Structural damage detection using FRFs and machine learning methods

Dianelys V. Ruiz[1], Bernardo L. Poncetti[1], Marcos M. Futai[1]

[1] *Dept. of Structural and Geotechnical Engineering, Universidade de São Paulo*
Avenida Professor Almeida Prado Travessa 2 Nº 83, 05508-070, São Paulo/SP, Brazil
dvegaruiz@usp.br, bernardoponcetti@usp.br, futai@usp.br

Abstract. In this paper, experimental tests and numerical simulations are conducted to evaluate the performance of different models for structural damage identification and quantification. For this purpose, an aluminum beam in Laboratory conditions is utilized as a test structure. Firstly, impact tests are performed to identify the modal parameters and frequency response functions (FRFs) of the healthy structure. Then, different damages are induced in the beam by means of rectangular notches, and FRFs from each damage scenario are measured. Meanwhile, a simplified numerical model of finite elements of the beam is developed and calibrated with respect to experimental data. The calibrated model is used to generate a set of simulations representing the different damage scenarios induced experimentally. The damage is introduced in the numerical model by reducing the cross-sectional area. Normalized FRF amplitudes are used as the damage indexes. To increase the predictive capability of the models, uncertainties are introduced considering the FRF amplitudes as random variables. Afterward, different datasets are constructed and several well-established machine learning classifiers such as Decision Tree, SVM and KNN are trained to perform damage identification and quantification. Finally, experimental data measured on the damaged beam are used as input variables to evaluate the prediction capacity of the trained classifiers. Undamaged and damaged data are correctly classified by most of the classifiers. However, to quantify the degree of damage some shortcomings are found.

Keywords: vibration-based method, damage detection, damage quantification, machine learning

1 Introduction

The detection of structural damage in civil engineering structures has been a concern for researchers and engineers in the last decades. Therefore, numerous damage detection approaches have been developed with the aim of providing means of early warning against damage or any type of structural anomaly (Ahmadi et al. [1], Dilena et al. [2], He and Zhou [3], Aied et al. [4], Li et al. [5]). Vibration-based methods, which rely on the global dynamic behavior of the structure to assess its condition and identify structural damage have received considerable attention lately.

Vibration-based damage detection is performed by first extracting a reference pattern from the vibration response and then applying a pattern recognition method to compare the damaged pattern with that of the reference condition (Avci et al. [6]). Commonly, damage to the structure has been explained as changes in modal parameters such as the natural frequencies, mode shapes, and structural damping. Therefore, several researchers have studied damage detection techniques based on changes in structural dynamic characteristics.

Among a variety of vibration-based approaches, one of the dynamic features adopted by many researchers is the frequency response function (FRF). It has been demonstrated that FRF can be used as a damage index with success (Bandara et al. [7], SAMPAIO et al. [8], Lee and Shin [9]). In addition, the development of machine learning approaches in recent years have allowed promising results in damage detection. A great variety of algorithms has been employed by several researchers. Each algorithm has its own technique and hyperparameters which can be turned to achieve a better performance in anomaly detection. However, the main challenge is to rely on enough data from undamaged and damaged structural conditions. These data are usually produced through computational simulations, employing analytical or numerical models. Although high-fidelity computational models represent the structural behavior more realistically, the higher computational cost justifies the adoption of simpler models.

Thus, this study aims to evaluate some well-established machine learning approaches for structural damage detection using numerical and experimental data. For this purpose, a series of experimental tests on an aluminum beam in laboratory conditions are conducted to measure FRFs from undamaged and damaged conditions. The normalized FRF amplitudes are utilized as damage indexes. Damages are induced on the beam by means of rectangular notches at different locations. At the same time, a simplified numerical model of the beam is used to generate datasets to train and validate different machine learning classifiers, with the goal of identification and quantification of damages. The damage is modeled as a partial reduction of the cross-sectional area. Moreover, uncertainties coming from different sources are considered assuming that FRF amplitudes are random variables. Datasets considering two levels of uncertainties and reductions in cross-sectional area are considered. A set of machine learning models, including Support Vector Machine (SVM), Decision Tree (DT), and K-Nearest Neighbors (KNN) are tested and validated to perform damage identification and quantification. The results show that the presence of damage can be successfully identified. However, the quantification of damages has shown some shortcomings.

