Analysis of dynamic response on aircraft runway and taxiway bridges

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Abstract. This paper presents a computational analysis of the dynamic responses of an aircraft bridge subjected to a moving mass and corresponding load in conditions of taxiway and runway, moving through its entire length. It aims to understand the dynamic behavior of the bridge and to determine its transversal response. The Euler-Bernoulli beam model is here, discretized in finite elements and a dynamic algorithm was applied, using numerical integration by Newmark's Method for the solution of ordinary differential equations to obtain the transversal displacements of the structure in the time domain, to evaluate its responses comparing the results with different velocities, and the effect of contribution of moving mass. The main objective is to apply the results obtained with the method to obtain displacements due to moving masses and corresponding loads contributing to the design of aircraft bridges with reliability.

Keywords: Aircraft bridges. Structural Dynamics. Moving loads.

1 Introduction

The necessity of space for the development of runways and taxiways cause the need to build aircraft bridges, there are notable cases, such as the example of Funchal airport, on the island of Madeira in Portugal, where the runway landing area was extended across a section of the bay in an elevated manner, with the construction of a bridge, 57m above a shallow-water bay, extending it for 1000m as shown in Figure 1. Multiple taxiway bridges cases are observed, such as Phoenix Sky Harbor International Airport, where a series of taxiway bridges over roads between terminals and connect the north and south airfields, or Frankfurt Airport, where there are a few taxiway bridges connecting to the northwest runway bridge.

Some of the loads that structures are submitted vary as function of time, like the motion of people and vehicles, wind loads, impact, earthquakes, and even projectiles or rockets. The development of trustful models that evaluate the responses of structures is essential to warrant the security of them. The displacement of loads in structures causes dynamic responses in the form of vibrations that can produce excessive displacements, which can cause the collapse of the structures.

Bajer and Dyniewicz [1] point out that Saller [2] made the first study to address the problem of mobile masses, and although these studies began in the early part of the last century and there are references such as Fryba [3] and Rao [4] present analytical solutions to the moving load problems applied in models with different boundary conditions aiming to find the responses of these structures. Even today there are few resources for analysis of mobile loads implemented in commercial software.

This paper presents a case study, in which a three span continuous beam is subjected to a moving load and corresponding moving mass in longitudinal motion and aims to find the transverse responses of the structure for different velocities to find the maximum displacement of the structure.

Figure 1. Funchal Airport runway extension[5].

2 Modelling

The studied system is shown in Figure 2. It consists of an Euler-Bernoulli beam model with six spans, with length L, modulus of elasticity E, cross section area A, moment of inertia about the y-axis I and density ρ. Along the beam, the aircraft is considered as a moving mass M travelling with constant velocity v(t).

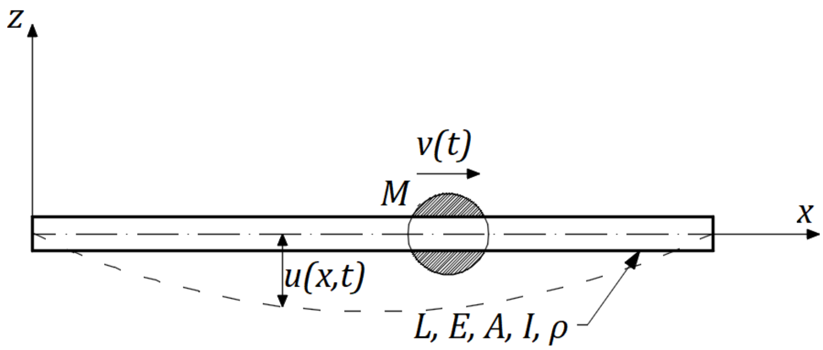


Figure 2. Simply supported beam subjected to a moving mass system.

The model is discretized and analyzed using the finite element method where the real structure is represented as a model consisted of several elements, as shown in Figure 3, with several degrees of freedom. Each element has mass, stiffness and corresponding damping, leading to the generation of matrices of mass, stiffness and damping of each element.

