Optimization of materially nonlinear trusses subjected to dynamic loads

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Abstract. The structural optimization of trusses has been extensively studied over the past few decades. However, investigations incorporating material nonlinearity are still relatively scarce. This work aims to propose a formulation for optimizing truss structures, considering the material nonlinearity of the structure under dynamic loading. To address this problem, the Particle Swarm Algorithm (PSO) integrated into the Ansys program was utilized for nonlinear dynamic structural analysis. To validate the optimization approach, a problem from the literature, considering material nonlinearity was analyzed. Additionally, the real impact of material nonlinearity on optimization results were evaluated.

Keywords: Material Nonlinearity, Optimization, Dynamic Load, Particle Swarm Algorithm

1 Introduction

The optimization of trusses has been widely studied mainly in the field of dimension and topological optimizations considering static loads and a linear elastic behavior of the material. However, the optimization of trusses with dynamic loads and considering the nonlinearity of the material are scarce in the literature.

Ramos and Paulino [1] performed a topological optimization of trusses based on the ground-structure approach, considering a material nonlinearity. The hyperelastic models of Hencky, Saint-Venant, Neo-Hookean and Ogden were used. Finally, the nonlinearity effect on the optimization problem was evaluated.

Zhang, Ramos Jr and Paulino [2] proposed a topological optimization of truss structures using a ground structure approach with a filter algorithm. Structure examples in 2D and 3D with multiple load cases and nonlinear material behaviorwere considered. For an example with approximately 1 million design variables, the proposed algorithm is more than 40 times faster than the standard ground structure approach.

Viet-Hung and Seung-Eock [3] developed a reliability-based design optimization method for truss structures by integrating nonlinear inelastic analysis, a structural reliability analysis method, and a proposed optimization method based on a differential evolution algorithm. Several examples were considered to evaluate the efficiency of the method.

Qin et al. [4] optimized complex truss structures with non-uniform discrete design variables combined with the finite element method. A nonlinear mechanical analysis was performed to reach the accurate design of these structures. The structure was optimized using 14 design variables.

When it comes to both nonlinearities in the same problem, some works are highlighted. Ju et al. [5] minimized the weight of a lightweight FRP composite triangular truss under nonlinear structural response constraints. Gradient-based and genetic algorithm optimization processes were selected and implemented in matlab.

Ha, Vu, and Truong [6] presented an effective method to optimize the stay cables of steel cable-stayed bridges using nonlinear inelastic analysis and a micro-genetic algorithm (μGA). To estimate the nonlinear inelastic behaviors of the bridge, the influence of cable sag, large displacement and second-order effect were considered. A practical advanced analysis (PAA) method was employed to capture both geometric and material nonlinearities.

Karimi and Kani [7] used genetic algorithms to find the worst imperfection pattern in shallow lattice domes considering geometric and material nonlinearities. The worst imperfection pattern was analyzed, calculated, and plotted. Later, Javidi, Salajegheh and Salajegheh [8] studied geometric and material nonlinear behavior in the ideal design of space structures, where weight and collapse energy were considered as fitness function. The optimization problem was considered as multi-objective. Two algorithms were developed, the multi-objective crow search algorithm (MOCSA) and the multi-objective modified crow search algorithm (MOMCSA).

Among the metaheuristics optimization algorithms, the Particle Swarm Optimization (PSO) algorithm proposed by Kennedy and Eberhart [9] stands out, as it is easy to implement computationally and due to its robustness in the search for the optimal solution. Some works of truss optimization using PSO can be found in Erlacher et al. [10], Silva et al. [11], Domingues et al. [12], and others.

This work aims to present a formulation for an optimization problem of trusses subjected to dynamic loads considering material nonlinearity. For nonlinear dynamic analysis, Ansys software was used and the solution to the optimization problem was obtained via PSO with an integration of Ansys-PSO via Matlab.

2 Optimization Problem Formulation

The optimization problem aims to define the cross-sectional areas of the bars that minimize the final weight of the structure, by imposing constraints on axial stresses. The design variables that set the optimization problem are the cross-sectional area of each bar of the structure, included in vector $\boldsymbol{A}$, where bn is the total number of bars.

$$\boldsymbol{A} = \{A_1, A_2, \dots, A_{bn}\} \tag{1}$$

These variables were considered discrete and continuous, with the discrete variables assuming area values from a commercial catalog of tubular structural profiles.

