STONEHENGE – A toolbox for nonlinear vibration energy harvesting

Americo Cunha Jr[1], João Pedro Norenberg[2], João Victor Peterson[1], Vinicius Gonçalves Lopes[1], Roberto Luo[1], Leonardo de la Roca[1], Marcelo Pereira[1,3], José Geraldo Telles Ribeiro[1]

[1]*Rio de Janeiro State University, Brazil*
Rio de Janeiro - RJ, 20550-900, Brazil
americo.cunha@uerj.br
[2]*São Paulo State University, Brazil*
Ilha Solteira - SP, 15720-000, Brazil
[3]*SENAI Institute, Brazil*
Rio de Janeiro - RJ, 20911-210, Brazil

Abstract. This paper presents STONEHENGE (https://stonehengesuite.org), a comprehensive suite for the nonlinear analysis of energy harvesting systems, specifically designed to study piezoelectric vibration energy harvesters. These systems leverage nonlinearity, introduced through strategically placed magnets, to enhance energy harvesting efficiency across a broad frequency spectrum, though at the cost of increased system complexity. The STONEHENGE toolbox offers a robust set of tools for analyzing and characterizing these complex dynamics, featuring modules for initial value problem analysis, dynamic animation, nonlinear analysis, sensitivity analysis, stochastic simulation, and chaos control. Building on this foundation, recent enhancements have been made to include uncertainty quantification and non-convex optimization, as well as novel models with asymmetries and amplifiers. These additions further extend the toolbox's capabilities, enabling more comprehensive and reliable analysis of nonlinear energy harvesting systems. The paper discusses both the traditional and novel features of STONEHENGE, highlighting its utility in advancing the development and refinement of sustainable energy harvesting technologies.

Keywords: Energy harvesting, Nonlinear dynamics, Optimization, Sensitivity analysis, Chaos control

1 Introduction

In this work, we discuss **STONEHENGE - Suite for Nonlinear Analysis of Energy Harvesting Systems**, a comprehensive toolbox designed to facilitate the study of nonlinear piezoelectric vibration energy harvesters [1]. These harvesters harness kinetic energy from the environment, converting it into electricity through the piezoelectric effect [2]. By introducing nonlinearity via strategically placed magnets, these systems achieve enhanced efficiency across a broad frequency spectrum [3, 4]. However, this improvement comes with the trade-off of increased complexity in system dynamics [5, 6]. To address this complexity and fully realize the potential of nonlinear energy harvesting, the STONEHENGE library was developed [1]. It is tailored to analyze the harvesting performance and dynamic behavior of these systems, enabling users to explore and characterize system dynamics across a wide range of physical parameters and excitation conditions through advanced numerical simulations. Originally, the toolbox encompasses six modules (see Figure 1):

- *Initial Value Problem:* Analyzes system behavior from initial conditions, allowing for a detailed examination of transient and steady-state dynamics.
- *Dynamic Animation:* Provides visual representations of system dynamics, aiding in intuitive understanding and interpretation.
- *Nonlinear Tools:* Equips users with specialized tools to dissect and comprehend the nonlinear aspects of vibration harvesting systems, which is essential for a comprehensive analysis [7].
- *Sensitivity Analysis:* Facilitates the assessment of how system behavior responds to variations in parameters, offering insights into the system's robustness and performance under different conditions [8].
- *Stochastic Simulation:* Enables the exploration of system behavior under stochastic excitation, crucial for understanding real-world operating conditions [9].
- *Chaos Control:* Provides methods for mitigating chaotic behavior within the system, enhancing predictability and stability [10, 11].