This study preceded the one published in Ruiz et al. [10].

2 Experimental program

An aluminum beam with dimensions of 1200mm×76.2mm×6.35mm, and it is installed at Lab-Infra from University of São Paulo. The geometric and mechanical properties of the beam are shown in Tab. 1. The beam is tied in by two clamps at its ends, as can be seen in Fig. 1a. Thus, the free span of the beam is 1120 mm. Five piezoelectric accelerometers with 5g capacity are installed at the bottom of the beam at different locations. These accelerometers' positions are established to coincide with the anti-nodes of the first three vertical vibration modes, previously obtained numerically by means of modal analysis. A Data Acquisition System from BDI company is used to sample the acceleration signals at 1000 Hz. The ambient temperature conditions at the Laboratory remained stable during the tests. Therefore, it is assumed that the experimental measurements were not affected by temperature effects in this study.

Table 1. Geometric and mechanical properties of the beam.

Magnitude	Unit	Value
Torsional constant	m^4	$2{,}36\times10^{-7}$
Moment of inertia around $y-$axis	m^4	$2{,}34\times10^{-7}$
Moment of inertia around $z-$axis	m^4	$1{,}63\times10^{-9}$
Polar moment of inertia	m^4	$2{,}36\times10^{-7}$
Cross-sectional area	m^2	$4{,}84\times10^{-4}$
Material density	kg/m^3	2700
Modulus of elasticity	N/m^2	$5{,}77\times10^{10}$
Shear modulus	N/m^2	$4{,}66\times10^{10}$

2.1 Impact tests on healthy structure

To identify the modal parameters, free vibration tests are performed. The beam is excited by an impact applied with an instrumented hammer at three different positions, which are shown in Fig. 1b. These positions are chosen to excite the first three vibration modes that have the maximum vertical displacements at these points. Using the acceleration signals recorded by the accelerometers, the modal frequencies and damping rates are obtained using the Short Time Fourier Transform (STFT) technique. The natural frequencies of the three first vibration modes are f_1 =21,6 Hz, f_2 = 59,3 Hz and f_3 =121,3 Hz, respectively. The damping rates are ξ_1 =0,72 %, ξ_2 =0,51 % and ξ_3 =0,66 %, respectively. The FRFs at each accelerometer position are computed and established as benchmark data from the healthy beam.

2.2 Impact tests on damaged structure

In order to induced different damages, rectangular notches are created on the beam at different locations. Each notch is approximately 3 mm wide and 2 mm deep. Each notch created represents a damage scenario. Thus, nine damage scenarios are recreated in total, from one notch until nine notches on the beam. This is better illustrated in Fig. 2. Then, in the same way as described in the previous section, impact tests are performed and acceleration responses and impact forces are recorded simultaneously. Finally, the FRF at each accelerometer's position for each damage scenario is computed and established as benchmark data from the damaged beam.

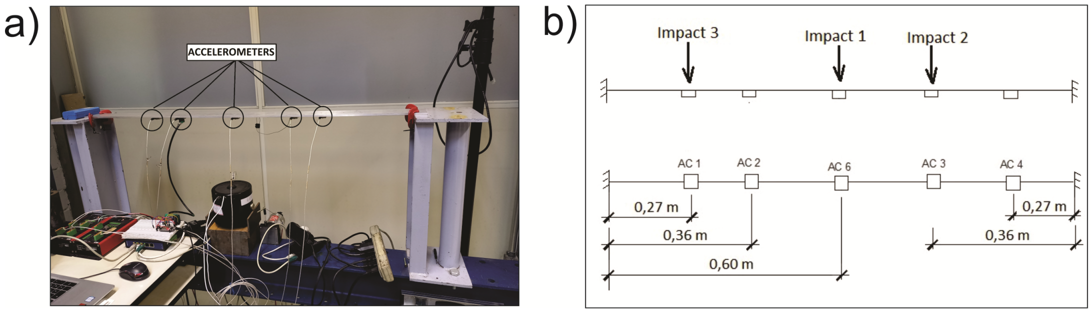

Figure 1. Experimental setup for dynamic testing (Ruiz et al. [10]). a) test structure, b) accelerometers' positions and excitation points.