The objective function calculates the total weight of the truss by the sum of the weight of each bar:

$$Minimize\ f = \sum_{i=1}^{nb} \rho\, A_i L_i \tag{2}$$

where ρ is the specific mass of the steel, A_i is the cross-sectional area of bar i, and L_i is the length of bar i.

The constraints imposed to the optimization problem are:

$$\frac{\sigma_{T_{max}}}{\sigma_{T_{lim}}} - 1 \le 0 \qquad \frac{\sigma_{C_{max}}}{\sigma_{C_{lim}}} - 1 \le 0 \tag{3}$$

where $\sigma_{T_{max}}$ and $\sigma_{C_{max}}$ are the maximum values of tensile and compressive axial stresses, respectively; $\sigma_{T_{lim}}$ and $\sigma_{C_{lin}}$ are the allowable limits values of tension and compression, respectively. All values of stresses are obtained considering a material nonlinear analysis with a dynamic load. The allowable limit values of tension and compression are defined by Kim and Park [13].

It is highlighted that the constraints given by Eq. (3) require a nonlinear dynamic analysis for each iteration of the optimization process. This analysis was performed via Ansys software using the LINK180 element and the Newmark method for solving the dynamic problem:

$$[\mathbf{K}]\{\boldsymbol{d}\} + [\mathbf{C}]\{\dot{\boldsymbol{d}}\} + [\mathbf{M}]\{\ddot{\boldsymbol{d}}\} = \boldsymbol{F}(\boldsymbol{t}) \tag{4}$$

where **[K]** is the stiffness matrix of the structure, which depends on the cross-sectional area of the bar, **[M]** is the mass matrix of the structure, which also depends on the cross-sectional area of the bar, **[C]** is the damping matrix, which when considered is a linear combination of the former two. On the right-hand side of the equation, there is the time-dependent loading $\boldsymbol{F}(\boldsymbol{t})$. Figure 1 shows how the optimization process works.

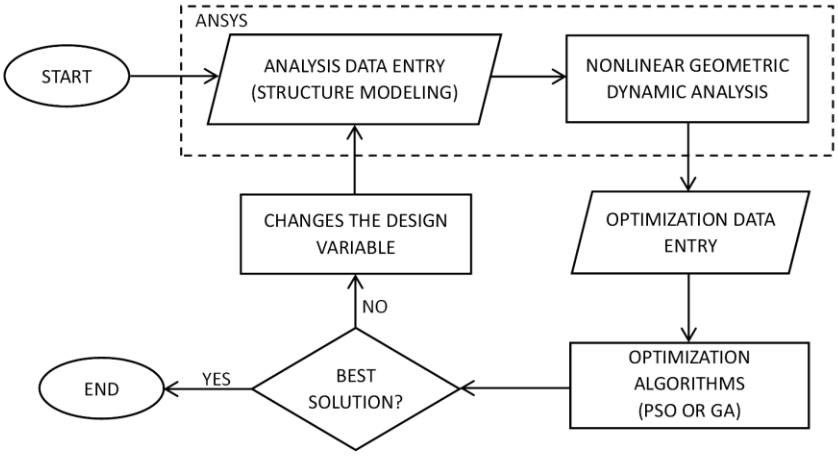
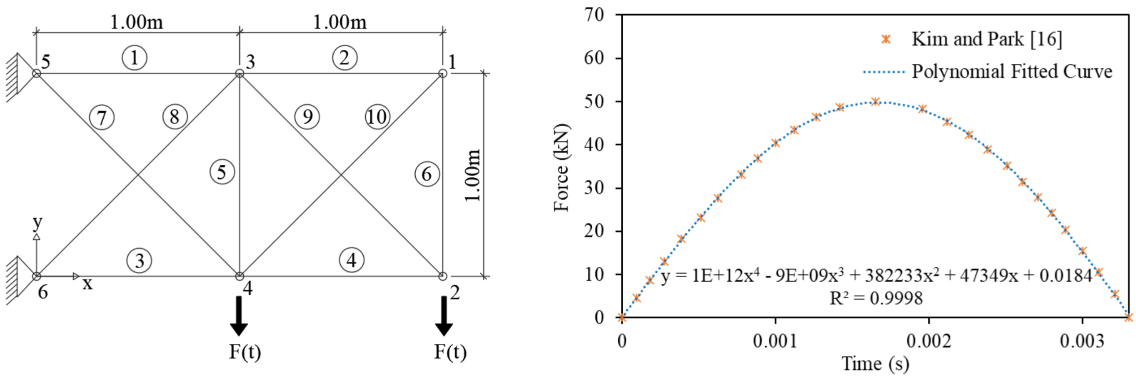