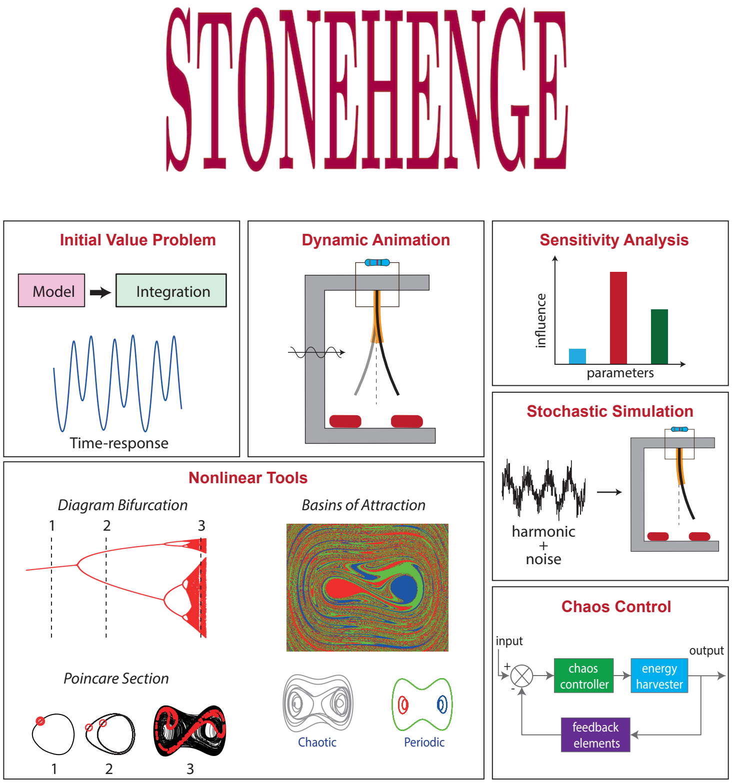

Figure 1. Schematic of the six original modules of STONEHENGE package. For further details about the functionalities and usage, the reader can visit the suite website: https://stonehengesuite.org

Utilizing a bistable oscillator as a benchmark [12, 13], STONEHENGE serves as a valuable resource for the development and refinement of both existing and emerging nonlinear vibration-based energy harvesting systems. By providing a comprehensive toolkit for analysis and characterization, STONEHENGE seeks to catalyze advancements in this field, ushering in an era of more efficient and sustainable energy harvesting technologies [14].

Building upon this robust foundation, we have recently introduced novel enhancements to the STONEHENGE suite, which expand its capabilities, enabling more comprehensive analyses of complex energy harvesting systems. The following sections provide an overview of these enhancements and discuss the new insights they have provided into the dynamics and optimization of nonlinear vibration energy harvesting systems.

2 STONEHENGE Website

Now STONEHENGE suite has a dedicated website, accessible at https://stonehengesuite.org, which serves as a comprehensive resource for users. The site offers an overview of the suite, detailing its features and usage, along with thorough documentation to guide users through its functionalities. It includes sections on reproducibility to ensure consistent and replicable results, as well as information on how to cite the suite in academic work. The website also provides details about the authors and contributors, institutional support, and funding sources that have enabled the development of STONEHENGE. Users can find contact information for support, and a list of references that have utilized STONEHENGE in their research, showcasing its impact in the field. The suite is distributed under the MIT license, ensuring that it is freely available for use, modification, and distribution.

3 Recent Enhancements

Since the initial release of STONEHENGE [1], several targeted enhancements have been made to expand the suite's capabilities in nonlinear energy harvesting analysis. These updates provide advanced tools for optimization, chaos control, probabilistic mapping, uncertainty quantification (UQ), and the analysis of complex systems.

One of the significant additions is the capability to analyze asymmetric bistable energy harvesters, which are characterized by non-symmetric potential wells. These systems exhibit complex dynamic behaviors that can be harnessed for improved energy harvesting performance [15]. To complement this, the global sensitivity analysis module has been expanded, allowing for a detailed evaluation of how variations in system parameters influence performance [8].

The suite has also been enhanced with probabilistic modeling module, enabling the generation of probabilistic maps that provide insights into the dynamic responses of bistable energy harvesters under aleatory uncertainty. This addition is crucial for designing systems that can reliably operate in random environments [16]. Furthermore, the UQ capabilities have been strengthened, offering robust tools for assessing the reliability and performance of energy harvesting systems under various conditions.

To address challenges in optimizing energy harvesting systems, a cross-entropy method has been incorporated into STONEHENGE, facilitating the identification of optimal parameter sets that maximize energy output while managing chaotic dynamics [17]. In parallel, a chaos control module based on digital extended time-delay feedback has been added, providing effective techniques for stabilizing chaotic systems [11].

Additionally, the suite now includes models for piezomagnetic vibration energy harvesters equipped with amplifiers, which enhance energy conversion efficiency by incorporating advanced amplification mechanisms [18]. This feature allows for more refined design and optimization of piezomagnetic systems.

These enhancements broaden the functionality of STONEHENGE, offering researchers and engineers advanced tools for the analysis, optimization, and control of nonlinear energy harvesting systems. The suite's continued evolution ensures it remains a leading resource in the rapidly advancing field of energy harvesting.

