System identification of a lead-acid battery based on experimental data

Raphaela C. Machado[1], Pedro G. Gonçalves[1], Hugo N. Ferraz[1], Saulo A. G. Salcedo[2], David C. F. Zúñiga[3], Jeferson J. Lima[4], Teófilo M. de Souza[1], Jose M. Balthazar[4,5], Luiz C. S. Góes[6]

[1]*Dept. of Electrical Engineering, Paulista State University*
Av. Ariberto Pereira da Cunha, 333 - Portal das Colinas, 12.516-410, Guaratinguetá-SP, Brazil
raphaela.machado@unesp.br, pedro.maria@unesp.br, hugo.jr@unesp.br, teofilo.souza@unesp.br
[2]*Dept. of Mechanical Engineering, Universidad de Concepción*
Calle Edmundo Larenas, 219, 4070409, Concepción-Bío Bío, Chile
saulogomez@udec.cl
[3]*Dept. of Aeronautical Engineering, Paulista State University*
Av. Profª Isette Corrêa Fontão, 505 - Jardim das Flores, 13.876-750, São João da Boa Vista-SP, Brazil
david.castillo-zuniga@unesp.br
[4]*Dept. of Electrical Engineering, Federal University of Technology*
R. Doutor Washington Subtil Chueire, S/n - Jardim Carvalho, 84.017-220, Ponta Grossa-PR, Brazil
jefersonlima@utfpr.edu.br
[5]*Dept. of Electrical Engineering, Paulista State University*
Av Eng. Luiz Edmundo C. Coube, 14-01- Bauru, 17.033-360, Bauru-SP, Brazil
jose-manoel.balthazar@unesp.br
[6]*Dept. of Aeronautical and Mechanical Engineering, Technological Institute of Aeronautics*
Rua Marechal-do-Ar Eduardo Gomes, 50 - Vila das Acácias, 12.228-615, São José dos Campos-SP, Brazil
goes@ita.br

Abstract. The purpose of this work is to determine a battery model by means of experimental data collected during a lead-acid battery's discharge. To precisely ascertain the battery's electrical properties and examine how it discharges during operation, it is crucial to create a mathematical model that perfectly represents the system. This paper presents the identification of a lead-acid battery electrical model using collected data in this way. In addition to having the characteristics characterized as resistive and capacitive elements and being well fitted to the experimental data, Jackey's model was chosen to represent the battery dynamics because it has the benefit of not being overly complex. The goal is to use optimization techniques to determine Jackey's model's parameters. Ultimately, the results show that the identified mathematical model accurately represents the system, and, therefore, the tool is a viable alternative for the mathematical modeling of lead-acid batteries.

Keywords: lead-acid battery, electrical systems, modeling, system identification

1 Introduction

Electric batteries play the key role of ensuring that critical systems of an aircraft remain operational in the event of an engine failure, or emergency. Thus, they are responsible for providing energy to keep the lighting system running, they also power sensors, actuators, and other subsystems. The correct operation of the batteries requires constant supervision to replace them on time and take advantage of all their useful capacity. Some threats caused by the use of batteries are battery leakage, failure of battery charging, excessive discharge ratio, short circuits, and others. These risks can be reduced by investing in testing, performing preventive maintenance, and using new materials.

According to the intended application, a battery management system (BMS) is required. It protects the battery and increases its lifespan. The BMS consists of sensors, actuators, and controllers, which can assess the battery parameters, estimate its state, and report data. In this sense, obtaining a representative mathematical model for the system is important in order to determine the electrical characteristics of the battery, and analyze its discharge behavior during its use. In addition, the model is useful for the BMS design, according to the results presented in [1], where the hardware and firmware are described, ensuring the operation of all tasks and functionalities in real time. The state estimation algorithm was based on a mathematical model for the battery and the application of the Kalman filter. The methodology was validated using experimental data and the results presented were satisfactory.

About system modeling, even though there are simple and sophisticated models, the degree of complexity of the model will be determined by the application for which it will be used. Regarding the functioning of lead-acid batteries, although there are complex chemical reactions and laws from physics that can be employed to describe the system dynamics, mathematical models obtained from identification techniques can be an alternative to modeling and understanding their behavior.

