Identification of Single and Multiple Damage in Beams Using Natural Frequencies and Bayesian Data Fusion

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Abstract. The identification of damage is one of the most important procedures in the investigation of structural health. Searching for more accurate and non-destructive ways of doing these analyses, vibration-based methods have been widely used. Here, three different ways of modeling damage will be presented, verifying the relationship between damage and changes in natural frequencies. Four cases are investigated with three support types. A Bayesian data fusion algorithm is introduced to detect damage. The results show good accuracy in locating the damage for most cases analyzed.

Keywords: Structural health monitoring, Experimental modal analysis, Vibration, Bayesian data fusion

1 Introduction

Structural Health Monitoring (SHM) is the process of identifying damage in structures using various methods. This process involves the observation of a structure or system over a period of time using measurements of dynamic response periodically spaced, data extraction from the measurements to obtain damage-sensitive parameters, and statistical analysis of these parameters, as stated by Inman *et. al* [1] and Farrar and Worden [2]. It is of great significance in aerospace, civil and mechanical engineering, and exhibits an increasing importance in many other fields. SHM has a crucial relevance in determining the remaining useful life of a structure, which is essential in avoiding catastrophic failure. However, these analyses can be complicated and sometimes require destructive methods in order to achieve accurate results. Nonetheless, vibration-based methods have shown promising results as non-destructive approaches, relying primarily on variations of the modal parameters. Cawley and Adams [3] developed a method of studying structural integrity using natural frequencies, and were successful in monitoring damage growth. Later studies published by Hearn and Testa [4] showed the relationship between natural frequencies variation and damaged structures. Hassiotis and Jeong [5] introduced a damage identification method depending only on natural frequencies, but investigations on more complex structures were not completed. In more recent years, the implementation of optimization algorithms have led to more accurate results and faster analysis. Among all the possible methods, Bayesian approaches have been successfully applied regarding structural analysis. Yin *et. al* [6] used incomplete modal parameters approximated by Bayesian methods. In the present work, a beam is analyzed where three different damage modeling methods are presented, in order to verify the relationship between the presence of damage, local stiffness reduction and variation in natural frequencies. Bayesian data fusion is then introduced to locate the damage in four different cases for three types of support.

2 Theoretical background

2.1 Damage and natural frequencies

Consider a system with multiple degrees of freedom. It can be modeled through a simple equation of motion given by $M\ddot{x}+C\dot{x}+Kx=0$, where M stands for the mass matrix, C the damping matrix, K the stiffness matrix, and x is the displacement vector. For a system where no damping can be assumed, the equation can be simplified to $M\ddot{x}+Kx=0$. In order to obtain the natural frequencies we must assume a harmonic solution, and considering the equation of motion we find the relation given by:

$$(K - \omega_j^2 M)\phi_j = 0. \tag{1}$$

We establish that the natural frequencies are represented by ω_j, and the mode shapes are represented by ϕ_j. The j index serve as a mode number indicator. Motion only exists if $\phi_j \neq 0$, and a set of coupled equations arise, rendering the solution to the differential equation quite complex. One of the most common ways of solving that problem is using modal analysis. We find that the following relations are true, given that Λ is the matrix of squared natural frequencies and I the identity matrix:

$$[\phi]^T M[\phi] = I. \tag{2}$$

$$[\phi]^T K[\phi] = \Lambda. \tag{3}$$

Deterioration of structures reduces the stiffness of said structure, resulting in a change of the stiffness matrix K. Let this difference be denoted by δK. It is also assumed that the perturbation is small enough such that there is no decrease in mass. For Equation (3) to remain true, a change in the natural frequencies must follow.

2.2 Bayesian data fusion

Consider a finite element (FE) model of a beam. Let ω_j be the natural frequency of the jth vibration mode of the undamaged beam, and ω_j^d the analogous for the damaged beam. The relative natural frequency change (RNFC) is given by:

$$\Delta\omega_j = \frac{\omega_j - \omega_j^d}{\omega_j}. \tag{4}$$

For a constant damage severity, the RNFC curve can be normalized to the [0,1] range, denoted by $\Delta\bar{\omega}_j = normalization(\Delta\omega_j)$. The RNFC curve can also be obtained using a different approach, namely, mode shapes. Let ϕ be the mode shape vector, the mode shape curvature is denoted by the double derivative of the mode shape, ϕ''. The square of the curvature, $(\phi'')^2$, is further normalized to the [0,1] range, denoted by $g = normalization((\phi'')^2)$. Consider the damage at the ith element, this curve is a function of the location of damage alone, such that $g = g_j(\zeta_i)$.

