's load-bearing capacity, θ is the inclination of the concrete strut, σ_{column} is the compressive stress in the strut, $\sigma_{column,lim}$ is the maximum allowable stress, σ_{pile} is the stress on the pile, $\sigma_{pile,lim}$ is the pile's stress limit, and e_x and e_y are the horizontal and vertical distances between piles, with $e_{x,min}$ and $e_{y,min}$ being the minimum distances between them in these directions. Finally, L_{max} represents the maximum possible length for the pile, and d_{min} and d_{max} are the available intervals for its diameter.



(a) Top view (b) Side view

Figure 2. Views of the 3-pile cap and its design variables

From the parameters already presented, those that will be used as design variables for optimization through PSO are: the steel reinforcement area A_s (cm²), the cap height h (cm), the concrete compressive strength F_{ck}(MPa), the spacing between piles $e_{x,y}$ (cm), and the pile diameter d_{pile} (cm).

3 Numerical Results

To demonstrate the formulations mentioned here and the application of PSO, the optimization process of the 3-pile cap will be presented. For this, the geotechnical profile shown in Fig. 3 and the loads as per Table 1 are considered, along with the dimensions a_p and b_p of the column that unloads onto the pile cap, which is 160 cm and 30 cm, respectively. The optimal designs regarding CO_2 emissions and total cost will then be compared with each other and also with the results obtained by Rodrigues et al. [9], who studied the optimization of pile caps using the Genetic Algorithm (GA) and presented a numerical example for 3-pile caps under the same loads and conditions, but analyzing only the reduction of CO_2 emissions.

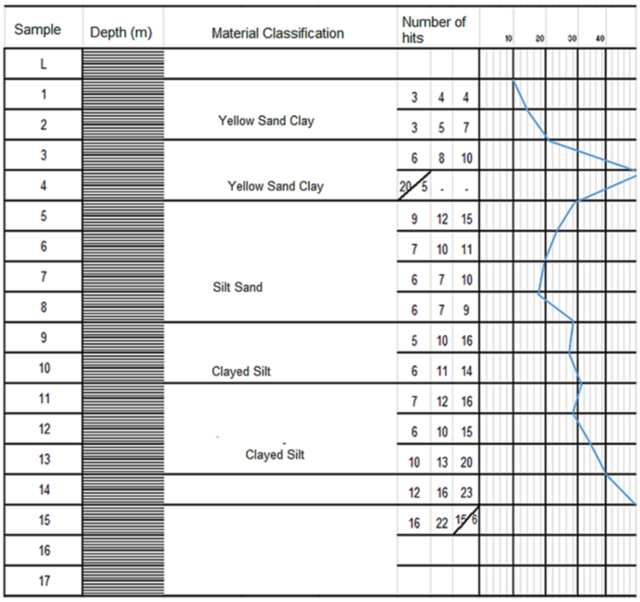


Figure 3. Geotechnical profile of the soil

Table 1. Pile cap loads

Loads	Value
Axial force	P = 4650 kN
Bending moment in X	M_x = 750 kN.m
Bending moment in Y	M_y = 50 kN.m

For the optimization, standard market values from the Brazilian market, obtained through the National System of Costs and Indices of Civil Construction (SINAPI) [17] and local sellers, were used for the cost analysis of the main materials involved in the pile capes construction: concrete, steel, and woodwork. The CO_2 emission data for each of these materials were obtained as described by Santoro and Kripka [4]in Table 2.