In the work of [2], the general characteristics of a lead-acid battery and some mathematical models to represent the operating dynamics of a lead-acid battery for various applications were presented. The obtained identification results were based on the Jackey's model, due to its simplicity and ability to fit well with experimental battery discharge data. The work of [3] describes the internal features of lead-acid, Nickel-Cadmium, Nickel, and lithium-metal hybrid batteries, as well as their advantages and disadvantages. In addition, simulations of the charging and discharging process of some types of batteries were presented.

Modeling of lead-acid batteries is described in the current work. Here, experimental data are sampled for different battery discharge ratios using the Arduino. Then, the optimization tools of Matlab environment were used for system identification. This paper is organized as follow. In Section 2, a concise overview of the system is provided, presenting the operating principle of the batteries, as well as the main electrical characteristics. A description of the battery model used in this work and the main parameters to be identified are also presented. The identification process used in this study is discussed in Section 3 and the system identification outcomes are shown in Section 4. Lastly, the work's conclusion is provided in Section 5.

2 System Description

This section explains how a lead-acid battery works. Additionally, a discussion is held regarding the primary electrical factors that characterize it. Additionally, a description of the mathematical model used to depict a lead-acid battery's discharge behavior is provided.

2.1 Lead-acid battery

The lead-acid batteries, like other battery types, transform chemical energy that has been stored into electrical energy. They contain electrodes made up of lead plates immersed in an acidic electrolyte, typically sulfuric acid. Sulfuric acid is used both as a component of the active mass and as an electrolyte ingredient. In each of these applications, dilute sulfuric acid is obtained by mixing concentrated sulfuric acid, with a relative density of 1,835 g/cm^3 with water, up to the desired value. The concentrations of sulfuric acid most frequently used in the manufacture of batteries correspond to a relative density range ranging from 1,050 to 1,400 g/cm^3. For example, the density of the filling electrolyte of Moura Stationary - MVA Series batteries is 1,300 g/cm^3 at 25ºC with the element fully charged.

The charging process of lead-acid batteries must be carried out with low electrical currents, which leads to a high charging time. During the charging process, overloads must be avoided to mitigate their sensitivity to current overloads. This kind of battery should not be fully discharged, it would bring a loss of charging capacity or avoid completely a new charge. These batteries are sensitive to variations in operating temperature, high temperatures increase capacity but decrease lifespan and low temperatures decrease their capacity.

2.2 Electrical features of a lead-acid battery

The main features of a battery are the following:

Nominal voltage. Or the electrical potential available at the battery terminals. It is calculated by multiplying the number of cells and the voltage value of each cell. For example, typically lead-acid batteries have a voltage rating of 12V.

Nominal capacity. Corresponds to the electrical load that the battery can provide in ampere-hours. For example, lead acid batteries used to have capacities of C20, C10, C5, C3, and C1. Mathematically, the capacity is expressed by $C = I\Delta t$, where I is the rated current and Δt is the associated time interval.

Open circuit voltage (OCV). It is the voltage measured at the terminals when the battery is not connected to any circuit. The open circuit voltage is a function of the temperature as well as the concentration of the electrolyte.

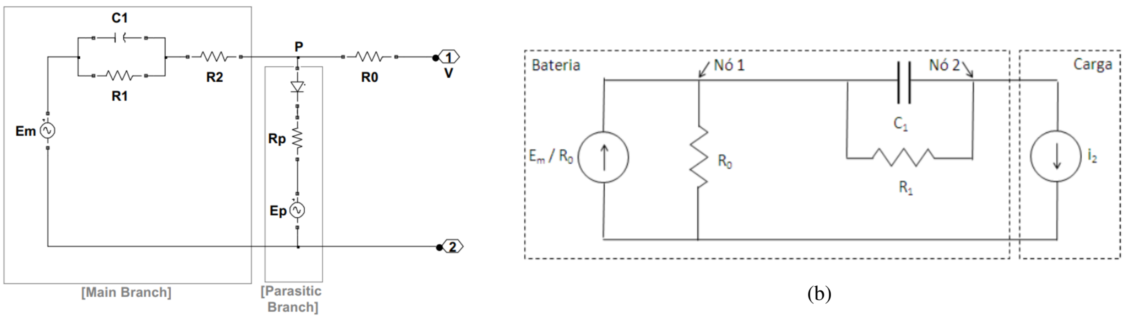
State of charge (SOC). It indicates the percentage of accumulated energy (charge) on the battery. It can be estimated through the OCV measured at the poles of the battery. In math terms, the state of charge is defined according to [4]:

