Hydraulic Modeling of Water Distribution Networks: Hardy-Cross Method Implemented in Python

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Abstract. Distribution network is a part of the water supply system that, formed by pipes and accessory parts, aims to provide potable water to consumers continuously with recommended quantity and pressure [1]. However, due to the exponentially growing demand for potable water, the design of network layouts has become increasingly complex nowadays. In this regard, due to regional parameters such as topography and supply size, the use of meshed networks is feasible. Thus, this study aims to evaluate the performance of a meshed distribution network of a fictitious layout using the Hardy-Cross method through system modeling in a Python language program. Furthermore, the computational program was validated by comparing its results with those provided by the EPANET software. The methodology was based on the bibliographic review of issues related to solving meshed networks using the Hardy-Cross method, where a fictitious problem was modeled with parameters specified in NBR 12218/2017 [2]. Thus, this method was implemented in a computational program in Python language, and from this, the results related to the established layout were obtained. Additionally, the same layout was built in the EPANET software, and its results were used to validate the computational program. Therefore, it was possible to conclude that the computational program presented satisfactory results when compared to the EPANET software data. The recent performance advances in the Python language, adds greater ease of use, result accuracy, modeling problem speed, and expands the possibilities of evaluating networks. Finally, it was found that both modeling approaches are in accordance with the reference values established by Tsutiya [1] and recommended by NBR 12218/2017 [2].

Keywords: Mesh networks; Optimization; EPANET.

1 Introduction

Water is an extremely important resource for the continuity of life on Earth. Thuas, Kiliç [3] shows that every decision to be taken about this resource is vital, in a way that it is impossible to dissociate mankind and nature. According to the author, the hydric resources are in danger from many factors, such as climate changes, severe pollution of supply sources and the uneven distribution of these resources.

As stated by Rai and Sanap [4], imagining life on Earth without water is an impossible task, once it is an essential element to the human being. In this vein, the authors affirm that the water is indispensable for many daily activities, such as cooking, washing, drinking, taking showers, maintaining basic sanitation and practicing agriculture.

In light of this, Sousa and Cunha [5] affirm that the distribution networks are one of the most important parts of public water supply systems. In this way, investments linked to their constructions demand that the solutions adopted have minimal cost, that is, optimized solutions that fulfill the minimal requirements established for reglementations and norms of water distributions with the minimal cost associated.

According to Tsutiya [1], water distribution networks are a part of a supply system destined to furnish drinkable water to the consumers in a continuous and adequate way. In this sense, still according to the author, the distribution networks can be classified in three different types: branched networks, mashed networks or mixed networks.

Demir, Yetilmezsoy and Manav [6] assert that the most built water distribution method around the world are the meshed networks. Therefore, the authors show that this specific hydraulic design displays a series of complexities attached to its conception and operation, demanding more sophisticated mathematical modelings.

One of the most suggested approaches in the literature is the Hardy-Cross method, which is based on interactive calculation to find the water flows in pipes that form the mashes. The method uses a correction term to update the water flow in each mesh [7]. Dumka et al. [8] exhibit that the Hardy-Cross method is a type of network analysis that cannot be solved analytically for each node, once its set of equations must be solved for each mesh.

Sodek et al. [9] presents that, with the most recent advances on digital computation, it is possible to develop significant improvements in the proposed method, since the resolution of the method's set of equations can be done faster and in a more efficient way using computational methods.

Therefore, the present work has the objective to develop a computational program in Python language in order to characterize hydraulic parameters, such as water flow, velocity and pressure, in a meshed network with a single loop, while comparing the performance of the program with the software "EPANET", once the modeling of this type of problem in Python promotes the interdisciplinarity within Civil Engineering education and can be more user friendly, as the usage of EPANET requires a specific type of knowledge that enables the application of the software in practical problems.

2 Methods

The present work's methodology is based on a bibliographic revision on problems involving mashed distribution networks and how to solve them utilizing the Hardy-Cross method. In this way, a hydraulic simulation of a fictitious problem was carried out using the Python language. Additionally, for comparison purposes, the same problem, with the same parameters, was modeled in the EPANET software. The parameters used for the problem modeling were based on the provisions of the NBR 12218/2017 standard [2].