Table 2. Cost and emission CO_2 from materials

Material	Cost	CO_2 Emission
Concrete 20MPa	463.14 R$/m³	129.85 $kgCO_2/m^3$
Concrete 25MPa	474.87 R$/m³	142.71 $kgCO_2/m^3$
Concrete 30MPa	491.01 R$/m³	153.68 $kgCO_2/m^3$
Concrete 35MPa	504.22 R$/m³	163.25 $kgCO_2/m^3$
Concrete 40MPa	518.15 R$/m³	171.73 $kgCO_2/m^3$
Concrete 45MPa	532.09 R$/m³	189.6 $kgCO_2/m^3$
Concrete 50MPa	546.02 R$/m³	199.72 $kgCO_2/m^3$
Steel	10.51 R$/kg	1.05 $kgCO_2/kg$
Woodwork	67.37 R$/m²	1.78 $kgCO_2/m^2$

Based on this information and the proposed model, the design parameters shown in Table 3 were obtained using PSO associated to the method of Blévot and Frémy [11], as well as the constraints imposed by ABNT NBR 6118 [10], to obtain the optimal parameters of the 3-pile cap by analyzing cost and emissions. Table 4 shows the final values of these two parameters, as well as a third result regarding the emissions obtained by Rodrigues et al. [9], who used GA to minimize the emissions of the pile cap, while Fig. 4 presents the individual values of emissions and costs for each of the materials involved.

Table 3. Optimized design to reduce cost and emission with PSO algorithm

Solution	Cap											Piles		
	f_{ck} (MPa)	H (cm)	A (cm)	B (cm)	As_x (cm²)	As_y (cm²)	e_x (cm)	e_y (cm)	θ (º)	V (m³)	Steel (kg)	L (m)	d (cm)	V (m³)
Rodrigues et al. [9] (GA)	30	105	252	225	38.5	36.4	175.0	178.3	46	7.86	368,6	5.0	70.0	5.77
Cost Min.	30	115.0	254.3	224.1	26.5	49.3	175.0	178.3	49.6	6.8	317.8	5.0	70.0	5.77
CO_2 Emission Min.	30	105.0	254.3	224.1	29.0	53.90	175.0	178.3	47.0	6.4	347.5	5.0	70.0	5.77

Table 4. Cost and emission for the 3-pile cap

Solution		Cap + Piles	
		Cost (R$)	Emission ($kgCO_2$)
Authors (PSO)	Cost Min.	8484.10	1867.69
	CO_2 Emission Min.	9222.86	1830.81
Rodrigues et al. [9] (GA)	CO_2 Emission Min.	-	2049.52

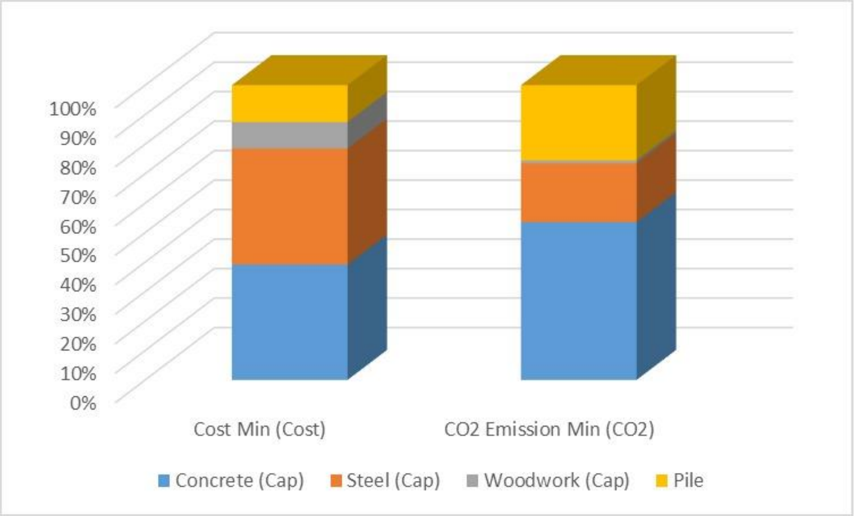


Figure 4. Cost and emission for each material

As observed in Table 4, the results obtained via PSO were better than those presented by Rodrigues et al. [9] in terms of final emissions. When the cost was minimized and emissions were calculated, the value was 9.11% lower, and when emissions were minimized, it was 10.67% lower. It can also be observed that despite the difference between the values when the functions were minimized separately, these values were relatively close.

