FREQUENCY AND STRESS OPTIMIZATION OF AN ENGINEERED WOOD PORTAL FRAME SUPORTING A ROTATING MACHINE

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Abstract. We consider a portal frame with 3 prismatic bars in "engineered" wood. The horizontal beam, of length L, is articulated at the top of the vertical columns, of height h, embedded in the base. The cross sections are rectangular with thickness b, constant, and width d, which can be different in the beam (d_b) and in the columns (d_c), always greater than or equal to the thickness. The material is considered linear elastic and homogeneous, with given modulus of elasticity (E) and density r. In the middle of the beam span, a motor of mass M is mounted, rotating at a frequency of N rpm. We present the minimization of the mass of the structure so that the motor rotation frequency is always at least 20% above any of the first 2 undamped free vibration frequencies of the frame. Normal stresses due to bending moments generated by unbalanced forces and gravity are also checked.

Keywords: frequency, optimization, enginerered wood, portal frame.

1 Introduction

According to Bertoldo et. al [1] the use of wood has been highly recommended due to its sustainability and diverse applications as it has considerable mechanical resistance. With the rise in interest in the application of wood in civil construction and the expansion of new technologies, engineered wood emerged, which is an entire class of construction products and materials, manufactured through the union of pieces of raw wood, wood scraps, fibers wood crushed wood and/or sawdust with adhesives to create products that look and act like wood, but are designed to be stronger and more resilient.

The present article presents a dynamic analysis and mass optimization of a simple portal frame by means of a model composed of three prismatic bars in "engineered" wood: a horizontal beam articulated to vertical columns clamped in their bases. In this example, we disregard geometric and material nonlinearities.

The objective of this work is to find the minimum cost in the design of the structure via minimization of its mass by definition of the design variables (and), which will be done, as suggested by Brasil and Silva [2], using the computational method provided in Excel Solver, considering the frequency and resistance constraints of inequality, as suggested by Brasil and Silva [3] and Mazzilli [4].

In this work, the dynamic properties of stiffness and mass of the system's 2 vibration modes are approximated by applying Rayleigh's Method, with appropriately assumed shape functions. With these values, the vibration frequencies of these modes are determined and compared to the proposed limits in relation to the motor rotation frequency in steady state. Likewise, a Balancing Quality Degree is adopted for the motor, given by ISO 1940-1:2003, in order to determine, based on the mass of its rotor, the unbalancing force that generates, together with gravity effects, the bending moments and corresponding normal stresses in the structural parts, to be compared with the allowable stress of the material.

2. Mathematical model and numerical parameters

We consider the model in Figure 1, an engineered wood frame composed of three bars of constant rectangular sections of thickness b = 12 cm; eight d can assume different dimensions in the beam (d_b) and in the columns (d_c) always greater than or equal to b or less than or equal to 3b; the beam is 6 m long and is articulated at the meeting with the columns of 3 m high each clamped in the base. The material is considered elastic and homogeneous with modulus of elasticity E=10 GPa, density ρ = 800 kg/m³ and allowable stress $\bar{\sigma}$ =10 Mpa. On the beam shaft is mounted a motor with a total mass M = 150 Kg, a rotor with a mass $m_0 = 50$ Kg, a steady-state operating frequency of N=360 RPM and a degree of balancing quality G = 6.3 mm/s ISO 1940-1:2003 [05]; adopted damping rate ξ = 2.5 % in both vibration modes. We consider the action of the force of gravity g = 10 m/s² acting on the entire system. Frequency constraints ω_b ≥1.2 ϖ and ω_c ≥1.2 ϖ are considered.