$$SOC(t) = SOC(0) - \frac{1}{C}\int_0^t i_2(t)dt \tag{1}$$

where C is the nominal capacity of the battery and $i_2(t)$ is the discharge current.
Internal resistance. It determines the amount of power that can be delivered in a given time interval. Experimentally, it is possible to determine the internal resistance using a DC load, measuring the voltage and current values at two different time instants and applying Ohm's law: $R_{cc} = \frac{V_1 - V_2}{I_1 - I_2}$.
Discharge ratio. It expresses the rate of discharge or charge of the battery compared with its maximum capacity. Mathematically, the discharge ratio can be obtained from the capacity definition of a battery. For example, a battery with a nominal capacity of 7 Ah can supply 7A for 1 hour. If the same battery is discharged for 20 hours, its current becomes 350 mA, but maintains the same nominal capacity.

2.3 Battery model

Several models that characterize the performance of the battery are reported in the literature. The analytical models are based on physics processes, whereas the stochastic approaches use random systems [1]. Nonetheless, the primary factor in choosing the model is its applicability. In this work, the Jackey's model was chosen, as shown in Figure 1, to represent the battery discharge behavior, due to all parameters being a function of electrical quantities such as voltages and currents.



(b)

(a)

Figure 1: Jackey's model: (a) Structure of the battery nonlinear equivalent circuit [5]. (b) Simplified battery equivalent circuit [2].

Disregarding the parasitic current branch and simplifying the nonlinear Jackey's model, presented in the Fig. 1(a), is reduced to a more simple form, Figure 1(b) From the equivalent circuit shown in Fig. 1(b), it is possible to write the dynamic equations of the battery, according to Equations 2 and 3 for nodes 1 and 2, respectively, using Kirchhoff's law of knots.

$$\frac{E_m}{R_0} - \frac{V_1}{R_0} - C_1\frac{d}{dt}(V_1 - V_2) - \frac{(V_1 - V_2)}{R_1} = 0 \tag{2}$$

$$C_1\frac{d}{dt}(V_1 - V_2) + \frac{(V_1 - V_2)}{R_1} - i_2 = 0 \tag{3}$$

Then, equations 2 and 3 are rewritten in matrix form 4:

$$\begin{bmatrix} -C_1 & C_1 \\ C_1 & -C_1 \end{bmatrix} \begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \end{bmatrix} = \begin{bmatrix} -\frac{E_m}{R_0} \\ i_2 \end{bmatrix} + \begin{bmatrix} \left(\frac{1}{R_0} + \frac{1}{R_1}\right) & -\frac{1}{R_1} \\ -\frac{1}{R_1} & \frac{1}{R_1} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{4}$$

In the equation 4 there are four unknown parameters, which are the electromotive force E_m, the resistance R_0 seen by the battery terminals, the capacitance C_1 that simulates the voltage delay and the resistance R_1 that varies according to the depth of the charge. These parameters, however, cannot be determined directly because they are all dependent on other parameters, including open-circuit voltage, state of charge, and electrolyte temperature, as described in [5].

Unknown parameters, according to [2], are defined as:

$$E_m = E_{m0} - K_E(273 + \theta)(1 - SOC) \quad (5)$$
$$R_0 = R_{00}[1 + A_0(1 - SOC)] \quad (6)$$
$$R_1 = R_{10}e^{(1-kSOC)} \quad (7)$$
$$C_1 = \frac{\tau_1}{R_1} \quad (8)$$

where the new unknown parameters, which will be identified in this work, are the constant A_0, the resistance R_{10}, the time constant τ_1, the initial load state $SOC_{t=0}$ and the constant k.

Each constant of the equations above was empirically determined, such as the electrolyte temperature θ (ºC), the constant K_E (V/ºC), and the open-circuit voltage at full charge E_{m0} (V). Also, the first value of resistance $R_0(0)$ was estimated according to [2].

3 System identification method

The process of identifying a dynamics system entails obtaining a model—a mathematical representation of the system's behavior. According to [6], these kinds of models can be grouped into three categories:

- White box: The behavior of the system is described from a knowledge *a priori* of the phenomena involved in the process, and it is necessary to define what the physical variables will be and apply the equilibrium relations and physical laws to obtain the mathematical model. However, this is not always feasible, due to the high degree of complexity of the system. In white box modeling, the analysis of the dynamic behavior of the system is done, for example, through the solution of the differential equations obtained.
- Black box: The dynamics of the system is done from experimental data, without any information *a priori*, which is an advantage when one has no knowledge of the process involved or when analytically describing the dynamics of the system is a very complex task.
- Gray box: In addition to experimental data, auxiliary information related to the process is collected to describe the system.

