Model Updating in Dynamic Finite Element for Damage Detection on an Aircraft Wing

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Abstract. SHM is a fundamental area for aerospace engineering. Within this broad area, there are non-destructive ways to assess integrity. This paper presents an approach to damage detection through optimization of a finite element model, where the design variables are the diameters of the circular sections forming the components of the structural system, and the objective function is the relative difference between the frequencies of the damaged model and the adjusted one. This allows for the detection of damages that caused changes in the modal parameters of the structure through an optimal combination of design variables that generate responses obtained in the modal analysis of the damaged structure once the model is calibrated. The practical application used is the detection of damages in a simplified aircraft wing model.

Keywords: Damage Detection, Structural Dynamics, Finite Elements Method, Optimization, Structural Health Monitoring.

1 Introduction

Structural Health Monitoring (SHM) is a broad multidisciplinary field that deals with the balance between monitoring and maintenance [1], and it is responsible for the engineering approach concerned with ensuring the correct performance of structures in their given functions. Among the techniques used for monitoring the integrity of structures, the methods can generally be divided into destructive and non-destructive. The focus of the present work applies to the latter, with a particular interest in integrity monitoring using vibration-based techniques.

One of the possible ways to assess integrity is by attempting to identify damage, an evaluation that [2] categorized into 4 levels of depth.

- Level 1: damage detection;
- Level 2: damage localization;
- Level 3: damage quantification;
- Level 4: estimation of remaining service life.

The scope of this article encompasses a global damage detection using modal parameters, that is, Level 1.

Vibration-based detection methods, being non-destructive techniques, are much less invasive, sometimes requiring no more than instrumentation followed by tests that are harmless to the performance of the structure. From this data collection, modal parameters, understood as dynamic signatures of the system, can be extracted and analyzed [3].

The modal parameter used as an integrity or damage signature in this article is the natural frequency, employing an optimization with bio-inspired search (genetic algorithms) to determine the optimal distribution of characteristics of structural elements that would generate the frequencies found in the computational modal analysis of the structure.

This branch of study involving experimental dynamics applications for intelligent inspections in structural systems is a growing scientific area. Elsevier, through its scientific metric platform SciVal, confirms this, as can be seen in the annual publications graph obtained from the platform as shown Figure 1.

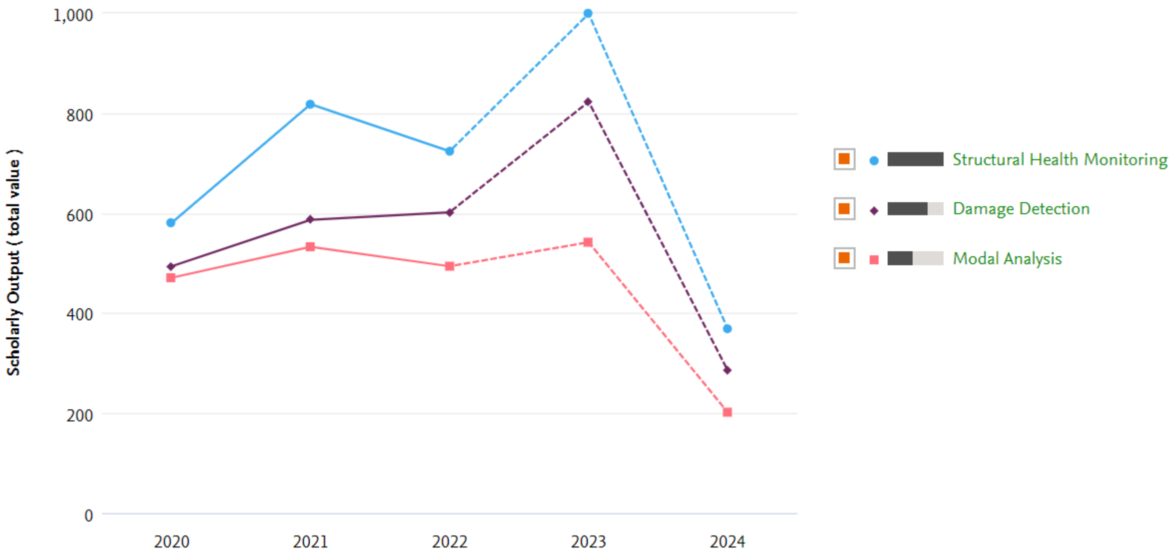

Figure 1. Growing publications of works in SHM, damaged detection and modal analysis.