4 Example of Application

To illustrate the capabilities of the STONEHENGE suite, we present an application focused on a bistable energy harvester system, which exemplifies the suite's proficiency in handling dynamic responses of systems exhibiting geometric nonlinearity. The MATLAB code provided in `main_pmehna_ivp.m` is employed to simulate the system's behavior, integrating essential parameters such as mechanical damping, piezoelectric coupling, excitation amplitude and frequency, and a nonlinear electromechanical coupling.The simulation process involves solving the system's equations of motion over a specified time interval using a Runge-Kutta (ode45) solver, with the initial conditions set to compute the system's response in terms of displacement, velocity, and voltage, as defined by the equation of motion implemented in `pmehna_eom.m`.

The simulation begins by defining the system parameters, including the mechanical damping ratio, piezoelectric coupling terms, excitation amplitude and frequency, nonlinear electromechanical coupling, and the asymmetric coefficient of the potential energy. These parameters are crucial in capturing the complex behavior of the bistable system, particularly in scenarios where asymmetry and nonlinearity play significant roles in the harvesting process.

The simulation results are graphically represented in Figure 2, which includes two critical plots that provide insight into the system's dynamic behavior. The displacement-time response, shown in Figure 2(a), indicates that the displacement of the harvester remains consistently positive and varies within a narrow range, oscillating between approximately 0.8 and 1.7 units. This behavior suggests that the system is operating in a stable regime, without transitions between potential wells, which is characteristic of a monostable response rather than the expected bistable behavior. The absence of large amplitude oscillations, typically associated with bistable systems that switch between stable states, further confirms this stable, monostable operation. In parallel, the voltage-time response, presented in Figure 2(b), shows the voltage output of the harvester oscillating between approximately -0.1. and 0.1 units. This steady-state periodic response indicates that the piezoelectric element is effectively converting mechanical vibrations into electrical energy. The consistency of the voltage fluctuations, without any chaotic or irregular spikes, reinforces the observation that the system is functioning within a stable, monostable regime. Together, these results suggest that the current configuration of system parameters supports a monostable operation, offering a clear picture of the system's performance under the given conditions.

The MATLAB code is structured to promote clarity and ease of use, with modular functions that allow for straightforward modification and extension. By adjusting the input parameters in `main_pmehna_ivp.m` and `pmehna_eom.m`, users can explore various scenarios, making this example a valuable tool for investigating the effects of nonlinearity and asymmetry in energy harvesting systems. This example demonstrates the interplay between the system's parameters and its dynamic response, offering valuable insights for the design and optimization of bistable energy harvesters. The resulting plots in Fig. 2 provide a visual understanding of the system's performance, facilitating analysis and interpretation of the results.

When the asymmetries in the system are set to zero, the response of the energy harvester changes significantly, as shown in Figure 3 which exhibits a chaotic behavior. This indicates that the system is now operating in a truly bistable regime, where the harvester switches between two stable states.

main_pmehna_ivp.m

```
clc
clear
close all


% program header
% -----------------------------------------------------------
disp(' ---------------------------------------------------')
disp('           IVP Bistable Energy Harvester            ')
disp('                                                    ')
disp(' by                                                 ')
disp(' Joao Pedro Norenberg / Americo Cunha Jr.           ')
disp(' jpcvalese@gmail.com  / americo.cunha@uerj.br       ')
disp(' ---------------------------------------------------')
% -----------------------------------------------------------

Xpar.ksi    = 0.01;  % mechanical damping ratio
Xpar.chi    = 0.05;  % dimensionless piezoeletric coupling term (mechanical)
Xpar.lambda = 0.05;  % dimensionless time constant reciprocal
Xpar.kappa  = 0.5;   % dimensionless piezoeletric coupling term (eletrical)
Xpar.f      = 0.083; % amplitude of excitation
Xpar.Omega  = 0.8;   % frequency of excitation
Xpar.beta   = 0.0;   % nonlinear electromechanical coupling term
Xpar.delta  = 0.15;  % asymmetric coefficient of potential energy
Xpar.phi    = 15;    % bias angle

% time interval integration
tspan = 0:0.01:1000;

% initial condition
x0    = 1;
xdot0 = 0;
v0    = 0;
IC    = [x0 xdot0 v0];

% system response function
[time,Y] = pmehna_eom(Xpar,IC,tspan);

% serial time response
Qdisp = Y(:,1);      % displacement-time of system
Qvelo = Y(:,2);      % velocity-time of system
Qvolt = Y(:,3);      % voltage-time of system

% post-processing
disp(' ---------------------------------------------------')
disp('           post-processing: plotingg                ')
disp(' ---------------------------------------------------')

% plot displament
figure(1)
plot(time,Qdisp,'b','LineWidth',1)
set(gca, 'FontName' , 'Arial' , 'FontSize' ,13 );
xlabel('     time     ', 'FontSize' , 15 , 'FontWeight' , 'bold');
ylabel(' displacement ', 'FontSize' , 15 , 'FontWeight' , 'bold');
grid

% plot voltage
figure(2)
plot(time,Qvolt,'b','LineWidth',1)
set(gca, 'FontName' , 'Arial' , 'FontSize' ,13 );
xlabel('    time  ', 'FontSize' , 15 , 'FontWeight' , 'bold');
ylabel('  voltage ', 'FontSize' , 15 , 'FontWeight' , 'bold');
ylim([-1.2 1.2])
grid

disp(' ---------------------------------------------------')
disp('              successfully finished                 ')
disp(' ---------------------------------------------------')
```