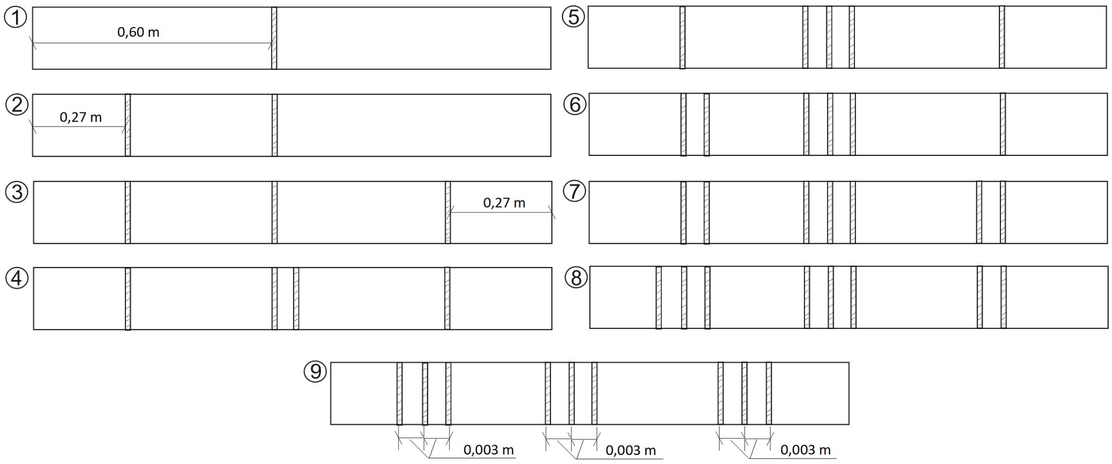

Figure 2. The nine damage scenarios induced on the beam.

3 Numerical model of the beam

The test structure is simulated as a fixed-fixed beam. To obtain its dynamic response, a simplified FE model composed by 120 beam elements and 121 nodes is developed in MATLAB [11]. Only the vertical direction is considered in the analyses. The structure is considered linear before and after damage, therefore, the modal superposition technique is applied. Thus, the dynamic response is obtained by the well-known equation of motion

$$\mathbf{\Phi^T M \Phi \ddot{Y}(t)} + \mathbf{\Phi^T C \Phi \dot{Y}(t)} + \mathbf{\Phi^T K \Phi Y(t)} = \mathbf{\Phi^T P(t)} \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the mass, damping and stiffness matrices of the structure, $\mathbf{\ddot{Y}}$, $\mathbf{\dot{Y}}$, and $\mathbf{Y}$ the generalized coordinates of the acceleration, velocity and displacement vectors in the direction of the applied force and $\mathbf{P(t)}$ is the excitation forced.

Then, applying the orthogonality properties of the modal shapes $\mathbf{\Phi^T}$ normalized in relation to the mass matrix

$$\begin{aligned} &\mathbf{\Phi^T M \Phi} = \mathbf{I} \\ &\mathbf{\Phi^T C \Phi} = 2\varepsilon_n \omega_n \\ &\mathbf{\Phi^T K \Phi} = \omega_n^2 \mathbf{\Phi^T M \Phi} \end{aligned} \tag{2}$$

By introducing eqs. (2) into eq. (1), the equation of motion for each vibration mode i, from i=1 to i=n, can be expressed as

$$\ddot{Y}_i(t) + 2\varepsilon_i \omega_i \dot{Y}_i(t) + \omega_i^2 Y_i(t) = \Phi_i^T P(t) \tag{3}$$