Therefore, the present work aims to evaluate a damage detection method by optimizing the cross-sections of the elements of a structure, minimizing the difference between the natural frequencies of the updated model and the damaged reference model. This optimization allows extracting information about the integrity of the structure from the finite element model after the adjustment.

2 Methodology

For the structural analysis intended, it was decided to use grillage element formulations, whose stiffness and mass matrices can be defined by equations 1 and 2 respectively [4]. Where the properties showed in the matrices below are: linear mass m', area A, bending inertia I, and torsional inertia J, element length l, Young's modulus E, shear modulus G.

$$[K]=\begin{bmatrix} \frac{12EI}{l^3} & 0 & -\frac{6EI}{l^2} & -\frac{12EI}{l^3} & 0 & -\frac{6EI}{l^2} \\ 0 & \frac{GJ}{l} & 0 & 0 & -\frac{GJ}{l} & 0 \\ -\frac{6EI}{l^2} & 0 & \frac{4EI}{l} & \frac{6EI}{l^2} & 0 & \frac{2EI}{l} \\ -\frac{12EI}{l^3} & 0 & \frac{6EI}{l^2} & \frac{12EI}{l^3} & 0 & \frac{6EI}{l^2} \\ 0 & -\frac{GJ}{l} & 0 & 0 & \frac{GJ}{l} & 0 \\ -\frac{6EI}{l^2} & 0 & \frac{2EI}{l} & \frac{6EI}{l^2} & 0 & \frac{4EI}{l} \end{bmatrix} \tag{1}$$

$$[M]=\frac{m'l}{420}\begin{bmatrix} 156 & 0 & -22l & 54 & 0 & 13l \\ 0 & 140J/A & 0 & 0 & 70J/A & 0 \\ -22l & 0 & 4l^2 & -13l & 0 & -3l^2 \\ 54 & 0 & -13l & 156 & 0 & 22l \\ 0 & 70J/A & 0 & 0 & 140J/A & 0 \\ 13l & 0 & -3l^2 & 22l & 0 & 4l^2 \end{bmatrix} \tag{2}$$

Using such elements, a free vibration analysis, also called modal analysis, can be performed, which is obtained as the solution of the matrix equation of motion given in equation 3, assuming that displacement solutions in time can be described by equation 4. The result of substituting 4 into 3 is shown in 5.

$$[M]\{\ddot{u}(t)\}+[K]\{u(t)\}=\{0\} \tag{3}$$

$$\{u(t)\}=\{\phi\}_n\cos(\omega_n t-\theta_n) \tag{4}$$

$$([K]-\omega_n{}^2[M])\{\phi\}_n=\{0\} \tag{5}$$

According to [5], the solution of the characteristic equation 5 yields a number n of eigenvalues and their associated eigenvectors equal to the number of degrees of freedom of the structure. It is possible to numerically truncate at a value where it is judged to contain the relevant responses for the analysis. In this work, an analysis up to the 6th eigenvalue and associated eigenvector was performed.

Physically, the eigenvalue $\omega_n{}^2$ found by the numerical solution has the physical meaning of the square of the natural angular frequency of the system, while the amplitudes $\{\phi\}_n$ represent the modal shapes that the structure assumes when vibrating at this frequency mode. As a comparison between models, the cyclic natural frequencies f_n, in Hz, were used, which can be calculated from ω_n as shown in equation 6.

$$f_n=\frac{\omega_n}{2\pi} \tag{6}$$

In general, according to [6], a structural optimization problem in dynamics can be written as determining the vector b that minimizes f over a time T defined by equation 7. Where the displacement vector, given by the state variable u, must satisfy the motion equation defined in 3.

$$f(b,T)=\bar{f}(b,T)+\int_0^T \tilde{f}(b,u,\dot{u},\ddot{u},t)dx \tag{7}$$

Currently, a widely diffused and established method of doing this is by using Genetic Algorithms (GA), which generate populations of individuals (combinations of variables used in optimization) randomly generated from generation to generation (iterations) with the introduction of crossover between the best individuals and mutations in random characteristics, to find the best global solution to the optimization problem [7].