In this work, the gray box model is used for identification. An optimization algorithm will be used to minimize a cost function, which is a function of the error between the model and the measured output.

3.1 Optimization tool

An optimization tool was utilized for the Jackey's model identification process, whose objective is to minimize the error the computed and measured discharge curves. The algorithm for identification was performed using the Matlab environment and performed in two stages. In the initial stage, the initial conditions for the problem are identified through the application of a genetic algorithm (GA). In the second stage, a nonlinear optimization is employed just to fit the model, adjusting the model's parameters to optimize match between the model's predictions and the experimental data.

The Genetic Algorithm (GA) is a robust optimization method and search algorithm inspired by evolutionary biological systems, as proposed by Holland (1975) [7, 8]. It employs the Darwinian survival-of-the-fittest strategy as a search algorithm. The genetic algorithm comprises several essential steps to complete the search task. It begins with the initialization of the population with N chromosomes, then progresses to the evaluation phase, wherein it calculates the fitness for each chromosome. An essential step is mutation, which enables the algorithm to avoid stopping at local minimal points. It then proceeds to the substitution step and repeats this process until it reaches the stopping criterion.

In Matlab, the **fmincon** function belongs to a class of nonlinear programming algorithms, which find the optimal point of a given problem by minimizing your objective function subject to constraints. In this work was applied the *active-set* algorithm to perform the optimization. The objective function in this study is expressed in terms of the outputs from the model.The voltage V_2 in Equation 4, and the measure of the discharge voltage. The objective function, or cost function, was defined as the mean squared error (MSE), according to Equation 9.

$$J(\theta) = \frac{1}{N}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2 \tag{9}$$

where y is the measured output (discharge voltage), $\hat{y}$ is the estimated output by the model (voltage V_2) and N is the number of samples used for identification. Also, the objective function J is a function of the unknown parameters $\theta = \begin{bmatrix} A_0 & R_{10} & \tau_1 & SOC_{t=0} & k \end{bmatrix}$.

Overall, five parameters were tuned, given an initial value by using of the genetic algorithm of Matlab from the **ga** function. The minimum and maximum constraints for optimization was not a trivial process. So, several simulation using the dynamic equations of battery were executed for different parameters. In this work was used the **ode15s** solver to compute the model's state of the Equation 4.

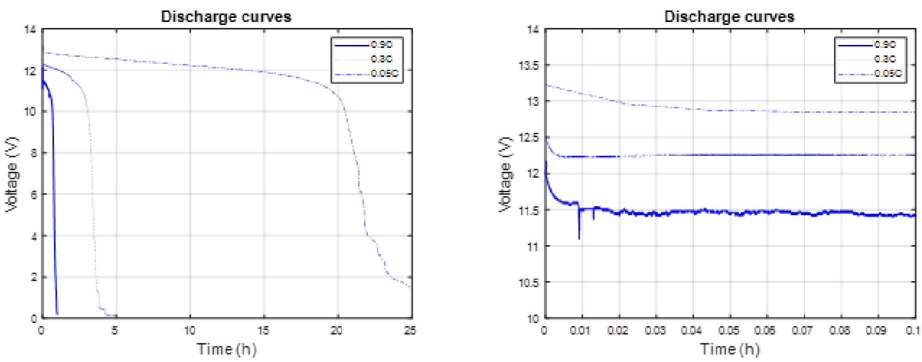
4 Experimental trial results

The outcomes of a lead-acid battery's system identification will be shown in this section. The process includes analyzing the experimental battery discharge voltage data using an optimization method to determine the unknown parameters, as mentioned in subsection 2.3.

4.1 Data gathering for system identification

Experimental discharge curves of a lead-acid battery with a nominal capacity of 7Ah were obtained using Arduino for data acquisition. The data were collected and interpolation was performed to adjust the sampling period of all curves in 80 s. Figure 2(a) shows three different discharge ratios, 0.9 C, 0.3 C and 0.05 C. To obtain the discharge curves, three different resistive loads were used.