Then, by solving eq. (3) using the Runge–Kutta fourth order integration method, the acceleration, velocity, and displacement at the required points of the structure are obtained, respectively, as

$$\ddot{\mathbf{v}}_{\mathbf{e}} = \sum_{i=1}^{n} \ddot{\mathbf{Y}}\Phi_i \qquad \dot{\mathbf{v}}_{\mathbf{e}} = \sum_{i=1}^{n} \dot{\mathbf{Y}}\Phi_i \qquad \mathbf{v}_{\mathbf{e}} = \sum_{i=1}^{n} \mathbf{Y}\Phi_i \tag{4}$$

Finally, the FRF is estimated with the help of MATLAB functions as

$$\mathrm{FRF} = \left| \left(\mathrm{S}_{\mathbf{vp}} \odot \left| \frac{1}{\mathrm{S}_{\mathbf{vp}}} \right| \right) \odot \sqrt{\mathrm{S}_{\mathbf{vv}}} \odot \frac{1}{\mathrm{S}_{\mathbf{pp}}} \right| \tag{5}$$

where $\mathrm{S}_{\mathbf{vp}}$ is the cross power spectral density of the acceleration response to the excitation force, and $\mathrm{S}_{\mathbf{vv}}$, $\mathrm{S}_{\mathbf{pp}}$ are the power spectral densities of the acceleration and force, respectively.

4 Training dataset

The computational model presented in the previous section is first adjusted to match the experimentally measured FRFs. Figure 3 shows the numerical-experimental comparison of FRFs.

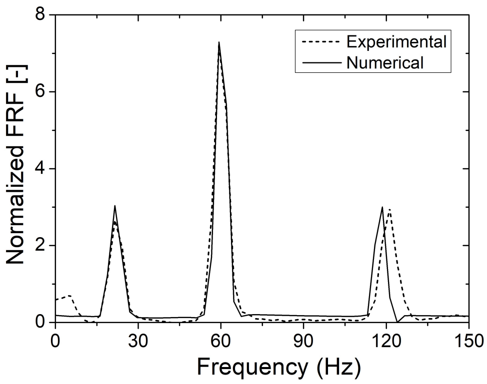

Figure 3. Comparison between numerical and experimental FRFs.

Then, the adjusted model is used to simulate the system response for the different damage scenarios. The damage is introduced in the model through the local reduction of the cross-sectional area (A). Thus, a reduction coefficient r that varies from 0-1, where r =0 represents a completely lost cross-sectional area and r =1 the condition of the healthy structure is used. This type of penalization coefficient is adopted based on the literature (Ritto and Rochinha [12]). In the local stiffness matrix (corresponding damaged element), the value of r is multiplied by A.

In this study, two reduction levels of cross-sectional area are considered, 30 % and 70 %. When 30% damage is considered in a certain element, r assumes a value of 0.7 (1-0.30) and when 70% damage is considered r assumes a value of 0.3 (1-0.70). Thus, the local stiffness matrix is modified. It is important to highlight that, although the reduction of cross-sectional area in the real beam is much smaller, the used of a simplified unidimensional FE model justifies the adopted values. The focus of this study is to evaluate the potential of machine learning approaches for damage detection based on a simplified model, which consumes much less simulation time.

The FRFs of each damage scenario are calculated and normalized by means and standard deviation. The normalized FRF amplitudes of the three first vibration modes are used as damage indexes. To consider in some form uncertainties or noise coming from different sources, it is assumed that the normalized amplitudes are random variables that follow a normal distribution, with mean equal to the nominal value and a certain standard deviation. Two different standard deviations values are considered, 1% and 2 % of nominal values, which are referred to 1% uncertainty and 2 % uncertainty henceforth. Thus. the FRF amplitudes are randomly generated within these distributions.

Multiple simulations are performed to construct several datasets to train machine learning classifiers. Different datasets are constructed, dataset 1 considers 30% reduction of cross-sectional area with 1% uncertainty, dataset

2 considers 30% reduction of cross-sectional area and 2% uncertainty, dataset 3 considers 70% reduction of cross-sectional area with 1% uncertainty, dataset 4 consider 70% reduction of cross-sectional area with 2% uncertainty, and dataset 5 considers all data. Each dataset comprises 50 random samples of each of the nine damage scenarios. At first instance, it is considered nine damage levels, from one to nine damaged elements. These are labeled as 1%,2%...9% damage. Then, to improve the classification process it is considered only three damage levels, labeled as 1% up to three damaged elements, 2% up to six damaged elements, and 3% up to nine damaged elements. In total, ten different datasets are used to train the machine learning classifiers (see Tab. 2). All samples of damaged and undamaged data are presented in Fig. 4, organized to consider nine damage levels (Fig. 4a) and three damage levels (Fig. 4b).