Figure 1. Scheme of the optimization process

The optimization problem solution was obtained via PSO using the Adaptive Penalties Method (APM) proposed by Barbosa and Lemonge [14] with a maximum number of iterations equal to 50, a population size of 50 individuals, and a tolerance: 10^{-6}.

3 Numerical Results

To validate the problem formulation, the 10-bar truss in Fig. 2(a), proposed by Kim and Park [13], was studied. The structure was subjected to the dynamic loading in Fig. 2(b) and has a material with nonlinear behavior, as seen in Fig. 3. First, the structural analysis was performed, and then the optimization, under the same conditions. A cross-sectional area A = 3.14 cm² and a specific mass ρ = 7860 kg/m³ were considered.



(a) 10 Bars Truss (b) Dynamic Load

Figure 2. Analyzed problem

The reference authors used the ESLs (Equivalent Static Loads) method for the nonlinear dynamic analysis. To obtain the transient response of the structure, a total analysis duration of 0.003 s and a time increment of Δt = 0.0002 s were adopted. For greater precision, in this work, the time increment was Δt = 0.00001 s. Damping parameters were not considered.

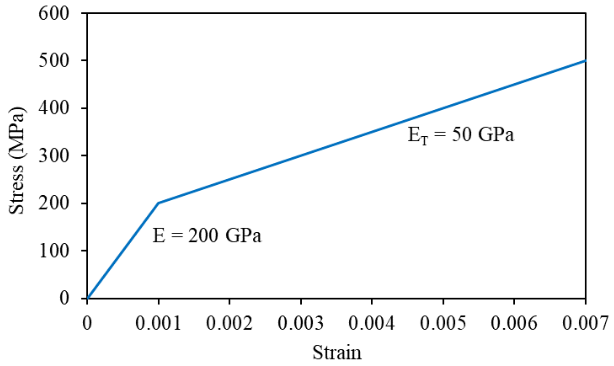
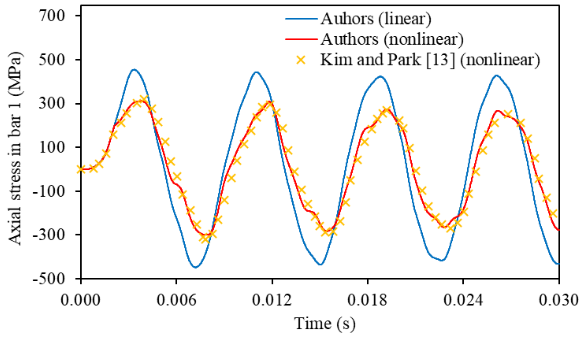
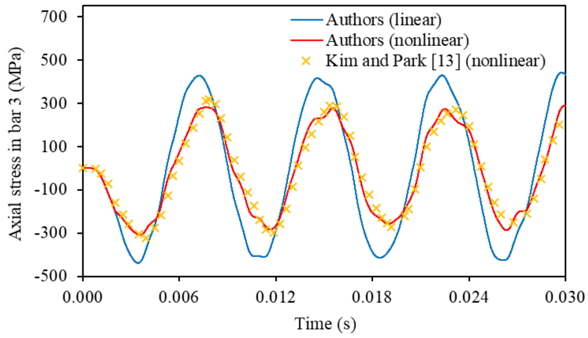


Figure 3. Nonlinear material curve

In addition to material nonlinear analysis, a linear analysis was also performed to compare the types of analysis, both using the same parameters. The axial stress results in bars 1 and 3 are shown in Figs. 4(a) and 4(b), respectively.