pmehna_eom.m

```
function [time,Y] = pmehna_eom(X,IC,tspan)
% check number of arguments
     if nargin > 3
         error('Too many inputs.')
     elseif nargin < 3
         error('Too few inputs.')
     end

% check number of parameters
    if numel(fieldnames(X)) > 9
        error('Too many parameters inputs.')
     elseif numel(fieldnames(X)) < 9
         error('Too few parameters inputs.')
     end

% model parameters
    ksi    = X.ksi;
    chi    = X.chi;
    lambda = X.lambda;
    kappa  = X.kappa;
    f      = X.f;
    Omega  = X.Omega;
    beta   = X.beta;
    delta  = X.delta;
    phi    = X.phi;

% equivalent dimensionless gravity of ferromagnetic beam
    p      = 0.59;

% ODE solver optional parameters
    opt = odeset('RelTol',1.0e-6,'AbsTol',1.0e-9);

% ODE right hand side (Z = [z; zdot])
    dYdt = @(t,y) [y(2);
               -2.*ksi.*y(2) + 0.5.*y(1).*(1.0+2*delta*y(1)-y(1).^2) + (1+beta*
                   abs(y(1)))*chi.*y(3) + f.*cos(Omega.*t)+p*sin(phi*pi/180);
               -lambda.*y(3) - (1+beta*abs(y(1)))*kappa.*y(2)];


% ODE solver Runge-Kutta45
    [time,Y] = ode45(dYdt,tspan,IC,opt);

end
```

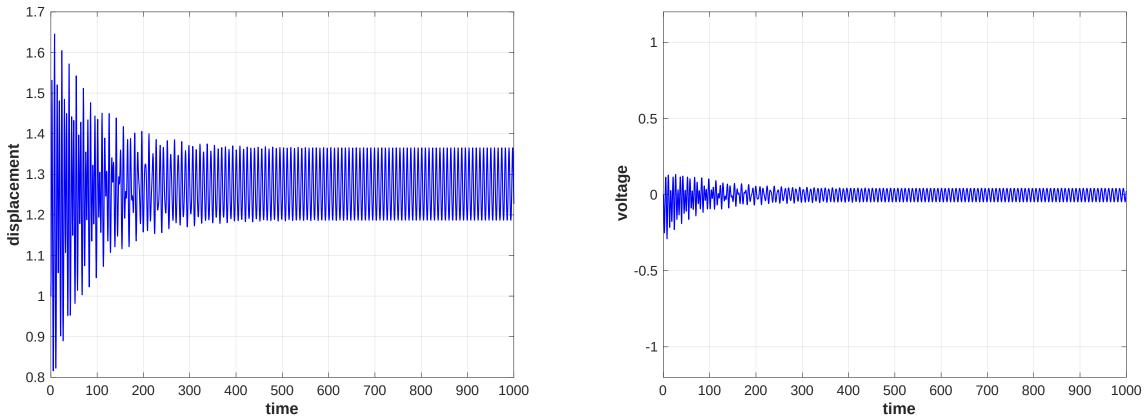

Figure 2. Time response of the bistable energy harvester with asymmetries: (a) Displacement and (b) Voltage.

In summary, this application underscores the effectiveness of the STONEHENGE suite in analyzing nonlinear dynamics and optimizing energy harvesting systems. The ability to simulate and visualize complex behaviors is essential for advancing the development of more efficient and robust energy harvesting technologies.