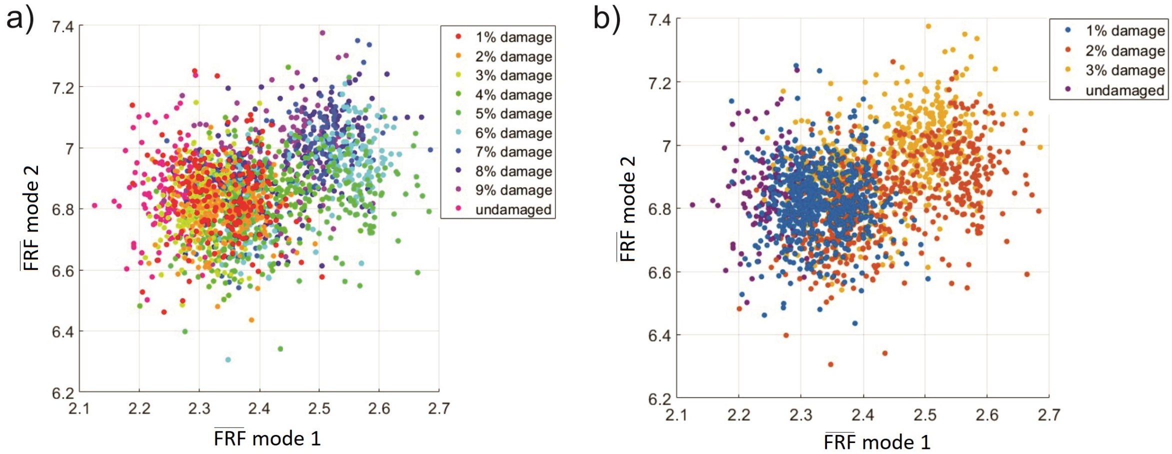

Figure 4. Damaged and undamaged samples generated from numerical simulations, a) dataset considering 9 damage levels and b) dataset considering 3 damage levels.

The main goals of this damage detection process are to identify the presence and to quantify the level of damage according to the number of damaged elements. For this purpose, several well-known classifiers are tested with the help of the statistics and machine learning toolbox from MATLAB, using the constructed datasets. Three types of classifiers are evaluated, which are Decision Tree (DT), Support Vector Machine (SVM), and K-Nearest Neighbors (KNN). A brief description of each algorithm is provided next.

DT is a decision support hierarchical model that uses a tree-like model of decisions and their possible consequences. The goal is to create a model that predicts the value of a target variable by learning simple decision rules inferred from the data features. A tree can be seen as a piecewise constant approximation. The capabilities of decision tree ensembles for structural damage detection have been demonstrated in Mariniello et al. [13].

SVM method is used to classify the samples by finding an optimal hyperplane in an n-dimensional space that distinctly classifies the data points. In the hyperplanes that can be classified, there are two hyperplanes that are in contact with two respective classes of data. The optimal hyperplane is between them and is the one that represents the largest separation or margin between the two classes (Hou et al. [14]). SVM maps the sample data to a high dimensional space through a kernel function takes low-dimensional input space and transforms it into higher-dimensional space.

The KNN algorithm is typically used as a classification algorithm. It assumes that similar things exist in close proximity. In other words, similar things are near to each other. The output of KNN determines, from the k-most similar instances, the class with the highest frequency. This is done on the basis of a majority vote, i.e., by having each instance vote for its class. The class with the highest number of votes is the predicted class.

5 Results and discussion

Table 2 shows the accuracy obtained with tested classifiers for each of the training datasets, which is calculated as the number of correct predictions out of total samples. Here, 5-fold cross validation was employed.