Considering the RNFC curves obtained by both methods previously described, a damage position function (DPF) is obtained, given by $DPF_{i,j} = 1 - |g_j(\zeta_i) - \Delta\bar{\omega}_j|$. A fusion of multiple DPFs is performed. Taking into account the basics of Bayes' theorem, let there be m information sources (S_m) and n events (A_n). Consider the source the jth vibration mode, and the event the damage in the ith element. The probability function, considering independent sources, is given by the following relation:

$$P(A_i|S_m) = \frac{\Pi_{j=1}^{m} P(S_j|A_i)P(A_i)}{\sum_{u=1}^{n}(\Pi_{j=1}^{m} P(S_j|A_u)P(A_u))}. \tag{5}$$

If the prior probability is defined as $P(A_i) = 1/n$, the conditional probability can be described as $P(S_j|A_i) = DPF_{i,j}$. Equation (5) then becomes:

$$P(A_i|S_m) = \frac{\Pi_{j=1}^{m} DPF_{i,j}}{\sum_{u=1}^{n}(\Pi_{j=1}^{m} DPF_{u,j})}. \tag{6}$$

The improvement is performed using $Q_i = \sqrt{P_i P_{n+1-i}}$, which is further standardized using Z-score. The probability damage indicator (PDI) is then calculated by considering that the PDI is equal to the z-score if $z - score \geq 0$ and zero if $z - score < 0$.

The PDI is then plotted along the length of the beam in a $location \times PDI$ graph. The peaks of the PDI indicate the location with the highest probability of presence of damage, as stated by Sha *et. al* [7].

3 Methodology

In order to identify the accuracy of modeled damage and its relationship with changes in natural frequency, a numerical analysis was performed and three different models were used. Consider a beam of length $l = 1.04m$, width $w = 0.02m$ and height $h = 0.009m$, made of a material with elastic modulus $E = 200GPa$ and density $\rho = 7855kg/m^3$. The problem is based on previous studies done by Sha *et. al* [7] and Khiem and Toan [8]. Damage is depicted as a decrease in stiffness from 10% in three possible locations (ζ_i) - $\zeta_1 = 0.2m$, $\zeta_2 = 0.45m$ and $\zeta_3 = 0.7m$. For the numerical analysis, two support types are considered, clamped-free (CF) and free-free (FF). Four cases are studied for each support type, detailed in Table 1.

Table 1. Details of studied cases in terms of stiffness reduction percentage in possible locations

Cases	ζ_1	ζ_2	ζ_3
Case 1 (0-0-0)	0%	0%	0%
Case 2 (0-10-0)	0%	10%	0%
Case 3 (10-10-0)	10%	10%	0%
Case 4 (10-10-10)	10%	10%	10%

The first model is constructed in ANSYS®Academic Research APDL, Release 23.2. A beam with 100 elements is considered, with elements of BEAM188 type. The damage is modeled as a reduction in stiffness proportional to the damage intensity. The second model is built in ANSYS®Academic Research Mechanical, Release 23.2, with the beam modeled as a solid body - damage is introduced similarly to the first model. The third model is built on the same environment as the second model, however, damage is modeled as an indent in the beam proportional to the severity of damage. Additionally, a beam with the same dimensions and properties was used in an experimental setup, however, with a clamped-clamped (CC) configuration and the same four cases. The results for natural frequencies were obtained from studies done by Khiem and Toan [8].

4 Results and discussions

The results for the first six natural frequencies for the four cases and all three models are shown in Table 2 for CF support, and Table 3 for FF support. Notice that all three models produce similar results for the frequencies.

Table 2. Natural frequencies for the three models clamped-free

FIRST MODEL CF						
Frequencies	1	2	3	4	5	6
Case 1 (0-0-0)	6.8986	42.949	121.30	235.81	392.15	583.20
Case 2 (0-10-0)	6.8948	42.529	120.97	234.73	389.95	582.35
Case 3 (10-10-0)	6.9247	42.438	119.98	232	386.53	580.66
Case 4 (10-10-10)	6.8671	42.399	119.18	231.7	385.41	575.58
SECOND MODEL CF						
Case 1 (0-0-0)	6.7886	42.529	119.02	233.04	384.85	574.19
Case 2 (0-10-0)	6.7880	42.519	119.01	233.01	384.78	574.16
Case 3 (10-10-0)	6.7860	42.519	119	232.96	384.70	574.11
Case 4 (10-10-10)	6.7831	42.493	118.92	232.86	384.46	573.74
THIRD MODEL CF						
Case 1 (0-0-0)	6.7886	42.529	119.02	233.04	384.85	574.19
Case 2 (0-10-0)	6.7871	42.509	119	232.97	384.70	574.13
Case 3 (10-10-0)	6.7820	42.509	118.99	232.87	384.53	574.03
Case 4 (10-10-10)	6.7822	42.490	118.91	232.85	384.44	573.71