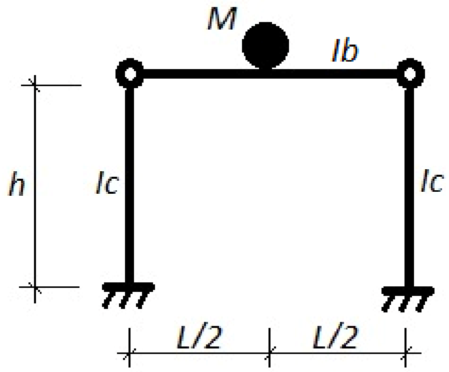

Figure 01: The mathematical model

3. Application of the Rayleigh's method

According to Yserentant [6], Rayleigh's variational method, based on principles of mechanical energy minimization, replaces the structural continuum, with infinite degrees of freedom, by a mathematical model with only one generalized coordinate to be determined, starting from shape functions that approximate the real deformation of the structural parts. Such functions must obey the geometric boundary conditions and assume a unit value in the direction of the adopted generalized coordinate. Thus, it makes it possible to determine scalar stiffness, mass and loading coefficients equivalent to those of the original continuous model. Currently, the Method has been one of the bases for formulating the Finite Element Method.

In what follows, we adopt shape functions $\phi(x)$,

3.1 Equivalent mass and stiffness formulation

Equivalent stiffness:

$$k = \int_0^L EI\,(\phi'')^2 dx \qquad (1)$$

Equivalent mass:

$$m = \int_0^L \rho A\,(\phi)^2 dx \qquad (2)$$

3.2 Symmetrical Mode Vibration Frequency (Beam)

Shape Function:

$$\phi = sen\left(\frac{\pi x}{L}\right) \qquad 0 \le x \le L \tag{3}$$

Equivalent stiffness of beam and mode:

$$k_b = \frac{\pi^4 EI}{2L^3} \cong \frac{48\, EI_b}{L^3} \tag{4}$$

Equivalent mass of beam:

$$m = \frac{\rho A_b L}{2} \tag{5}$$

Symmetrical Mode Equivalent Mass:

$$m_b = M + \frac{\rho\, A_b\, L}{2} \tag{6}$$

Symmetrical Mode Frequency:

$$\omega_b = \sqrt{\frac{k_b}{m_b}}\ \text{(rad/s)} \tag{7}$$

3.3 Vibration frequency of "Sway" mode (columns)

Shape Function:

$$\phi(x) = \frac{3x^2}{2h^2} - \frac{x^3}{2h^3} \qquad 0 \le x \le h \tag{8}$$

Equivalent stiffness of columns and mode:

$$k = \frac{3\, EI_c}{h^3} \tag{9}$$

Equivalent Mass of column:

$$m = \frac{33}{140} \rho\, A_c h \cong \frac{\rho\, A_c h}{4} \tag{10}$$

Equivalent Mode Stiffness:

$$k_c = \frac{6\, EI_c}{h^3} \tag{11}$$

Equivalent mass of the "sway" mode:

$$m_c = \frac{\rho\, A_c h}{2} + M + \rho\, A_v\, L \tag{12}$$

Symmetrical Mode Frequency:

$$\omega_c = \sqrt{\frac{k_c}{m_c}} \qquad \text{(rad/s)} \tag{13}$$

4. Calculation of bending moments and normal stresses

4.1 Force due to unbalance [05] (ISO 1940-1:2003)

$$F = m_0\, e\, \varpi^2 = m_0\, G\, \varpi,\ (G = e\, \varpi) \tag{14}$$

where $\varpi = \pi\, N/30$ (rad/s),

G must be given in m/s, mass in kg, so that forces will be given in Newton.

4.2 Maximum bending moment and normal stress in beam (steady state):

$$V = F \frac{1}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}} \tag{15}$$

$$M_{fb} = \frac{(V + m_b\, g)L}{4} \tag{16}$$

$$\frac{M_{fb}}{W_b} \leq \bar{\sigma}, \qquad \text{where} \qquad W_b = \frac{b\, d_b^2}{6} \tag{17}$$

4.3 Maximum bending moment and normal stress in columns (steady state):

$$H = F \frac{1}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}} \tag{18}$$

$$M_{fc} = H\, h \tag{19}$$

$$\frac{M_{fc}}{W_c} \leq \bar{\sigma}, \qquad \text{where} \qquad W_c = \frac{b\, d_c^2}{6} \tag{20}$$