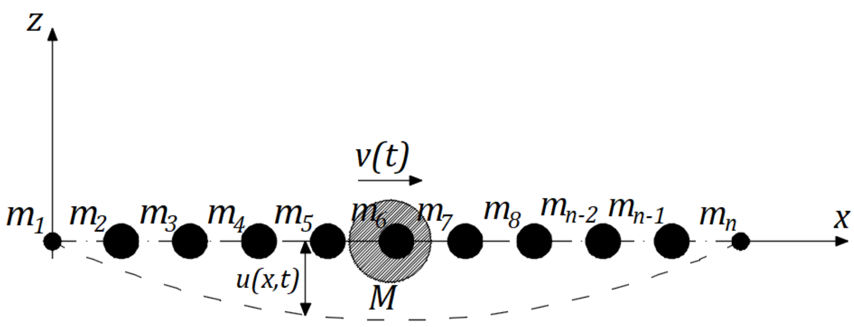


Figure 3. Discretized model

For the application of the forces in the regions between the nodes, the equivalent distribution of the forces is performed as a function of the distance between the load and the nodes, as shown in Figure 4

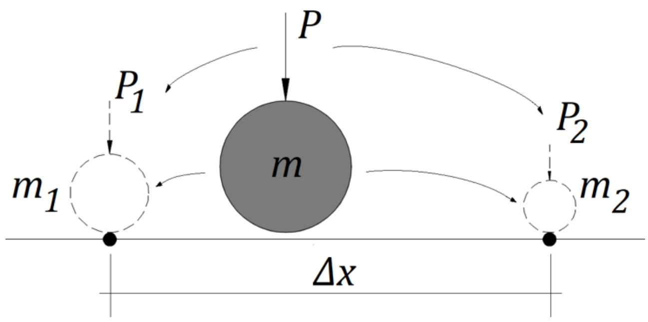


Figure 4. Distribution of masses and loads between nodes

When discretizing the structure, considering a Euler-Bernoulli beam element, each of the elements is associated with a local stiffness matrix

$$[k] = E\begin{bmatrix} \frac{A}{L} & 0 & 0 & -\frac{A}{L} & 0 & 0 \\ 0 & \frac{12I}{L^3} & \frac{6I}{L^2} & 0 & -\frac{12I}{L^3} & \frac{6I}{L^2} \\ 0 & \frac{6I}{L^2} & \frac{4I}{L} & 0 & -\frac{6I}{L^2} & \frac{2I}{L} \\ -\frac{A}{L} & 0 & 0 & \frac{A}{L} & 0 & 0 \\ 0 & -\frac{12I}{L^3} & -\frac{6I}{L^2} & 0 & \frac{12I}{L^3} & -\frac{6I}{L^2} \\ 0 & \frac{6I}{L^2} & \frac{2I}{L} & 0 & -\frac{6I}{L^2} & \frac{4I}{L} \end{bmatrix} \tag{1}$$

As the local mass matrix,

$$[m] = \rho AL \begin{bmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{13}{35} + \frac{6I_z}{5AL^2} & \frac{11L}{210} + \frac{I_z}{10AL} & 0 & \frac{9}{70} - \frac{6I_z}{5AL^2} & -\frac{13L}{420} + \frac{I_z}{10AL} \\ 0 & \frac{11L}{210} + \frac{I_z}{10AL} & \frac{L^2}{105} + \frac{2I_z}{15A} & 0 & \frac{13L}{420} - \frac{I_z}{10AL} & -\frac{L^2}{140} - \frac{I_z}{30A} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{9}{70} - \frac{6I_z}{5AL^2} & \frac{13L}{420} - \frac{I_z}{10AL} & 0 & \frac{13}{35} + \frac{6I_z}{5AL^2} & -\frac{11L}{210} - \frac{I_z}{10AL} \\ 0 & -\frac{13L}{420} + \frac{I_z}{10AL} & -\frac{L^2}{140} - \frac{I_z}{30A} & 0 & -\frac{11L}{210}\frac{I_z}{10AL} & \frac{L^2}{105} + \frac{2I_z}{15A} \end{bmatrix} \tag{2}$$

where the second parcels of the elements of this matrix correspond to the contribution of rotational inertia and, according to Venâncio [6], can be disregarded in cases where this effect is not considered.