It can be observed in Tab. 2 that, on average, SVM (quadratic) is the classifier that best performed in terms of accuracy. The best performance using numerical data is obtained considering three damage levels, 70 % of area reduction and 1 % of uncertainty. It can also be observed that, as expected, the larger the level of uncertainty the

Table 2. Accuracy obtained by different classifiers for each of the training dataset.

Classifier		Accuracy	(9 damage levels)		
	30% area red. 1% uncert.	*30% area red. 2% uncert.*	*70% area red. 1% uncert.*	*70% area red. 2% uncert.*	*all data*
Decision Tree	64.0 %	36.0 %	84.6 %	34.8 %	52.8 %
SVM (linear)	84.0 %	51.4 %	97.8 %	50.0 %	57.5 %
SVM (quadratic)	83.0 %	49.2 %	97.0 %	46.2 %	65.2 %
SVM (cubic)	78.4 %	45.4 %	96.8 %	43.4 %	67.0 %
SVM (fine gaussian)	49.8 %	26.8 %	82.4 %	25.0 %	51.1 %
SVM (medium gaussian)	85.0 %	49.6 %	97.6 %	48.8 %	68.2 %
SVM (coarse gaussian)	81.2%	51.8 %	97.0 %	51.8 %	54.8 %
KNN (fine)	73.4%	40.0 %	91.6 %	38.4 %	58.6 %
KNN (medium)	77.8%	43.4 %	94.6 %	43.8 %	63.7 %
KNN (coarse)	81.0 %	47.2 %	81.2 %	46.2 %	63.6 %
		Accuracy	(3 damage levels)		
Decision Tree	85.2 %	57.8 %	90.0%	75.4 %	73.5 %
SVM (linear)	96.2 %	76.2 %	96.8 %	83.6 %	71.0 %
SVM (quadratic)	95.8 %	76.6 %	99.4 %	88.4 %	77.3 %
SVM (cubic)	95.0 %	75.2 %	98.0 %	87.6 %	80.4 %
SVM (fine gaussian)	65.2 %	53.2 %	88.4 %	64.8%	69.5 %
SVM (medium gaussian)	96.6 %	79.0 %	99.2 %	89.8 %	81.3 %
SVM (coarse gaussian)	94.8%	75.2 %	96.4 %	84.0 %	70.0 %
KNN (fine)	95.0%	69.0 %	98.4 %	82.6 %	76.8 %
KNN (medium)	93.0%	72.6 %	97.6 %	87.4 %	80.8 %
KNN (coarse)	84.2 %	68.0 %	77.6 %	71.2 %	78.8 %

worse the accuracy, and the larger the cross-sectional area reduction the better the accuracy.

Finally, to test the prediction capability of trained classifiers with new data, the normalized FRF amplitudes from experimental tests are used as input to evaluate the prediction of those classifiers with the higher accuracy for each dataset. In other words, the classifier that best performed for each of the training datasets is used to make predictions using the experimentally measured FRFs. As a result, nine out of ten classifiers succeed in identifying the presence of damage. However, the performance is low in quantifying the level of damage. Classification results of the classifier that provided the best predictions are shown in Tab. 3. These were obtained using SVM (medium gaussian), trained with dataset using all data and 2 % of uncertainty, and considering three damage levels in the classification process. It can be seen in Tab. 3, that six data samples are correctly classified out of ten considered, which means 60 % of accuracy.

Table 3. The best prediction results using experimental data as input (Medium Gaussian SVM classifier).