Table 3. Natural frequencies for the three models free-free

FIRST MODEL FF						
Frequencies	1	2	3	4	5	6
Case 1 (0-0-0)	43.714	121.25	235.83	392.23	583.33	816.52
Case 2 (0-10-0)	43.555	121.01	234.67	389.99	582.49	809.80
Case 3 (10-10-0)	43.565	120.93	233.31	387.66	581.31	809.75
Case 4 (10-10-10)	43.574	120.09	233.03	386.53	576.24	808.13
SECOND MODEL FF						
Case 1 (0-0-0)	43.148	118.88	232.90	384.64	573.93	800.51
Case 2 (0-10-0)	43.134	118.87	232.87	384.57	573.90	800.29
Case 3 (10-10-0)	43.133	118.85	232.79	384.47	573.84	800.30
Case 4 (10-10-10)	43.100	118.75	232.67	384.22	573.46	800.02
THIRD MODEL FF						
Case 1 (0-0-0)	43.148	118.88	232.90	384.64	573.93	800.51
Case 2 (0-10-0)	43.117	118.87	232.83	384.49	573.87	800.07
Case 3 (10-10-0)	43.112	118.81	232.67	384.27	573.75	800.05
Case 4 (10-10-10)	43.091	118.74	232.65	384.18	573.42	799.98

The results for damage identification are displayed in Figs. 1, 2 and 3 for CF support, Figs. 4, 5 and 6 for FF support, and Fig. 7 for the experimental CC setup. The blue line is the PDI, the peaks representing the predicted location of damage, and the red dashed line indicates the actual location of damage. Notice how the peaks of the PDI coincide with the location represented by the dashed line, therefore, the algorithm locates damage with good accuracy for most observed cases.

The results for models 2 and 3 are reasonably similar, whereas model 1 produces more distinct results, specially for FF support. The CC support analysis (Fig. (7)) leads to misguided results due to a peak near the edges, which may be explained by the clamped-clamped configuration, but further investigation is needed. Results for triple damage also need enhancement for better accuracy.

Furthermore, it is worth noticing that a symmetry problem arises, and the algorithm is not able to properly detect damage beyond the beam symmetry axis. This happens due to the fact that similar damage in different locations result in the same amount of change in natural frequencies, and further processing of results is needed.

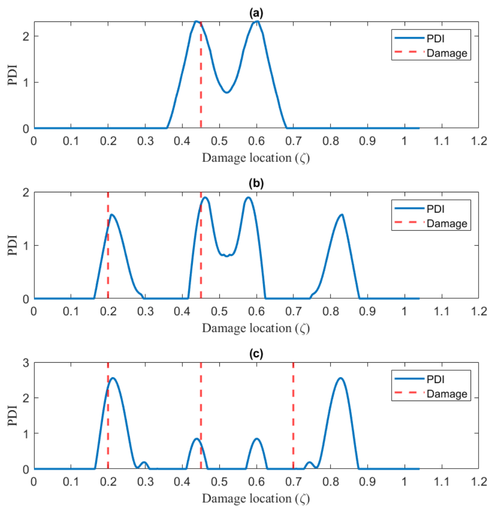

Figure 1. PDI for the first model CF cases 2 (a), 3 (b), and 4 (c)

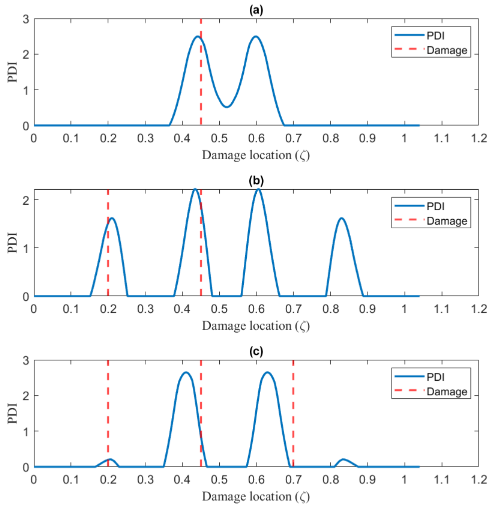

Figure 2. PDI for the second model CF cases 2 (a), 3 (b), and 4 (c)

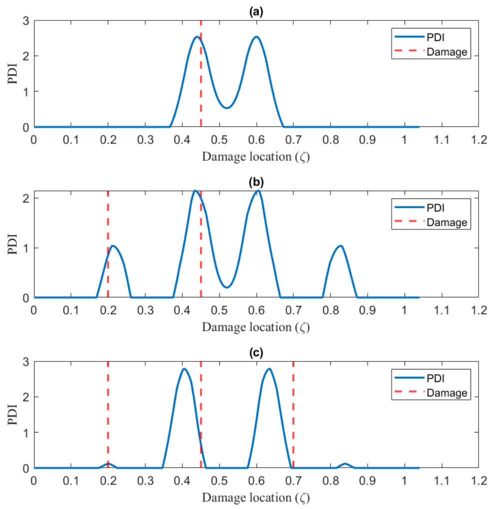

Figure 3. PDI for the third model CF cases 2 (a), 3 (b), and 4 (c)