5. Mass optimization

5.1 Objective (cost) Function:

$$f(\mathbf{x}) = \rho\, b\, (L\, x_1 + 2\, h\, x_2) \tag{21}$$

5.2 Project Variables:

$$x_1 = d_b \qquad x_2 = d_c \qquad \mathbf{x} = [x_1 \quad x_2]^{\boldsymbol{T}} \tag{22}$$

5.3 Inequality constraints:

5.3.1 Dynamic inequality (frequency) constraints:

$$g_1(\boldsymbol{x}) = 1{,}2\varpi - \omega_b \leq 0 \tag{23}$$
$$g_2(\boldsymbol{x}) = 1{,}2\varpi - \omega_c \leq 0 \tag{24}$$

5.3.2 Architectural inequality constraints:

$$g_3(\boldsymbol{x}) = x_1 - 3b \leq 0 \tag{25}$$
$$g_4(\boldsymbol{x}) = b - x_1 < 0 \tag{26}$$
$$g_5(\boldsymbol{x}) = x_2 - 3b \leq 0 \tag{27}$$
$$g_6(\boldsymbol{x}) = b - x_2 < 0 \tag{28}$$

5.3.3 Normal stress inequality constraints:

$$g_3(\boldsymbol{x}) = \frac{M_{fb}}{W_b} - \bar{\sigma} \leq 0 \tag{29}$$
$$g_4(\boldsymbol{x}) = \frac{M_{fc}}{W_c} - \bar{\sigma} < 0 \tag{30}$$

5.4 Application of a computational design optimization technique to find the mass and design variables ($x_1\ e\ x_2$):

According to Brasil and Silva [3], the Microsoft Excel Solver add-in allows to solve optimization problems, finding optimal values to determine variables subject to a set of constraints through nonlinear programming with the generalized reduced gradient (GRD) method.

Therefore, we found the optimal combination of beam height dimensions (d_b) and column (d_c) in values of 27,65 cm and 31,99 cm respectively, with correspond to a mass of 343,53 kg of structure.

6. Results

The results obtained through the modal analysis are presented in table 01 e results Forced vibration in table 02:

Table 01. Results Modal analysis - undamped free vibration

Acronym	**Value**	**Unit**	**Description**
d_b	0.2765	m	Beam height
d_c	0.3199	m	Column height
f	343.53	kg	Objective function
g1	0.0000000000		Symmetrical motor frequency (beam)
g2	0.0000000015		Symmetrical motor frequency (columns)
g3	-0.0834557849		Architectural conditions
g4	-0.1565442151		Architectural conditions
g5	-0.0401366852		Architectural conditions
g6	-0.1998633148		Architectural conditions
g7	-7710131		Stress acting on the beam
g8	-9943544		Stress acting on the columns

Table 02. Results Forced vibration

Acronym	**Value**	**Unit**	**Description**
ωv	45.2389	rad/s	Natural frequency of beam symmetric mode
ωc	45.2389	rad/s	Speaker Sway mode natural frequency
ϖ	37.6991	rad/s	Engine forcing frequency
βb	0.8333		Ratio between motor/beam frequencies
βc	0.8333		Ratio between motor/columns frequencies
uev	0.0005	m	Displacement in the beam when the engine is running
uec	0.0003	m	Displacement in the columns when the engine is running
Dv	3.2427		Factor dynamic amplication in the beam
Dc	3.2427		Factor dynamic amplication in the columns
σ	10000000	N/m²	Allowable normal voltage

For a better final interpretation of the results, we present the graph below that shows the effect of dynamic amplification on the structure (D) which is a function of β, for different values of ξ (figure 02):

