Dynamic relaxation for form finding of a pretensioned cable structure

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Abstract. In this paper, the numerical method of dynamic relaxation was adopted to find the final form of a simple pretensioned cable structure, such as those used in light roof structures known as “tensile structures” in which the external loads are balanced by tensile forces in the cables. The model consists of four same length and section cables arranged in an X-shape in a planar view, fixed at one end. The cables are not in a plane: two opposite cables have their supports in the same positive vertical height, and the other two opposite cables have their support in the same negative vertical height. There is a vertical force acting on the central node in the x3 direction modeling the weight of the structure and the four cables are pretensioned. The model admits large displacements so that a nonlinear dynamic approach is adopted. A numerical step-by-step finite differences time integration scheme is implemented for form finding.

Keywords: tensile structure, cable structure, dynamic relaxation.

1 Introduction

In engineering, there is an incessant quest to design ever lighter and more sustainable structures, while respecting the balance between structural optimization and safety. To achieve this goal, it is essential that calculation models are refined and reliable. Nature is often used as inspiration and when combined with mathematics to help understand the association between structural forms and the balance of forces, structurally rational designs can be achieved. One of the first researchers in tenso-structures using steel cables was the German architect Frei Otto, who carried out research into lightweight and adaptable construction, considering fundamental aspects of the relationship between architecture and nature [5].

Tensioned structures are light structures whose external loads are balanced by tensile forces. These structures are basically divided into two groups: cable structures and membrane structures, which can be combined to create mixed tensile structures. When applied to roofs, this structural model has the great advantage of being a lightweight structure [4]. Tensioned cable structures are mechanisms capable of withstanding stresses greater than the stresses allowed in conventional steel structures. The stiffness of the system tends to increase with increasing displacement as long as the cables remain tensioned, the consequence of which is that the use of linear analysis methods tends to overestimate displacements and stresses in rigid structures and underestimate them in flexible structures [3].

Dynamic relaxation is a numerical method, which, among other things, can be used to do "form-finding" for cable and membrane structures. The aim is to find a geometry where all forces are in equilibrium. In the past this was done by direct modelling, using hanging chains and weights (see Gaudi model Figure 1), or by using soap films, which have the property of adjusting to find a "minimal surface". The dynamic relaxation method is based on discretizing the continuum under consideration by lumping the mass at nodes and defining the relationship between nodes in terms of stiffness (see also the finite element method). The system oscillates about the equilibrium position under the influence of loads. An iterative process is followed by simulating a pseudo-dynamic process in time, with each iteration based on an update of the geometry [4].

Figure 1. Gaudi's model for Sagrada Familia

2 Mathematical Model

In the initial position displayed in Figure 2, Figure 3 and Figure 4, the coordinates of the points are: 1 (0, 0, 0); 2 (-L, 0, -h); 3 (L, 0, -h); 4 (0, L, h) and 5 (0, -L, h). The model consists of four symmetrical cables so the initial length for each bar is given by (1) and the final length is described as a function of the relative displacements in x_1, x_2 and x_3 directions, given by u_1, u_2 and u_3 respectively (2).