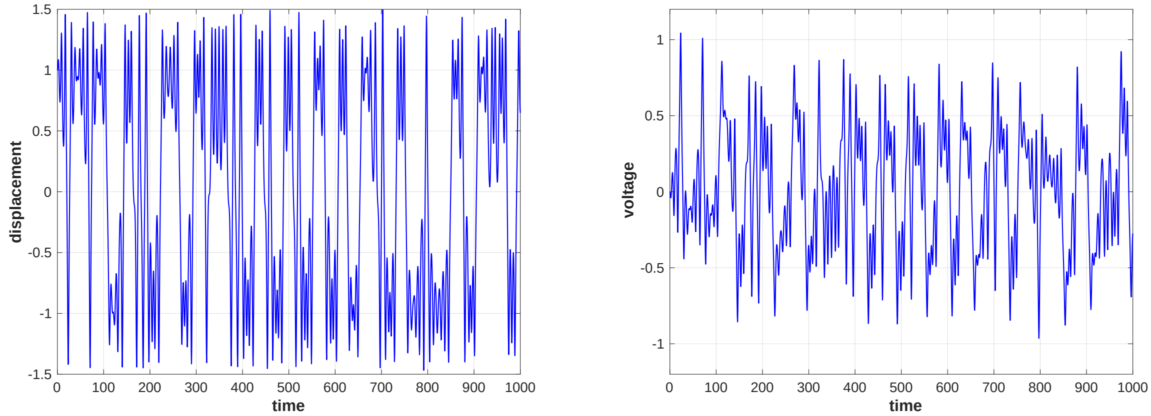

Figure 3. Time response of the bistable energy harvester without asymmetries: (a) Displacement and (b) Voltage.

5 Future Directions

Looking ahead, the STONEHENGE suite will continue to evolve to meet the growing demands of energy harvesting research. One of the key areas of future development is the inclusion of models for multifunctional energy harvesters. These advanced models will integrate the capabilities of energy harvesting with other functionalities, such as vibration mitigation and sensing, within a single device. This integration is particularly promising in the context of metamaterial beams, where bistable resonators can be employed for both energy harvesting and vibration control, as demonstrated in recent studies [19]. By expanding the suite to accommodate these multifunctional systems, STONEHENGE will provide researchers and engineers with powerful tools to design and optimize next-generation energy harvesters that offer enhanced performance and versatility.

6 Final Remarks

The STONEHENGE suite has emerged as a powerful and versatile tool for the analysis and optimization of nonlinear energy harvesting systems. By incorporating a range of modules designed to handle the complex dynamics associated with piezoelectric vibration energy harvesters, STONEHENGE enables researchers and engineers to explore and refine these systems with unprecedented detail and accuracy. The suite's ability to simulate and analyze both traditional and novel configurations, such as asymmetric bistable harvesters and systems with enhanced capabilities like chaos control and probabilistic mapping, makes it a valuable asset in the field.

The recent enhancements to the suite, including tools for uncertainty quantification, and non-convex optimization, and novel models, have further extended its utility, allowing for more comprehensive analyses. As the field of energy harvesting continues to evolve, STONEHENGE remains at the forefront, providing the necessary tools to address the increasing complexity of these systems.

Looking forward, the planned developments in multifunctional harvester models promise to expand the suite's applicability even further, integrating energy harvesting with other critical functions such as vibration mitigation and sensing. These advancements may contribute to the development of next-generation energy harvesters that are not only more efficient but also more adaptable to a wide range of practical applications.

In conclusion, STONEHENGE represents step forward in the study of nonlinear energy harvesting systems, offering a comprehensive suite of tools that continue to evolve in line with the latest research and technological advancements of this field. Its continued development will ensure that it remains an essential resource for researchers and engineers working to push the boundaries of energy harvesting technology.

Acknowledgements. The authors acknowledges the generous support from CAPES, Finance Code 001; CNPq, grant 305476/2022-0; and FAPERJ grants 210.167/2019, 211.037/2019, and 201.294/2021.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] J. P. Norenberg, J. V. Peterson, V. G. Lopes, R. Luo, L. de la Roca, M. Pereira, J. G. Telles Ribeiro, and A. Cunha Jr. STONEHENGE — Suite for nonlinear analysis of energy harvesting systems. *Software Impacts*, vol. 10, pp. 100161, 2021.