The local matrices are then converted to global matrices, and the equation of the system's motion is considered in the matrix form as

$$[\boldsymbol{M}]\{\ddot{\boldsymbol{u}}\} + [\boldsymbol{C}]\{\dot{\boldsymbol{u}}\} + [\boldsymbol{K}]\{\boldsymbol{u}\} = p(t) \tag{3}$$

where $[M]$ is the mass matrix, $[C]$ the damping matrix and $[K]$ the stiffness matrix of the system. Mazzilli et al. [7] report that it is a sufficient condition for the damping to be of the proportional type, the damping matrix $[C]$ to be a linear combination of the mass and stiffness matrices, expressed by

$$[C] = \sum_b a_b [M]([M]^{-1}[K])^b \tag{4}$$

where the particular case of the Rayleigh damping can be considered

$$[C] = a_0[M] + a_1[K] \tag{5}$$

where the factors a_0 e a_1 are obtained imposing arbitrary damping ratios ξ for two chosen modes frequencies, finding the solution for the system

$$\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \frac{1}{\omega_i} & \omega_i \\ \frac{1}{\omega_j} & \omega_j \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \tag{6}$$

To solve the problem of moving loads, the moving load P is considered changing position between the nodes of each element, changing the load vector $[P]$ at each time step, depending on the constant speed $v(t)$, solving the nonlinear problem in a linearized form.

In order to obtain system responses, it is necessary to integrate the equations of motion, so the Newmark method is used for direct integration, with the responses being calculated approximately according to coefficients b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7 that are fixed later in function of the constants. Brasil and Silva [8] report that, given the vectors of displacements, velocities and accelerations in a given time *t*, their values are determined in an instant $t + \Delta t$.

$$\dot{u}_{t+\Delta t} = \dot{u}_t + b_6\ddot{u}_t + b_7\ddot{u}_{t+\Delta t} \tag{7}$$

$$\ddot{u}_{t+\Delta t} = b_0(u_{t+\Delta t} - u_t) - b_2\dot{u}_t - b_3\ddot{u}_t \tag{8}$$

The coefficients b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7 are chosen in order to approximate the variation of the vectors,

$$b_0 = \frac{1}{\beta\Delta t^2}; \quad b_1 = \frac{\alpha}{\beta\Delta t}; \quad b_2 = \frac{1}{\beta\Delta t}; \quad b_3 = \frac{1}{2\beta} - 1; \quad b_4 = \frac{\alpha}{\beta} - 1; \quad b_5 \frac{\Delta t}{2}\left(\frac{\alpha}{\beta} - 2\right);$$

$$b_6 = \Delta t(1 - \alpha), b_7 = \Delta t\alpha \tag{9}$$

obtaining a system of algebraic equations that allows to find the increments of deflections in the step,

$$\widehat{K} u_{t+\Delta t} = \hat{p}_{t+\Delta t} \tag{10}$$

with tangent stiffness

$$\widehat{K} = b_0 M + b_1 C + K \quad (11)$$

and equivalent step load

$$\hat{p}_{t+\Delta t} = p_{t+\Delta t} + M(b_0 u_t + b_2 \dot{u}_t + b_3 \ddot{u}_t) + C(b_1 u_t + b_4 \dot{u}_t + b_5 \ddot{u}_t) \quad (12)$$

from which the deflections, velocities and accelerations of the next step are determined.

Assan [9] reports that the constants α and β can be considered respectively $1/2$ and $1/4$ for constant acceleration, and for linear variation of acceleration the values are $1/2$ and $1/6$.

For the model considering moving mass, its contribution is also considered in the mass matrix [M] at each time step, in function of the constant speed v(t).

3 Discussion and Results

For the study, a model was analyzed using Mathworks Matlab® to find the solutions, consisting of a continuous concrete beam, with three spans, inspired in taxiway bridge West of the Frankfurt Airport expansion, being the first span length of 30.00m, and the other two of 25.00m , with modulus of Young $E = 3.0105x10^{10}$, cross section area $A = 59.2\ m^2$, moment of inertia about y-axis $I = 14.65\ m^4$ and density $\rho = 2500kg/m3$. The moving mass $M = 404200\ kg$, corresponds to a Boeing 747-400, and moves at a constant speed. We analyzed the structure's response to different speeds, initially evaluating its behavior when subjected to a taxiway speed of aircraft, $v(t) = 10{,}00\ m/s$. Then, the structure is evaluated with higher speeds, simulating a runway speed of $v(t) = 90{,}00 m/s$. The studied model is shown in Figure 5.