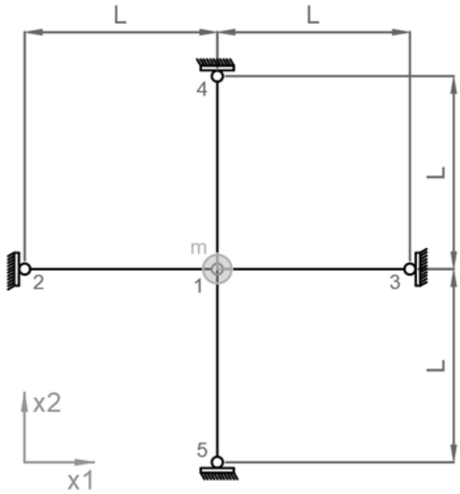

Figure 2. Schematic plan view – Initial position

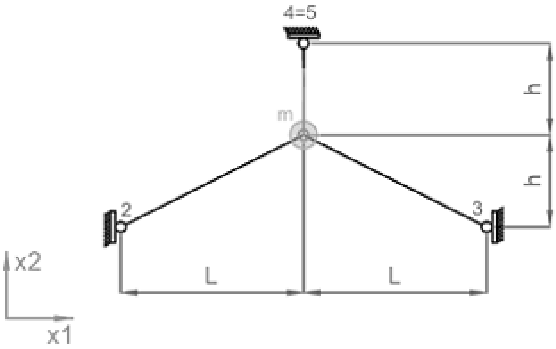

Figure 3. Schematic section X1 X2 view – Initial position

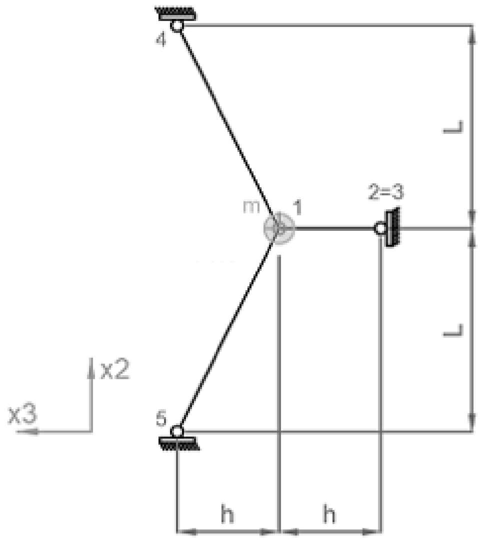

Figure 4. Schematic section X2 X3 view – Initial position

Number of elements (j) is 4.

The initial length is constant and calculate just once in this problem (*LIN*) and update length (*LL*) for each time step of the cable are represented by Eq. (1) and (2), respectively:

$$LIN = LI(j) = \sqrt{\sum_{i=1}^{3} [XI(j,i)]^2} \tag{1}$$

$$LL(j) = \sqrt{\sum_{i=1}^{3} [u(i) - XI(j,i)]^2} \tag{2}$$

Direction cosines of each cable at each time step (5):

$$C(j,i) = \frac{u(i) - XI(j,i)}{LL(j)} \tag{3}$$

Change in length of each cable at each time step (4):

$$DL(j) = LL(j) - LIN \tag{4}$$

It is possible to determine the normal elastic force (*N*) (5) in cable *j* (variable over time) by multiplying the axial stiffness (*K*) (6) by the length change (*DL*) given by the difference of the final and initial lengths:

$$N(j) = K\, DL(j) + N_0 \tag{5}$$

$$K = \frac{EA}{LIN} \tag{6}$$

where E is the Young's module, A is the cross-sectional area of the cable and are the same and constant for all the elements.

Components of the force N of each cable in each coordinate direction at each time (7):

$$CN(j,i) = N(j)\ C(j,i) \tag{7}$$

Sum of forces to be added to each of the three Equations of Motion, at each time step (8):

$$S(i) = \sum_{j=1}^{4} CN(j,i) \tag{8}$$

By direct dynamic equilibrium, according to Newton's second law, the three nonlinear Equations of Motion, adopting linear viscous damping C1, C2 and C3 for each direction.

$$M\ddot{u}_1 = Fx_1(t) - C_1\dot{u}_1 - \sum_{j=1}^{4} CN(j,i) \tag{9}$$

$$M\ddot{u}_2 = Fx_2(t) - C_2\dot{u}_2 - \sum_{j=1}^{4} CN(j,i) \tag{10}$$

$$M\ddot{u}_3 = Fx_3(t) - C_3\dot{u}_3 - \sum_{j=1}^{4} CN(j,i) \tag{11}$$

Next, we present a step-by-step numerical integration in time by finite [1] and [2]. Consider the system of 3 second-order, non-linear differential equations of motion (9), (10) and (11). For simplicity of exposition, one of these equations can be put in the form (12):

$$M\ddot{u} = F(t) - C\dot{u} - Su \tag{12}$$

where $S(u)$ is the sum of the components of the elastic restoring forces of the cables in the direction of a given generalized coordinate.

For its step-by-step numerical integration in time by finite differences, time is divided into very small intervals of duration h, in seconds. At a time t, the time derivatives indicated by superimposed points can be approximated by finite differences, as (13) and (14):

$$\dot{u}_t = \frac{u_{t+h} - u_{t-h}}{2h} \tag{13}$$

$$\ddot{u}_t = \frac{u_{t+h} - 2u_t + u_{t-h}}{h^2} \tag{14}$$

Replacing (13) and (14) in (12), at time *t*, and separating the displacements known at that time from the unknown future displacements, we have **Erro! Fonte de referência não encontrada.**.