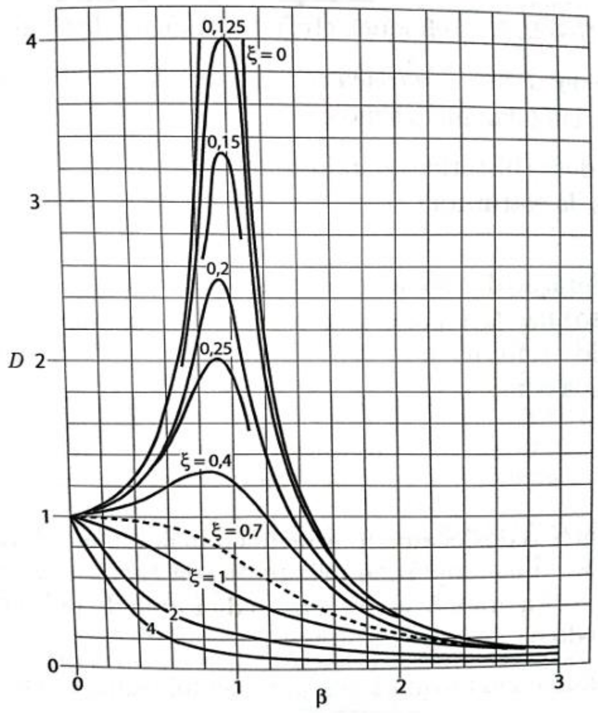

Figure 02: Variation of the dynamic amplification coefficient with damping and frequencies. Source Clough, R; Penzien, J., Dynamics of structures, 2nd Ed. New York: McGraw, 1993 apud Brasil and Silva [3].

7. Conclusions

Initially, we observed the ease with which a method as efficient as the Excel Solver offers in a few minutes the best objective function value for the structure. The mass and its design variables (dimensions d_b and d_c) were easily obtained considering the imposed design constraints (architectural, vibrations and stresses) that were met at the best optimum point, as shown in table 01.

We can also observe that que natural vibration frequencies of the structure (ωv e ωc) are directly related to the physical and mechanical properties of the structure and are not subject to the action of loading (motor). In this way, we establish as dynamic design inequality constrains that these vibrations are at least 20% greater than the engine vibration (ϖ). This condition was imposed so that the safety of the entire system was preserved by preventing the engine from entering into resonance with the structure, which is why second-order effects were not considered, as when resonance occurs the displacements are significant and must be considered.

Also analyzing the relationship β (motor forcing frequency/natural frequency of the structure), we observe in figure 02 what will happen to the structure as we change the loading frequency (motor). As the engine speed increases, the response of the ratio β approaches resonance (where $\beta = 1$). The values of β together with the damping rate ξ, directly influence the result of dynamic amplification on the structure, when the load is high frequency in relation to the structure the dynamic response will be small and when the damping rate is small the result of dynamic amplification will be great.

Finally, we can also conclude that the new dimensions found and presented in table 02, offer the structure normal stress values for the beam and pillars that are much lower than the allowable stresses, in forced vibration when the engine reaches its full operational stationary state, when this occurs We can say that security is being assured.

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References

[1] C. Bertoldo; G. A. Pereira; R. Gonçalves. Effect of Reducing the Size and Number of Faces of Polyhedral Specimen on Wood Characterization by Ultrasound. Materials, v. 16, p. 4870, 2023.
[2] R. M. Brasil and M. A. Silva, *Otimização de Projetos de Engenharia.* Blucher, São Paulo, 2019 (in Portuguese).
[3] R. M. Brasil and M. A. Silva, *Introdução à dinâmica das estruturas.* 2.ª ed. Blucher, São Paulo, 2015 (in Portuguese).
[4] C. E. N. Mazzilli, J. C. André, M. L. Bucalem, S. Cifú. *Lições em Mecânica das Estruturas.* Blucher, 2016 (in Portuguese).
[5] ISO 1940-1: Vibrations mécaniques — Exigences en matière de qualité dans l'équilibrage pour les rotors en état rigide (constant) — Partie 1: Spécifications et vérification des tolérances d'équilibrage. Second edition. Suiça, 2003 (in French).
[6] H. Yserentant, *A short theory of the Rayleigh–Ritz Method*. Computational Methods in Applied Mathematics, v. 13, n. 4, p. 495-502, 2013.