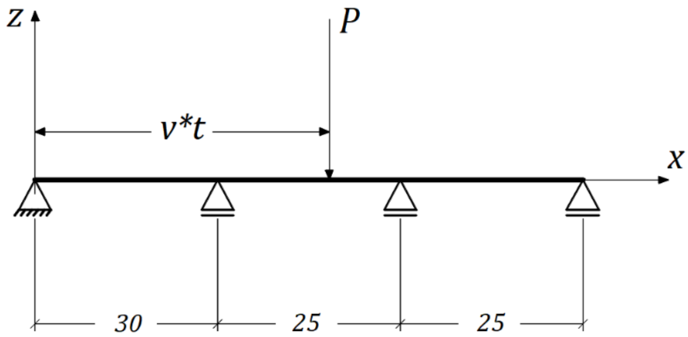


Figure 5. Beam model

The model is then discretized to perform computational analysis using the finite element method in a model with 32 elements and 33 nodes, each node with two degrees of freedom, relative to vertical displacement and rotation, and axial efforts can be disregarded.

Stiffness matrices and mass matrices are then generated for each moving load position.

Using the Newmark Method, the responses of the motion equation for each position of the mobile load are then calculated, as a function of the displacement speed, for constant speed of $10.00\ m/s$.

The transversal responses of the structure under moving load displacement speed of 10.00m/s are shown in Figure 6.