2.1 Hardy-Cross method

The Hardy-Cross method is an iterative procedure used for the measuring of looped networks. To solve this type of problem, it is necessary to establish a set of equations based on the principles of flow continuity at the nodes and energy conservation in each loop. These equations are explicit below:

$$\Sigma Q = 0 \tag{2.1}$$

$$\Sigma \Delta h = 0 \tag{2.2}$$

Expressions eq. 2.1 and eq. 2.2 forms a system of linear equations, which the solution is reached through iterations. Thus, the lengths, diameters, and materials of the pipes, as well as the water level of the reservoir, are provided. However, due to the unknown nature of the flow rates, an arbitrary flow guess is admitted in one of the pipes. In this way, by meeting the standards established by eq. 2.1, it is possible to estimate the flow rates throughout the network by adopting a positive convention for the direction of flow (clockwise or counterclockwise). Once all the parameters are known, it is possible to determine the head losses in each pipe using the Hazen-Williams equation, indicated by eq. 2.3:

$$\Delta h \ = \frac{10{,}641 \cdot Q^{1{,}85} \cdot L}{C^{1{,}85} \cdot D^{4{,}87}} \tag{2.3}$$

Furthermore, once the positions of the load points (nodes) are known, it is necessary to verify the compliance with eq. 2.1 and eq. 2.2. Generally, in the first iteration, eq. 2.2 is not satisfied. Hence, a corrective term is applied to the flow rates, as defined by eq. 2.4:

$$\Delta Q \ =- \frac{\Sigma \Delta h}{n \Sigma \frac{\Delta h}{Q}} \tag{2.4}$$

In this sense, the flow rates are adjusted in each pipe until both initial conditions established by eq. 2.1 and eq. 2.2 are satisfied.

2.2 Problem's design

The chosen problem for resolution involves a distribution system with a single mesh, supplied by a constant-level reservoir. The loop consists of four nodes and five pipes, arranged in the configuration shown in Figure 1:

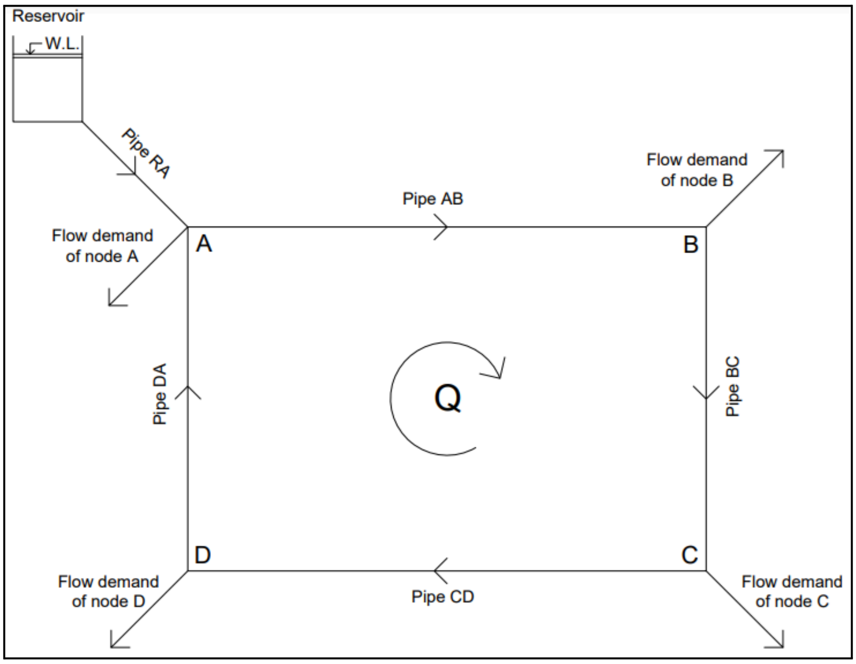

Figure 1. Meshed network scheme

The program can solve the type of problems with the same configurations. In this sense, to find a result for a real application, some parameters were established for solving the described problem. These parameters can be found below in Tables 1 and 2.

Table 1.Pipes' parameters

Pipe	Length (m)	Diameter (mm)	Hazen-Williams' constant
RA	100	400	120
AB	1200	300	120
BC	180	250	120
CD	1200	250	120
DA	180	250	120

Table 2. Nodes' parameters

Node	Level (m)	Demands (L/s)
W.L.	120	-
A	105	15
B	95	30
C	100	60
D	97	15

2.3 Implementation of Hardy Cross Method in Python

Remark 1: User interface. First, the input commands for the program's interactive user interface were modeled, gathering the necessary information to determine the flow rates and velocities in the pipes, besides the pressures in the nodes. At this stage, the user is responsible for providing the diameters, lengths, roughness coefficients of the pipes, the elevations and demands of the nodes, as well as supplying an initial guess to the flow rate on pipe AB. For the example solved, the initial guess adopted was 70 L/s.