The graphs that were obtained show that the battery discharge time increases with decreasing discharge ratio C. Also, according to Figure 2(b), the effect of the battery's internal resistance is observed at the initial instant of the discharge process, where the battery's no-load condition is changed to the charged condition, with a sudden decline in voltage from the battery in response to the applied load.



(a) (b)

Figure 2: Discharge curves: (a) for three different discharge ratio and (b) Figure 2(a) enlarged.

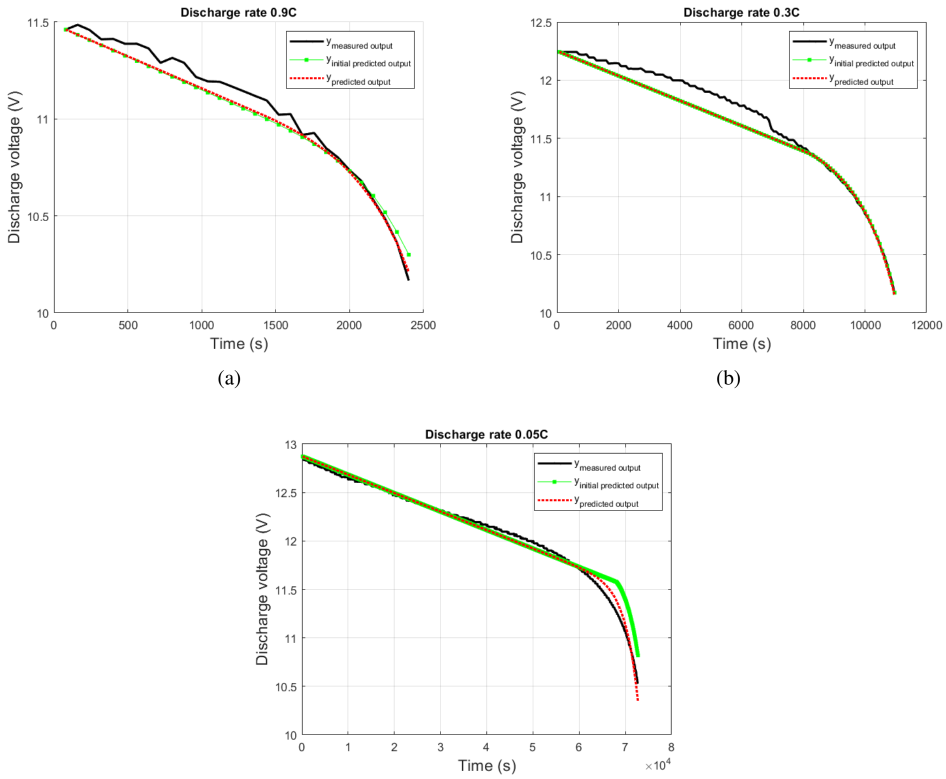
Also, current data were acquired, to observe the two electric variables at the time of battery discharge. It is worth mentioning that the value of the chain i_2 is considered known during the process of identifying the Jackey's model. In this work, the value of the discharge current was obtained by the average of the current values measured in the time window used for identification.

4.2 Performing the identification system

To carry out the identification, the Matlab software was used to implement the algorithm. As optimization tools applied in this work, the optimization function **ga** was used to obtain an initial estimate of the model parameters. Then, the initial estimate obtained by **ga** was used as the initial condition for the **fmincon** function. After numerous simulations to find suitable parameter values for the model, the following constraints were applied for both optimization methods:

$$\begin{bmatrix} 0.01 & 90 & 1\times10^{6} & 0.6 & 1 \end{bmatrix} \leq \begin{bmatrix} A_0 & R_0 & \tau_1 & SOC_{t=0} & k \end{bmatrix} \leq \begin{bmatrix} 10 & 1\times10^{10} & 1\times10^{13} & 1 & 30 \end{bmatrix} \quad (10)$$

After the optimization, the output prediction were plotted, as shown in Figure 3, for three different discharge ratios, whose average current values i_2 are 5.90 A, 1.94 A and 0.52 A, respectively. It is observed that the predictions obtained by the **ga** algorithm are slightly worse than the predictions obtained by the **fmincon** algorithm, which is expected.



(a) (b) (c)

Figure 3: Output predicted by the model for different discharge rates: (a) 0.9C (b) 0.3C, and (c) 0.05C.