[2] A. Erturk and D. Inman. Broadband piezoelectric power generation on high-energy orbits of the bistable duffing oscillator with electromechanical coupling. *Journal of Sound and Vibration*, vol. 330, pp. 2339–2353, 2011.

[3] F. Cottone, H. Vocca, and L. Gammaitoni. Nonlinear energy harvesting. *Physical Review Letters*, vol. 102, pp. 080601, 2009.

[4] A. Erturk, J. Hoffmann, and D. Inman. A piezomagnetoelastic structure for broadband vibration energy harvesting. *Applied Physics Letters*, vol. 94, pp. 254102, 2009.

[5] R. L. Harne and K. W. Wang. A review of the recent research on vibration energy harvesting via bistable systems. *Smart Materials and Structures*, vol. 22, pp. 023001, 2013.

[6] P. Mitcheson, P. Miao, B. Stark, E. Yeatman, A. Holmes, and T. Green. MEMS electrostatic micropower generator for low frequency operation. *Sensors and Actuators A: Physical*, vol. 115, pp. 523–529, 2004.

[7] V. Lopes, J. Peterson, and A. Cunha Jr. Nonlinear characterization of a bistable energy harvester dynamical system. In *Topics in Nonlinear Mechanics and Physics*, volume 228, pp. 71–88. Springer, Singapore, 2019a.

[8] J. P. Norenberg, A. Cunha Jr, S. da Silva, and P. S. Varoto. Global sensitivity analysis of asymmetric energy harvesters. *Nonlinear Dynamics*, vol. 109, pp. 443–458, 2022.

[9] V. G. Lopes, J. V. L. L. Peterson, and A. Cunha Jr. The nonlinear dynamics of a bistable energy harvesting system with colored noise disturbances. *Journal of Computational Interdisciplinary Sciences*, vol. 10, pp. 125, 2019b.

[10] L. de la Roca, J. V. L. L. Peterson, M. P, and A. Cunha Jr. Control of chaos via OGY method on a bistable energy harvester. In *Proceedings of the 25th International Congress of Mechanical Engineering (COBEM 2019)*, Uberlândia, Brazil, 2019.

[11] J. G. Telles Ribeiro, M. Pereira, A. Cunha Jr, and L. Lovisolo. Controlling chaos for energy harvesting via digital extended time-delay feedback. *European Physical Journal-Special Topics*, vol. 231, pp. 1485–1490, 2022.

[12] J. V. L. L. Peterson, V. G. Lopes, and A. Cunha Jr. On the nonlinear dynamics of a bi-stable piezoelectric energy harvesting device. In *Proceedings of the 24th International Congress of Mechanical Engineering (COBEM 2017)*, Curitiba, Brazil, 2017.

[13] L. Costa, L. Monteiro, P. Pacheco, and M. Savi. A parametric analysis of the nonlinear dynamics of bistable vibration-based piezoelectric energy harvesters. *Journal of Intelligent Material Systems and Structures*, vol. 32, pp. 699, 2020.

[14] C. Wei and X. Jing. A comprehensive review on vibration energy harvesting: Modelling and realization. *Renewable and Sustainable Energy Reviews*, vol. 74, pp. 1–18, 2017.

[15] J. P. Norenberg, R. Luo, V. G. Lopes, J. V. L. L. Peterson, and A. Cunha Jr. Nonlinear dynamics of asymmetric bistable energy harvesters. *International Journal of Mechanical Sciences*, vol. 257, pp. 108542, 2023a.

[16] J. P. Norenberg, A. Cunha Jr, S. da Silva, and P. S. Varoto. Probabilistic maps on bistable energy harvesters. *Nonlinear Dynamics*, vol. 111, pp. 20821–20840, 2023b.

[17] A. Cunha Jr. Enhancing the performance of a bistable energy harvesting device via the cross-entropy method. *Nonlinear Dynamics*, vol. 103, pp. 137–155, 2021.

[18] J. P. Norenberg, A. Cunha Jr, P. Wolszczak, and G. Litak. Piezomagnetic vibration energy harvester with an amplifier. *Theoretical and Applied Mechanics Letters*, vol. 13, pp. 100478, 2023c.

[19] J. P. Norenberg and A. Cunha Jr. Vibration mitigation and energy harvesting with bistable resonators in metamaterial beams. In *27th International Congress on Mechanical Engineering (COBEM 2023)*, Florianópolis, Brazil, 2023.