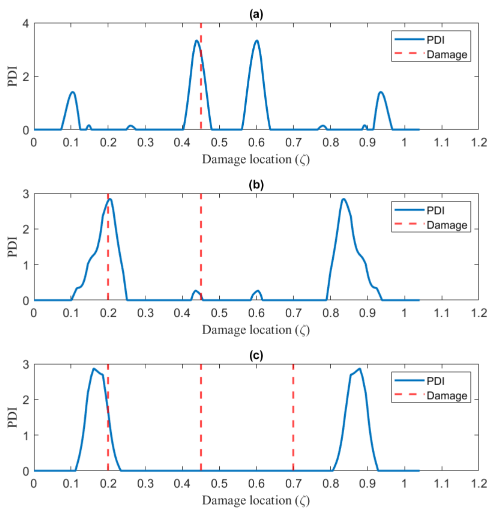

Figure 4. PDI for the first model FF cases 2 (a), 3 (b), and 4 (c)

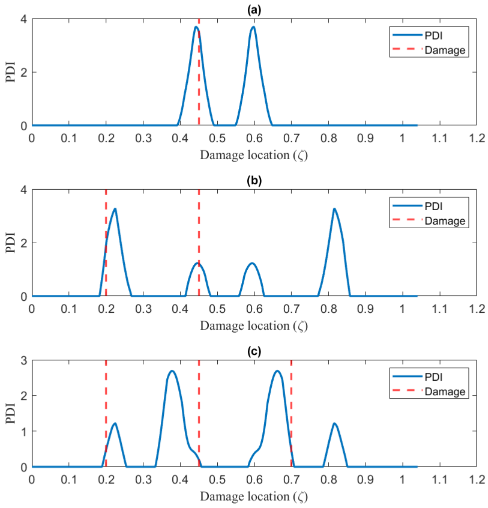

Figure 5. PDI for the second model FF cases 2 (a), 3 (b), and 4 (c)

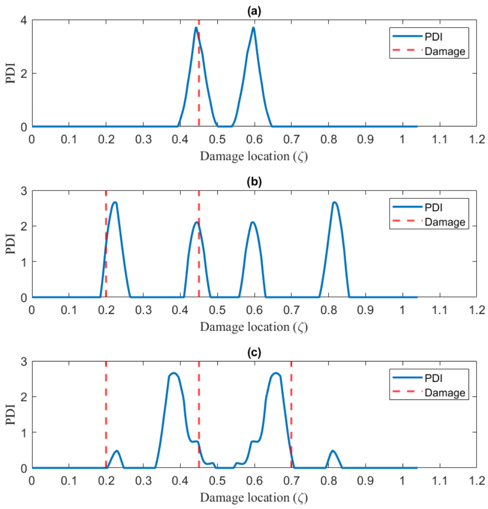

Figure 6. PDI for the third model FF cases 2 (a), 3 (b), and 4 (c)

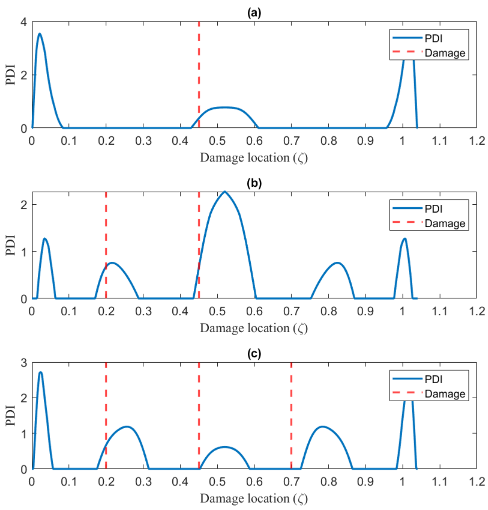

Figure 7. PDI for the experimental setup CC cases 2 (a), 3 (b), and 4 (c)

5 Conclusions

Damage can successfully be modeled as a local reduction in stiffness, as shown by the numerical analyses. The algorithm provides results for damage identification with good location accuracy for single and multiple damage scenarios and different types of support (clamped-clamped, clamped-free and free-free). An edge effect appears in CC cases which may distort results. A symmetry problem arises, which can be eliminated with additional analyses, such as wavelet or ultrasonic equipment, however, further studies are beyond this work's scope.

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