(a) Bar 1



(b) Bar 3

Figure 4. Axial stress in bars 1 and 3.

The nonlinear analysis resulted in lower stresses when compared to the linear analysis, around 30%, for both tension and compression. Furthermore, for the two bars, the maximum stress level was approximately 500 MPa, indicating that the material reached the second part of the nonlinear material curve.

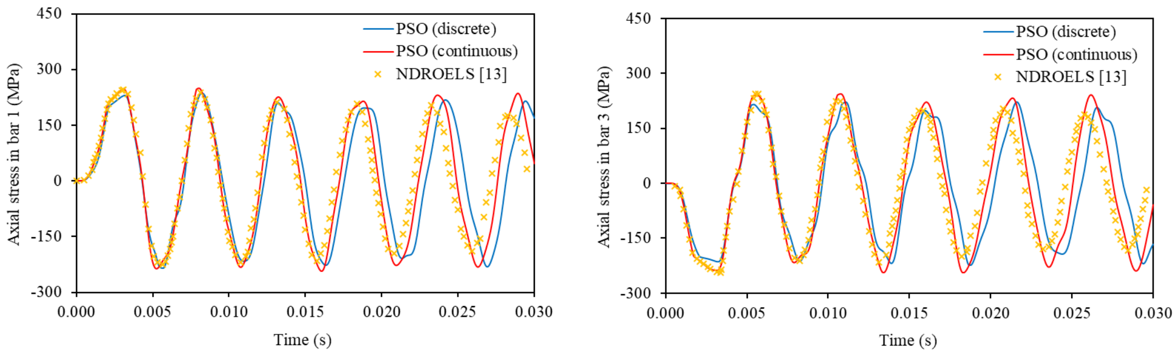
Kim and Park [13] also studied the optimization of this problem, maintaining the dynamic loading and the properties of the structure. The reference authors used only continuous variables and NDROESL (Nonlinear Dynamic Response Optimization Using Equivalent Static Loads) as an optimization algorithm. They considered 10 design variables and an allowable limit of stress equal to 250 MPa as the constraint of the problem, for both tension and compression. Tab. 1 presents the results obtained in this work, as well as those of Kim and Park [13].

Table 1. Truss optimization results

	Linear		Nonlinear	
	PSO (continuous)	PSO (discrete)	PSO (continuous)	Kim e Park [13] NDROESL
A_1 (cm²)	7.307	4.80	5.412	4.976
A_2 (cm²)	2.068	1.98	0.788	0.955
A_3 (cm²)	6.757	6.41	4.533	4.806
A_4 (cm²)	2.512	1.18	2.304	1.569
A_5 (cm²)	0.921	0.78	0.785	0.786
A_6 (cm²)	1.236	1.38	0.788	0.786
A_7 (cm²)	3.954	4.37	2.699	3.163

A_8 (cm²)	4.831	2.38	3.818	3.368
A_9 (cm²)	2.563	1.38	2.589	2.099
A_{10} (cm²)	1.810	2.18	0.785	1.138
$\sigma_{T_{max}}$ (MPa)	249.81	249.46	249.58	246.12
$\sigma_{C_{max}}$ (MPa)	248.53	244.82	248.52	243.48
Mean (kg)	31.57	25.24	23.60	-
Standard Deviation (%)	7.7	12.4	15.8	-
Best Solution (kg)	**30.98**	**24.45**	**22.48**	**21.77**