Remark 2: Determination of the results on an initial guess. In this stage of the code, the equations that determine the flow rates and head losses in the pipes of the loop were modeled. The program uses an initial flow guess in pipe AB so that, from eq. 2.1, it is possible to calculate the flow in the remaining pipes of the loop. The program then uses eq. 2.3 to calculate the head loss in the pipes. With the result of the initial guess, the program uses the 'SUM' and 'SUM2' functions to determine, respectively, the numerator and denominator of eq. 2.4. Subsequently, the program uses the previous result to calculate the flow correction term defined by eq. 2.4.

Remark 3: Application of Hardy-Cross method. Based on the results obtained from the initial guess, the program uses the 'while' function to iterate and redefine the flow rates and head losses until the sum of these reaches a value very close to zero, thereby satisfying eq. 2.2. Due to inherent limitations of the computational application, it is impossible to assume that the error is zero. For this reason, when the eq 2.2 is lower than 1.E-10, the program finishes the 'while' function and uses the results obtained by the interactive process from the meshed network to determine the flow rate from the reservoir to node A.

```
while abs(SUM)>1.E-10:
  for i in range(4):
    FLOWS[i]-=DELTA*1000

  for i in range (4):
    LOSSES[i]=((10.641*((abs(FLOWS[i])/1000)**n))/((RUG[i]**1.85)*(DIAMETERS[i]**4.87)))*LENGHTS[i]
    if FLOWS[i]<0:
      LOSSES[i]=LOSSES[i]*(-1)

  for i in range (4):
    RATIOS[i]=LOSSES[i]/(abs(FLOWS[i])/1000)

  SUM=0
  for i in range (4):
    SUM+=LOSSES[i]

  SUM2=0
  for i in range (4):
    SUM2+=abs(RATIOS[i])*n

  DELTA=SUM/SUM2
```

Figure 2. Hardy-Cross modeling

Remark 4: Determination of velocities and pressures. Once the flow rates and head losses are determined, it is possible to find the velocities and pressures in the pipes using the following equations:

$$V = \frac{Q}{A} \tag{2.5}$$

$$P_{downstream} = P_{upstream} + (H_{upstream} - H_{downstream}) - \Delta h_{pipe} \tag{2.6}$$

It is important to reiterate that, for the pressure calculations, the water level elevation was used as the reference point, meaning the pressure is considered zero at that level.

Remark 5: Results presentation. In possession of all results, the "prettytable" library was used to plot and present to the user the parameters calculated along the interactive process. All this data is presented in two separated sheets, one of them containing the pipes' information and the other containing the nodes' information. Beyond that, the output shows how many iterations the code did and the final value of 'DELTA' (eq. 2.4).

Remark 6: Treatment for more complex meshes As seen throughout this work, the developed program addresses only a single, simple problem with a well-defined topology. However, for the development of more complex problems involving multiple networks and arbitrary topologies, it is necessary to consider the existence of common pipelines between networks [8]. Additionally, when dealing with problems involving multiple networks, all loops must be analyzed simultaneously to achieve total convergence.

2.4 Implementation of the problem on EPANET software

In order to verify the reliability and effectiveness of the results obtained from the modeled code, the same problem was input into the EPANET software, as shown on the Figure 3:

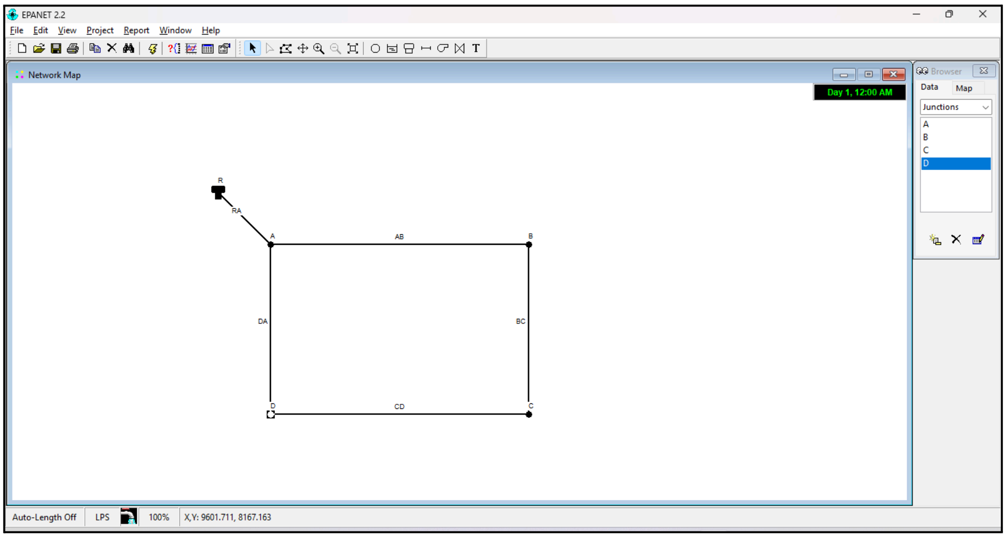

Figure 3. EPANET implementation

3 Results and discussions

3.1 Program's results

As a pedagogical demonstration of the application of the Hardy-Cross method, the flow rates at each iteration were obtained to understand how the method's application achieves the convergence of results. After data input, the result obtained by the developed program can be found on Figure 4:

The program found the solution after 5 iterations
The solution for the modeled network is presented in the following tables:

Iteration	Sum of losses (m)	Delta (L/s)	Flow AB (L/s)	Flow BC(L/s)	Flow CD(L/s)	Flow DA(L/s)
1	3.6862175874659995	-13.170655927067163	70.0	40.0	-20.0	-35.0
2	-0.2385306785420953	0.7568600264731961	56.8293	26.8293	-33.1707	-48.1707
3	-0.0007110421797767996	0.002269694187051641	57.5862	27.5862	-32.4138	-47.4138
4	-6.43091280227992e-09	2.05282750476921e-08	57.5885	27.5885	-32.4115	-47.4115
5	-2.220446049250313e-16	7.087940489392018e-16	57.5885	27.5885	-32.4115	-47.4115

Pipe	Flow (L/s)	Velocity (m/s)	Head Loss (m)
AB	57.59	0.81	3.256
BC	27.59	0.56	0.304
CD	-32.41	0.66	2.732
DA	-47.41	0.97	0.828
RA	120.0	0.95	0.26

Node	Pressure (mH2O)
A	14.74
B	21.48
C	16.18
D	21.91
W.L	0

Note: Positive and negative flows indicates clockwise and counterclockwise orientatios, respectively.

Figure 4. Results obtained by the program

3.2 EPANET's results

After modeling the problem, The result obtained by the EPANET software can be found on Figure 5:

Link ID	Flow LPS	Velocity m/s
Pipe RA	120.00	0.95
Pipe AB	57.58	0.81
Pipe BC	27.58	0.56
Pipe CD	-32.42	0.66
Pipe DA	-47.42	0.97

Node ID	Pressure m
Junc A	14.74
Junc B	21.53
Junc C	16.22
Junc D	21.92

Figure 5. Results obtained by EPANET

3.3 Discussions

After a critical analysis based on the results obtained, the excellence of the program in solving systems, like the problem proposed by the methodology of this paper, can be verified due to the slight divergence between the code and EPANET outcomes. Therefore, these divergences can be noticed more prominently in the calculation of the pressures at the nodes, with a maximum error of 0.25% at node C, compared to the reference value from the EPANET software. Nevertheless, based on the above data, the accuracy of the code is notable due to the very low residual error, inherent to the iteration process. Finally, it is worth noting that the flow rates, the reference parameter in the Hardy-Cross method, presented variations only in the second decimal place, which generated a maximum error of 0.04% in the pipe BC in relation to the reference value presented in EPANET, an extremely satisfactory data that verifies the reliability of the results obtained by the program.

4 Conclusions

In conclusion, it is possible to consolidate the usage of the program, since the developed model is capable of providing data consistent with the EPANET software by solving the system through the mathematical laws that model the simulated hydraulic system.

In agreement with the constructed code, it is evident that the knowledge required to reproduce the results involves topics covered throughout Civil Engineering courses. Thus, there is an interdisciplinarity between the concepts necessary for engineers and the appropriate knowledge associated with modeling water distribution systems, allowing this approach to be used as a practical teaching methodology in the classroom.

The methodology developed during this paper consists in a simplified application utilized to teach about the measurement of water distribution networks. For that reason, as a continuation of this research, the authors suggest the elaboration of algorithms for meshed networks with multiple loops, in addition to the development of a graphical interface for better understanding and interpretation of the results obtained.

Therefore, it is clear that the interface of the developed computational program is more comprehensive and accessible to the user, directly contrasting with the required knowledge base for using the EPANET software, which is much more specialized.

Finally, according to the methodology proposed by this article, it becomes feasible to reproduce and model any problem, in an extremely practical way, that follows the design used as the basis for the code developed throughout the work, allowing for the successful development of various practical applications.

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