Figure 5 compiles all this data into a single graph and presents four characteristic curves, where two of them refer to cost and CO_2 emissions when the objective function optimized cost reduction, while the other two refer to the same parameters but when the function optimized CO_2 emissions reduction. Since the magnitudes of each parameter are considerably different and aiming to improve data visualization, the axes were separated, and the total cost curve was inverted. Thus, the left vertical axis presents the total CO_2 emissions, and the right vertical axis presents the total costs for the execution of the 3-pile cap.

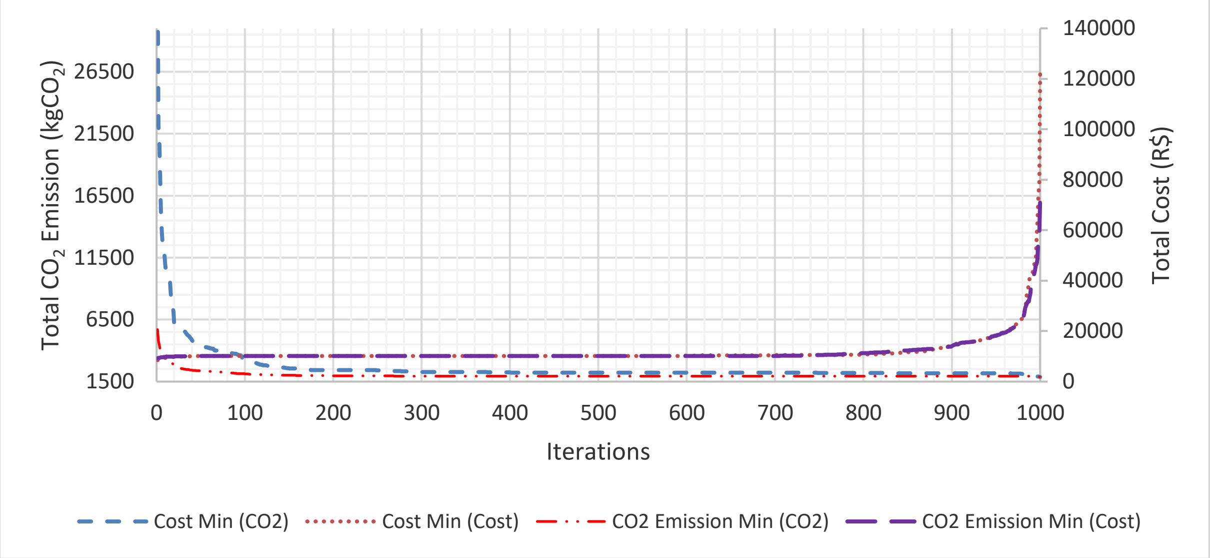


Figure 5. Emission of CO_2 and cost for both minimization problems

4 Conclusions

In the pursuit of a more sustainable and viable construction sector, the reduction of total costs and CO_2 emissions are highly relevant parameters that must be carefully analyzed. Thus, this study aimed to investigate the optimization of the design of a 3-pile cap, intending to meet the criteria defined by ABNT NBR 6118 [7] and to obtain the optimal characteristics for each of these aforementioned cases.

The final values obtained through PSO were similar, tending to favor the parameter that was optimized. When the cost was optimized, its final value showed a reduction of approximately 8.01% compared to the cost calculated when optimizing for CO_2 emission reduction. On the other hand, when optimizing for CO_2 emission reduction, the emission value was 1.97% lower than that obtained in the cost optimization counterpart. Furthermore, comparing the results obtained through PSO, with the data from the analysis performed using GA, it is noted that PSO offers a clear reduction in emissions calculation, proving to be an excellent alternative algorithm for this type of optimization problem.

Thus, it is concluded that the pursuit of more sustainable designs is not only essential but also feasible, without resulting in impractical cost differences. This further highlights the importance of environmental studies within civil engineering topics and the effectiveness of optimization techniques through metaheuristic algorithms as tools for reducing environmental impacts.

Acknowledgements. The authors would like to thank CAPES for the support given to the postgraduate program in civil engineering at UFES and FAPES for the research grant provided to the third author.

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