It is important to highlight that the discharge curves shown in Figure 2 were obtained for a very long time interval, which allowed the battery to discharge to very low voltage values, which in practice is not recommended, since this degrades the battery life. Also, for identification purposes, the curves were windowed in a shorter time interval, taking as reference the initial part of the data where the discharge current i_2 remained practically constant, which facilitated the application of the proposed model, which assumes that the discharge current is constant. Also, in Table 1 are presented the parameters of the identified models for these three discharge curves are presented. It is observed that the time constant identified is coherent and is associated with the discharge time of each curve, as well as the other parameters are within the expected range.

Finally, a model validation criteria, for system identification in the time domain performed in this paper to evaluate the quality of the identified model is the Mean Relative Squared Error (MRSE), according to Equation 11, which is defined as [9]. The comparison of the identified model performance as a function of the MRSE index, as shown in Table 2, indicates that the model predicted outputs in all cases present small prediction errors.

$$MRSE(\%) = \frac{1}{l}\sum_{i=1}^{l}\sqrt{\frac{\sum_{j=1}^{N_{val}}(y_j-\hat{y_j})^2}{\sum_{j=1}^{N_{val}}(y_j-\bar{y})^2}}\times 100 \quad (11)$$

where y is the measured output, $\hat{y}$ is the estimated output by the model, $\bar{y}$ is the average value of y, l is the output

Table 1: Model parameters determined by taking into account the three discharge ratios.

Discharge ratio	A_0	R_{10}	τ_1	$SOC_{t=0}$	k
0.9C	0.01	1.49×10^9	3.13×10^{12}	0.63	12.91
0.3C	0.01	4.34×10^7	4.47×10^{12}	0.60	12.85
0.05C	1.25	137.72	8.89×10^{12}	0.60	18.74

number of the system and N_{val} is the number of samples used for validation. A MRSE value equal to zero indicates a perfect model.

Table 2: Values of the MRSE index for system identification.

Discharge ratio	0.9C	0.3C	0.05C
MRSE(%)	1.32	1.90	1.34

5 Conclusions

The model identification of a lead-acid battery using experimental data was reported in this paper. The system's unknown characteristics were estimated through optimization, employing Jackey's model for gray box identification. Based on the output predictions achieved and the estimated parameters, one can conclude that the findings were good and that the recognized model is representative of the system, having a low MRSE index.

Acknowledgements. This research is supported by UNESP.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors or has the permission of the owners to be included here.

References

[1] da C. T. Silva. ***Sistema de gerenciamento de bateriais de lítio com estimação de estados com filtro de Kalman estendido adaptativo.*** PhD thesis, Escola Politécnica da Universidade de São Paulo - USP, 2022.

[2] V. G. C. Ogawa. **Proposta de identificação dos parâmetros do modelo de bateria para uso na modelagem de sistemas de partida de veículos automotivos**. Master's thesis, Escola Politécnica da Universidade de São Paulo - USP, 2011.

[3] N. M. C. Pereira. **Simulação de Regimes de Carga e Descarga em Baterias**. Master's thesis, Instituto Superior de Engenharia de Lisboa, 2016.

[4] B. Yang, J. Wang, P. Cao, T. Zhu, H. Shu, J. Chen, J. Zhang, and J. Zhu. Classification, summarization and perspectives on state-of-charge estimation of lithium-ion batteries used in electric vehicles: A critical comprehensive survey. *Journal of Energy Storage*, vol. 39, pp. 102572, 2021.

[5] R. A. Jackey. A simple, effective lead-acid battery modeling process for electrical system component selection. *SAE Technical Paper*, vol. , pp. 11, 2007.

[6] L. A. Aguirre. *Introdução à identificação de sistemas: técnicas lineares e não-lineares aplicadas a sistemas reais*. Editora UFMG, Belo Horizonte, Minas Gerais, 2007.

[7] J. H. Holland. Genetic algorithms and the optimal allocation of trials. *SIAM journal on computing*, vol. 2, n. 2, pp. 88–105, 1973.

[8] A. Altinten, S. Erdogan, F. Alioglu, H. Hapoglu, and M. Alpbaz. Application of adaptive pid control with genetic algorithm to a polymerization reactor. *Chemical Engineering Communications*, vol. 191, n. 9, pp. 1158–1172, 2004.

[9] R. C. Machado. **Métodos de subespaços para identificação de sistemas em malha fechada**. Master's thesis, Instituto Tecnológico de Aeronáutica, 2013.