In the present study, the objective function, also known as the fitness function in GA, was the mean relative difference of mode frequencies, calculated as 8. Here, f_i^u represents the natural frequency of the updated structure, f_i^d represents the natural frequency of the damaged structure for the i-th mode.

$$f(b)=\frac{1}{n}\sum_{i=1}^{n}\left|\frac{f_i^u}{f_i^d}-1\right|.100 \tag{8}$$

To interact with the finite element package used in the Julia language analysis, the following parameter values showed in Table 1 were chosen for GA, as recommended in the work of [8].

Table 1. Values of the parameters used in GA optimization

Parameter	Value
Population Size	100
Crossover Rate	0.8
Mutation Rate	0.1
Selection	Uniform Rank
Relative Tolerance	1e-04
Absolute Tolerance	1e-04
Maximum Generations	2000

It is worth noting the connection between dynamics and optimization in this study, as the optimization of the vector b, represented here by the diameter of the cross-sections of the bars, will generate different individuals with varying values of stiffness and mass matrices. These variations will result in natural frequencies that are closer to or further from the damaged reference model. This will make the algorithm use the modal signature of the model as a guide in searching for the best combination of design variables that correctly approximate the expected dynamic behavior.

3 Results and discussions

For the application of the previously discussed methodologies, a case study is proposed involving a simplified model of an airplane wing with a 17.5 m horizontal projection, with ribs every 3.22 m, forming a 0.91 m cantilever. The spars are spaced 0.6 m apart at the free end and 2.92 m apart in the region of the clamped support, represented by squares in Figure 2.

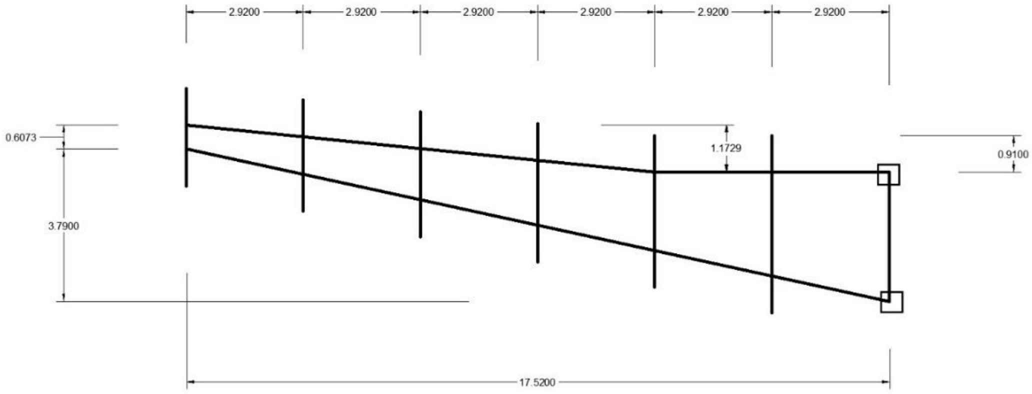

Figure 2. A schematic representation of the structural model used as an application.

The material comprising the structural elements is a steel with a Young's modulus E of 200 GPa and a shear modulus G of 78.9 GPa. The density ρ is 7850 kg/m³, and the original diameters b before the application of damage were 0.15 m.

Modal analysis was conducted by truncating the results of the characteristic equation up to the 6th mode of vibration. The results of the cyclic natural frequencies and the associated deformations are shown in Figure 3. Qualitatively, modes 1, 2, and 4 represent flexural vibration modes, while modes 3 and 5 represent torsion. Additionally, mode 6 presents a combination of simple flexural and torsional forms.