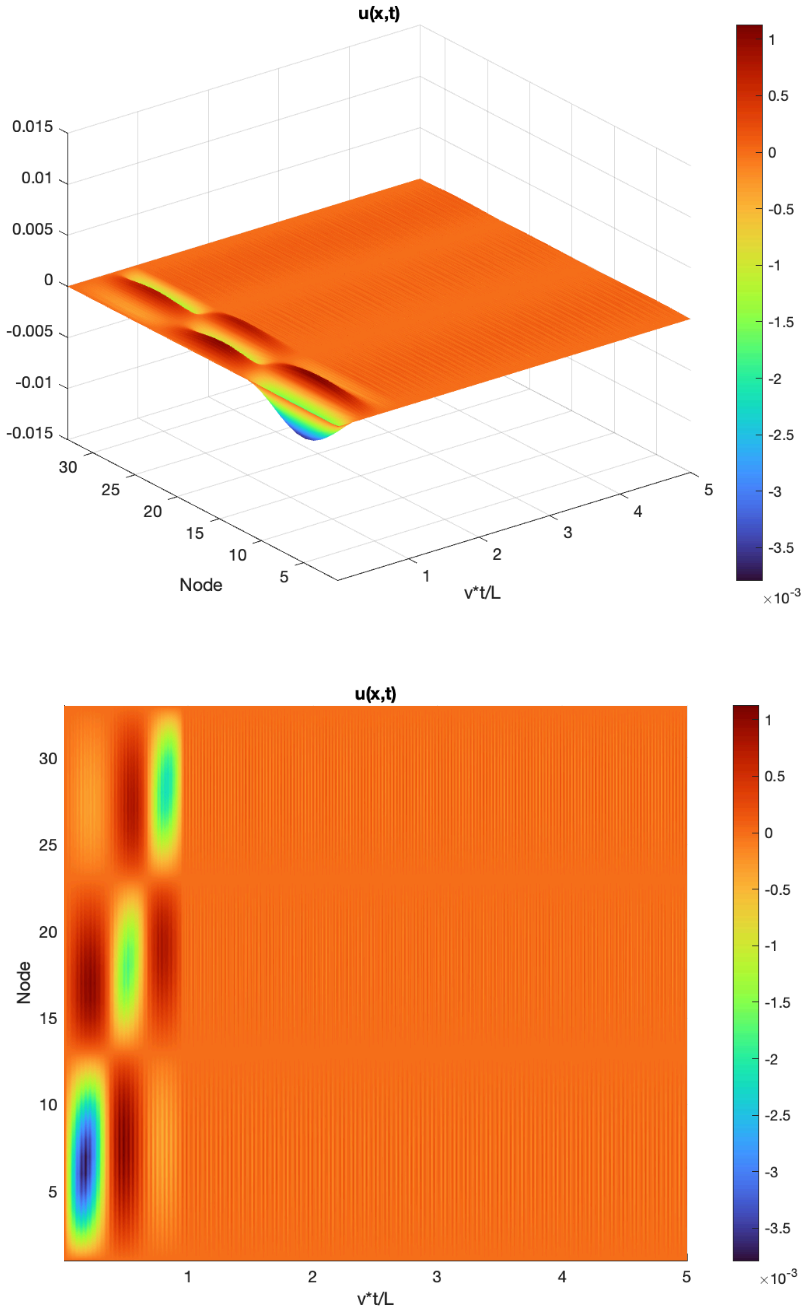
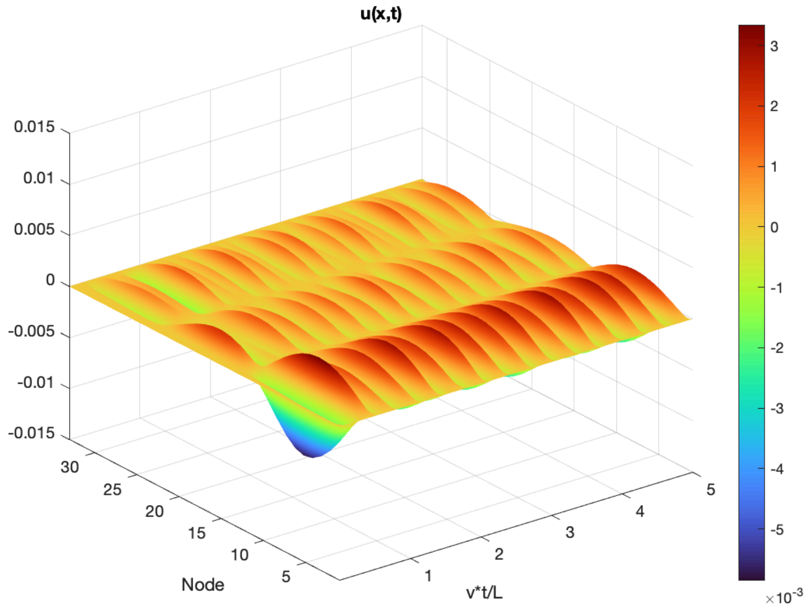


Figure 6. Transversal response of the undamped structure under mobile load, for displacement speed of 10.00m/s

The maximum displacement found for moving load displacement speed of 10.00 m/s, is in the order of 3.7862mm, when compared with the static deflection, have amplification is in the order of 1.0465.

Then, for academic evaluation of the model, it is analyzed considering the case of an aircraft taking off or landing in the same bridge, here the speeds of the moving mass and corresponding load are of 90.00 m/s. The transversal responses are shown in Figure 7.