Exp. data samples	True	Predicted	Label
1	undamaged	undamaged	correct
2	1% damage	2% damage	wrong
3	1% damage	1% damage	correct
4	1% damage	1% damage	correct
5	2% damage	2% damage	correct
6	2% damage	2% damage	correct
7	2% damage	2% damage	correct
8	3% damage	2% damage	wrong
9	3% damage	2% damage	wrong
10	3% damage	2% damage	wrong

6 Conclusions

In this paper, numerical studies and experimental tests were conducted for structural damage identification and quantification based on FRFs. The results confirm that normalized FRF amplitudes can be utilized as damage indexes. Uncertainties were introduced considering FRF amplitudes as random variables. Numerical results showed that the larger the level of uncertainty the lower the accuracy. On the other hand, the larger the reduction in the cross-sectional area the higher the accuracy. SVM (quadratic) was the classifier that best performed using numerical data, with 99,4 % of accuracy. Also, experimental FRF data were used as input samples to validate the trained classifiers. The results obtained demonstrated that even a simplified model, that represents the real structure to the extent possible, is able to detect the presence of damage. However, to quantify the level of damage more sophisticated computational models are needed, since the best accuracy obtained was 60 %, and considering three damage levels only in the classification process. In addition, the classifier with higher accuracy from numerical studies did not show the best performance in classifying experimental damage scenarios properly. This suggests that when simplified numerical models are used, it is important to consider uncertainties to improve the prediction capability.

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References

[1] H. R. Ahmadi, N. Mahdavi, and M. Bayat. A novel damage identification method based on short time fourier transform and a new efficient index. *Structures*, vol. 33, pp. 3605–3614, 2021.

[2] M. Dilena, M. Limongelli, and A. Morassi. Damage localization in bridges via the frf interpolation method. *Mechanical Systems and Signal Processing*, vol. 52-53, pp. 162–180, 2015.

[3] J. He and Y. Zhou. A novel mode shape reconstruction method for damage diagnosis of cracked beam. *Mechanical Systems and Signal Processing*, vol. 122, pp. 433–447, 2019.

[4] H. Aied, A. González, and D. Cantero. Identification of sudden stiffness changes in the acceleration response of a bridge to moving loads using ensemble empirical mode decomposition. *Mechanical Systems and Signal Processing*, vol. 66-67, pp. 314–338, 2016.

[5] J. Li, S. Law, and H. Hao. Improved damage identification in bridge structures subject to moving loads: Numerical and experimental studies. *International Journal of Mechanical Sciences*, vol. 74, pp. 99–111, 2013.

[6] O. Avci, O. Abdeljaber, S. Kiranyaz, M. Hussein, M. Gabbouj, and D. J. Inman. A review of vibration-based damage detection in civil structures: From traditional methods to machine learning and deep learning applications. *Mechanical Systems and Signal Processing*, vol. 147, pp. 107077, 2021.

[7] R. P. Bandara, T. H. Chan, and D. P. Thambiratnam. Structural damage detection method using frequency response functions. *Structural Health Monitoring*, vol. 13, n. 4, pp. 418–429, 2014.

[8] R. SAMPAIO, N. MAIA, and J. SILVA. Damage detection using the frequency-response-function curvature method. *Journal of Sound and Vibration*, vol. 226, n. 5, pp. 1029–1042, 1999.

[9] U. Lee and J. Shin. A frequency response function-based structural damage identification method. *Computers & Structures*, vol. 80, n. 2, pp. 117–132, 2002.

[10] D. V. Ruiz, de C. S. C. Bragança, B. L. Poncetti, T. N. Bittencourt, and M. M. Futai. Vibration-based structural damage detection strategy using frfs and machine learning classifiers. *Structures*, vol. 59, pp. 105753, 2024.

[11] MATLAB. *version 9.12.0.2039608 (R2022a) Update 5*. The MathWorks Inc., Natick, Massachusetts, 2022.

[12] T. Ritto and F. Rochinha. Digital twin, physics-based model, and machine learning applied to damage detection in structures. *Mechanical Systems and Signal Processing*, vol. 155, pp. 107614, 2021.

[13] G. Mariniello, T. Pastore, C. Menna, P. Festa, and D. Asprone. Structural damage detection and localization using decision tree ensemble and vibration data. *Computer-Aided Civil and Infrastructure Engineering*, vol. 36, n. 9, pp. 1129–1149, 2021.

[14] K. Hou, S. Guanghui, W. Haiming, Z. Le, Z. Qiang, W. Shuang, and H. Wei. Research on practical power system stability analysis algorithm based on modified svm. *Protection and Control of Modern Power Systems*, vol. 11, n. 3, 2018.