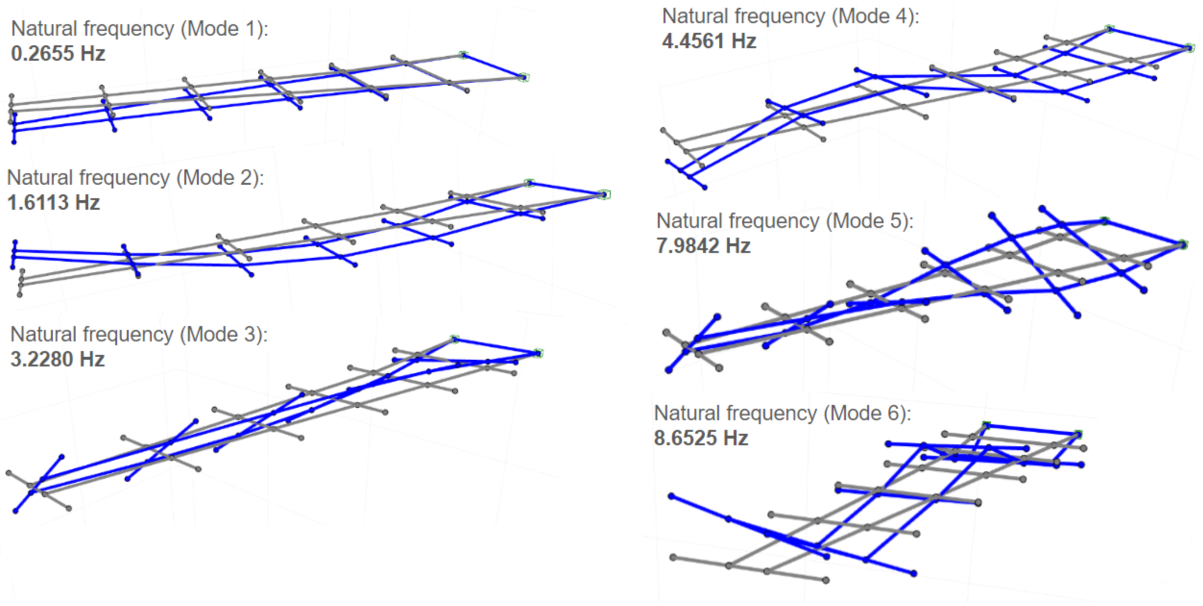

Figure 3. The first 6 modal shapes obtained from the finite element model of the wing.

In total, the model has 31 elements, 26 nodes, and 78 degrees of freedom. The damage imposed on the structure was done by considering a 30% reduction in the diameter value. Therefore, in the damaged elements (shown in red in Figure 4), the diameter became 0.105 m. A three-dimensional graphical representation of these damages on the wing has been plotted and is shown in Figure 4.

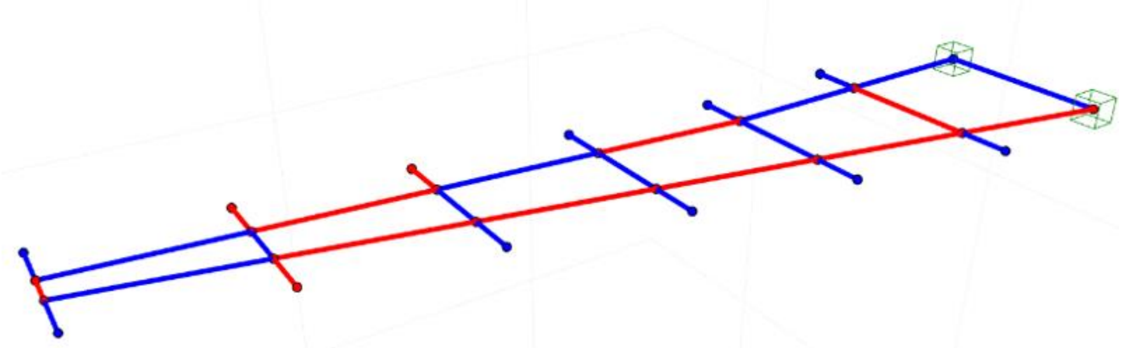

Figure 4. Graphical model of the structure with damaged bars shown in red.

With damage imposed on 15 elements, a new modal analysis considering the damaged structural characteristics was conducted, and the natural frequencies obtained, along with their differences compared to the intact model, are shown in Table 2. The average difference in frequencies caused was 8.06%.

Table 2. Comparison between cyclic natural frequencies of the undamaged and damaged models

Mode	Undamaged Model	Damaged Model	Relative Difference
1	0.2655 Hz	0.2469 Hz	7.02%
2	1.6113 Hz	1.4516 Hz	9.91%
3	3.2280 Hz	2.993 Hz	7.08%
4	4.4561 Hz	4.0361 Hz	9.42%
5	7.9842 Hz	7.3445 Hz	8.01%
6	8.6525 Hz	8.0547 Hz	6.91%

The optimization performed by GA reduced the fitness function defined in 8, is shown in the graph in Figure 5. It is evident that after 1000 generations, the algorithm met the convergence criterion and assumed that the result was optimal.