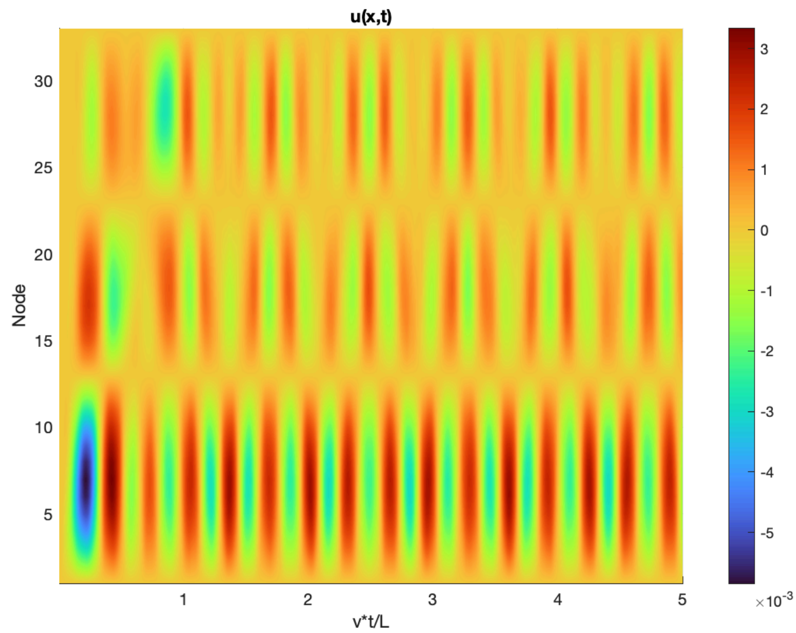


Figure 7. Transversal response of the undamped structure under mobile load, for displacement speed of 90.00m/s

The maximum displacement found for moving load displacement speed of 90.00 m/s, is in the order of 5.8410mm, when compared with the static deflection, have amplification is in the order of 1.6144.

Then we analyze the effect of considering the moving mass contribution in the mass matrix for the analysis evaluating the dynamic amplification coefficient for different velocities. The results are shown in Figure 8.

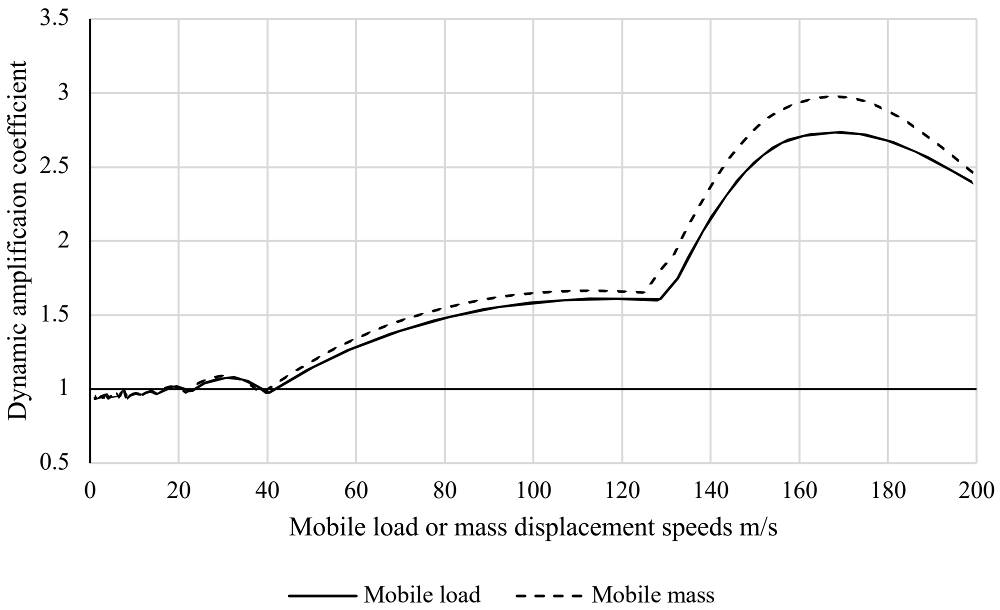


Figure 8. Relation between the displacement speed of the mobile load and mobile mass and the dynamic amplification coefficient for three span continuous beam

The maximum displacement is found for moving load displacement speed of 170.1m/s and its amplification is in the order of 2.73, and for moving mass displacement speed of 168.6m/s, and its amplification is in the order of 2.97.

4 Conclusions

It is possible to conclude that the application of an aircraft moving load in a continuous beam has a significant dynamic amplification of the responses of the structure. For taxiing speeds, the responses are amplified in around 5 percent.

Application of higher speeds cause responses of higher orders, in an analysis of speeds of landing or takeoff for the same bridge, the dynamic amplification factor reaches the value of 1.61.

The consideration of the moving mass in the global mass matrix promotes the increase of the responses when compared to the analysis only considering the moving load, in this study the dynamic amplification factor reaches values 9% higher for moving mass, in comparison with the model without consideration of the mass influence.

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