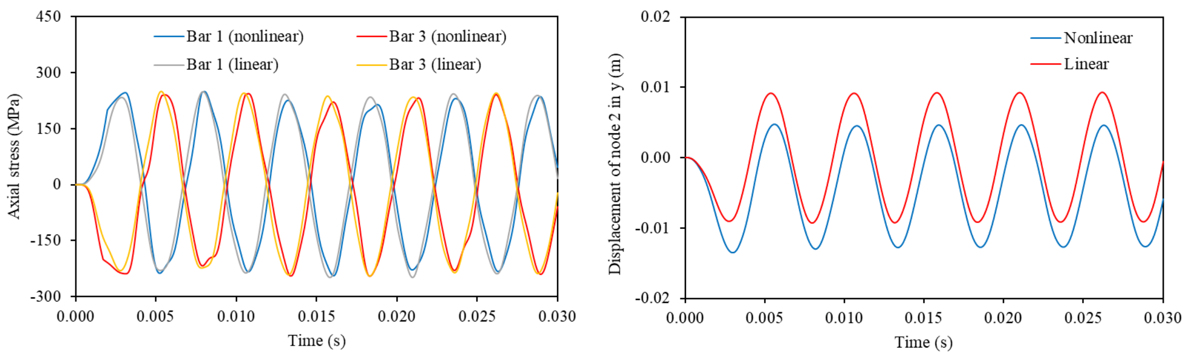
For optimization with discrete variables, there was a difference of 12.3% between the total weight values of this work and the reference, on the other hand, the use of continuous variables led to a closer result, with a difference of only 3.2 %. Figs. 5(a) and (b) show the stress curves in bars 1 and 3 in the nonlinear analysis, validated with the reference results. It is also possible to note the instant of time in which the maximum and minimum stresses occurred.



(a) Bar 1 (b) Bar 3

Figure 5. Axial stress of bars 1 and 3 of the optimized truss.

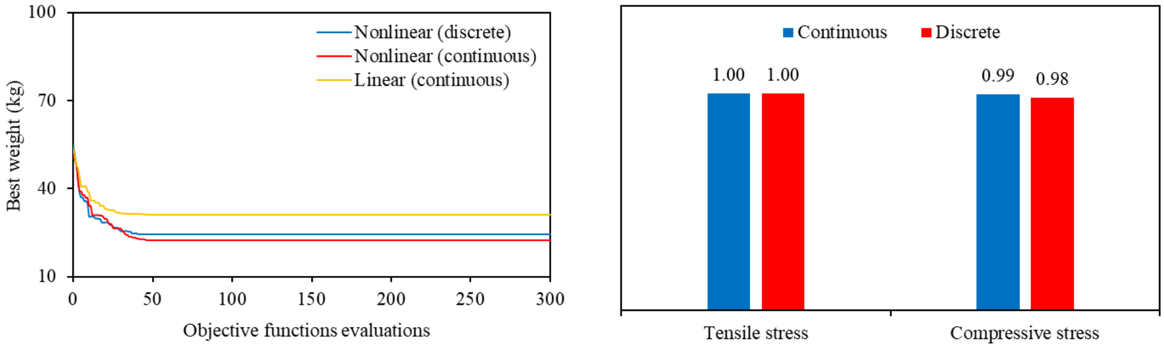
A comparison between the linear and nonlinear analyses with continuous variables was made in Figs. 6(a) and 6(b). In Fig. 6(a) the stresses in the bars had a similar behavior, although the best solution was different. On the other hand, Fig. 6(b) shows that the effect of nonlinearity had more influence on the displacement, varying over time.



(a) Axial stress (b) Displacement

Figure 6. Comparison between linear and nonlinear analyses

Fig. 7(a) presents the convergence curve of the optimization via PSO, the best solution found around iteration 46. The optimized structure considering the nonlinear analysis presented a weight 27.4% smaller than the linear one. Fig. 7(b) shows the critical constraints of the optimization, where both stresses have a similar influence on the problem.



(a) Optimization History (b) Critical constraints

Figure 7. Optimization results

4 Conclusions

According to the results presented, the interface optimization routine proposed between Ansys and PSO was effective in obtaining a solution to a problem with dynamic loading and nonlinearity in the material. Furthermore, there was a reduced maximum difference between the nonlinear solution proposed in the literature and the one obtained via PSO, showing good agreement between the results in the final weight, although the algorithms find different solutions for the bar areas. The structure optimized considering the nonlinear analysis presented a significant weight reduction when comparedto the linear one, despite the effects on stress and displacement being different over time. It was also noted that both stress constraints were active in the problem.

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