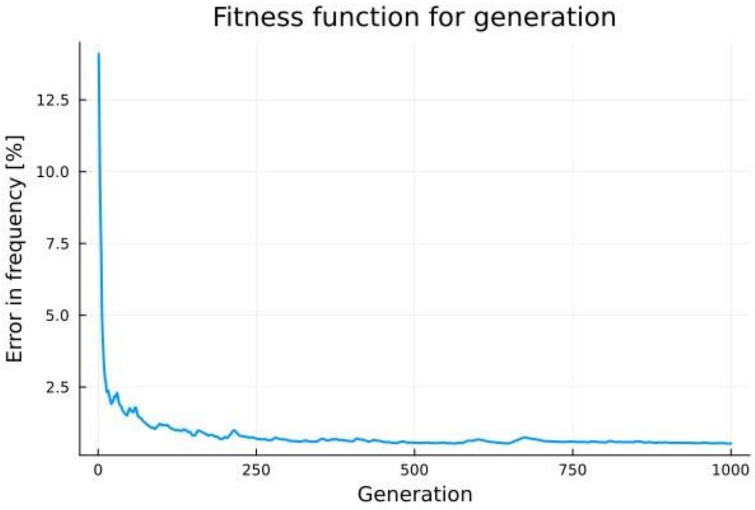

Figure 5. Minimization of the relative error in frequency across generations in genetic algorithms.

After the optimization, the natural frequencies approached the damaged reference model that was the objective, achieving an average relative difference value of 0.54%, with the frequencies shown in Table 3.

Tabel 3. Comparison between cyclic natural frequencies of the ajusted and damaged models

Mode	**Updated Model**	**Damaged Model**	**Relative Difference**
1	0.2468 Hz	0.2469 Hz	0.01%
2	1.4635 Hz	1.4516 Hz	0.81%
3	3.0003 Hz	2.993 Hz	0.03%
4	4.0392 Hz	4.0361 Hz	0.08%
5	7.3060 Hz	7.3445 Hz	0.53%
6	7.9154 Hz	8.0547 Hz	1.76%

It is still possible to discuss the localization and quantification of damage that this approach allows, albeit within certain limitations associated with uncertainties, as reported in the referenced work of [9].

Figure 6 presents the diameters found in the damaged and adjusted models on the vertical axis and to which element they refer on the horizontal axis. In this image, it is possible to verify that the red bars follow the behavior of the blue ones in most cases, demonstrating that the adjustment had a good ability to detect damage at a global level using the natural frequency as the optimized parameter.

However, it is still noticeable that there is a mismatch between the red and blue bars in some elements, meaning that the model did not detect damage occurring at this position. Additionally, there is an imperfect quantification, as the values of the damages in the adjusted model were greater than the actual damages, represented by larger blue diameters than red diameters in the elements with damage.

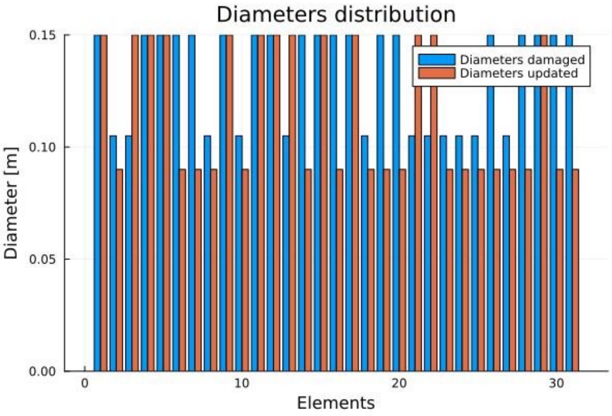

Figure 6. Comparison between the updated diameters in optimization and the diameters that were actually damaged

4 Conclusions

Structural Health Monitoring (SHM) is an incredibly important and growing area of scientific production in recent years, largely justified by advances in intelligent computational tools and the integration with classical numerical techniques. An example of this integration is vibration-based damage detection employing genetic algorithms.

This paper presented an approach to this specific category of evaluation in SHM using finite element model update to detect damage in an airplane wing using 6 natural frequencies as optimization parameters. The results showed that the adjustment was successfully performed, achieving a minimization of the average relative differences between the natural frequencies of the updated and damaged models, which decreased from 8.06% to 0.54%.

Along with this reduction, it was possible to identify the imposed damage using the adjusted model, which could be used to recommend to a manager a possible suggestion of repair or maintenance needs for such elements. However, the perfect localization of damage and its quantification are points that still require some level of increased precision through the improvement of this technique, remaining as recommendations for future work.

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