$$\left(\frac{M}{h^2} + \frac{C}{2h}\right) u_{t+h} = F(t) + \left(\frac{2M}{h^2}\right) u_t + \left(\frac{C}{2h} - \frac{M}{h^2}\right) u_{t-h} - S(u_t) \tag{15}$$

which can be written in the form **Erro! Fonte de referência não encontrada.**:

$$\widehat{\widehat{K}}\ u_{t+h} = \widehat{\widehat{p}}_t \tag{16}$$

where

$$\tilde{\bar{K}} = \frac{M}{h^2} + \frac{C}{2h} \quad (17)$$

and

$$\tilde{\bar{p}}_t = F(t) + \left(\frac{2M}{h^2}\right)u_t + \left(\frac{C}{2h} - \frac{M}{h^2}\right)u_{t-h} - S(u_t) \quad (18)$$

This is the algorithm to be implemented at each step. As it depends on the knowledge of u_{t-h}, some scheme must be employed for its initialization at the initial time. One of the most used is to consider constant acceleration in the first time interval.

3 Numerical Simulations

For numerical simulations, it was adopted numerical values for structural parameters and the formulation presented before was implemented in Matlab.

h = 2 m;
L = 1 m;
Which leads to the initial length of 2.23 m for each bar (Figure 5);
m = 50 kg;
F(x3) = mg = 50.10m/s² = 500 N
A1 = A2 = A3 = A4 = 1.98 cm² = 0.000198 m² (wire strand rope with 5/8" or 16mm diameter);
Density 7860 kg/m³;
E = 110 GPa = 1.1x10^7 N/cm²;
Damping ratio 0.5%;
N01 = N02 = N03 = N04 = 2000 N (initial traction forces);

For these values, this structure has frequencies 9.7 Hz and 8.9 Hz for initial configuration and Rayleigh damping of 4.65 Ns/m.

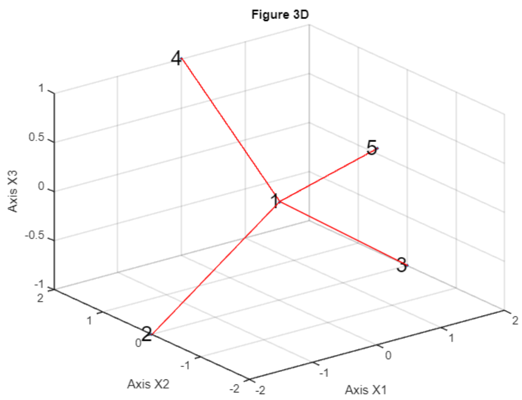

Figure 5. Schematic 3D view – Initial position

4 Results

The formulation presented previously was implemented in Matlab to determine the displaced position of the structure. The vertical displacement of the node 1 for the evolution of 3.5 seconds and the integration time step of 0.006 seconds is presented in Figure 6. The equilibrium position is 0.639mm.

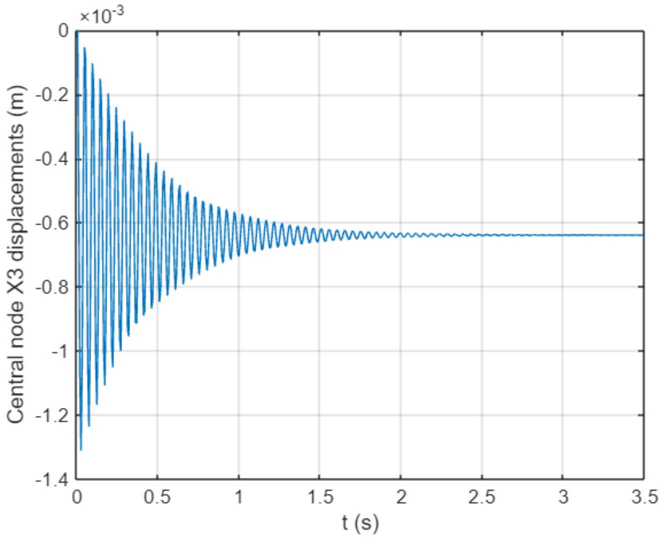

Figure 6. Vertical displacement – Central node

5 Conclusions

In the present work, a simple 3D structure was analyzed considering the system geometric nonlinearity and initial traction of the cables. The mathematical solution of the problem was implemented in the Matlab program allowing determining the displacements of the central node during a time variation.

The results achieved are within expectations because of the damping ratio of the system the displacement oscillates around the equilibrium position until the movement stops.

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