The Application of Monte Carlo Simulation and Finite Element Method in Structural Reliability of Aircraft Wing Structures

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Abstract. The aerospace industry increasingly demands performance and efficiency, therefore ensuring the structural integrity of aircraft components such as wings is crucial. In this context, reliability analysis methods offer a more robust assessment than classical deterministic approaches, once those consider the uncertainties present in the system. This work aims to determine the probability of failure of aircraft wing structures through the Monte Carlo Simulation (MCS), treating material properties and applied loads as random variables. Different profiles of wings were modeled in a computer-aided design (CAD) environment and their geometries were exported to MATLAB®, where the limit state function was evaluated using the finite element method (FEM).

Keywords: Structural reliability, Monte Carlo simulation, Aerospace structures

1 Introduction

Classical aircraft design methods rely on deterministic structural analysis, where both loading and structural resistance are assumed to be fixed values. This approach involves assessing various loading conditions that may occur during the structure's lifetime, setting maximum values for the loading and minimum values for the resistance, with the aim of achieving satisfactory structural safety in a "worst-case" scenario [1, 2]. A safety factor, usually 1.5, is also applied to provide an additional safety layer to cover cases where the loading exceeds the expected maximum or any inaccuracies in the design calculations [1]. This deterministic method of addressing structural safety corresponds to the level zero of reliability methods [3], which may not adequately address structural safety from scientific, probabilistic or economic perspectives, since stress and strain are not always linear and allowable stress does not ensure the same level of safety for all structural components [4]. In this context, the structural reliability evolved due to the need to combine concepts of safety and costs of engineering projects, and studies and applications of reliability methods in aircraft structures have been conducted.

Ning et al. [5] combined FEM and MCS to calculate the reliability of structural stress and determine the sensitivity of design variables for an aircraft wing under gust load. Pradlwarter et al. [2] proposed a method for reliability computation of aerospace structures, using a gradient estimation sampling procedure to reduce the computational cost compared to MCS. El Maani et al. [6] analyzed the fluid-structure interaction of an aircraft wing to determine its reliability using both FORM and SORM. The analysis focused on the first natural frequency of the coupled system, constrained by a certain value. Hraiba et al. [7] conducted a reliability study of an aircraft wing in the presence of uncertainties due to manufacturing tolerance or material inhomogeneity, with Young's modulus and material density as random variables. The authors used MCS and a reduced order model formulation to examine the dynamic response of the system. Wansaseub et al. [8] applied a surrogate-assisted method to perform reliability-based design optmisation (RBDO) of an aircraft wing structure, modeling a bi-objective optmization problem to minimise the wing mass and increase its reliability index, subject to structural and aeroelastic constraints. Kumar et al. [9] implemented an algorithm based on FORM for flutter reliability analysis of an aircraft wing in the frequency domain, where aeroelastic reliability of the wing is investigated considering geometric, structural and aerodynamic properties as Gaussian random variables.

In this work, we investigate the structural reliability of two types of aircraft wings using MCS and FEM. The study is conducted in the MATLAB® environment using the Partial Differential Equation Toolbox, with the wing geometries modeled in Solidworks®. Three cases are evaluated: static, modal, and transient.

2 Theory of strucutral reliability

The reliability of an engineering system is the probability that it will not fail, within a specified useful life and as long as its operating and design conditions are respected. When evaluating the reliability of a structural component, the aim is to determine the probability that the result load S acting on it will exceed its resistance R. This is called probability of failure and is defined as:

$$P_f = \mathrm{P}[S \geq R] = \int_{\Omega_f} f_{RS}(r,s)drds, \tag{1}$$

where Ω_f is the failure domain and f_{RS} is the joint probability density function of R and S [10].

2.1 Reliability index

The structural reliability can be determined by the safety margin $M = R - S$, where negative values indicate failure and positive values indicate survival. If R and S are random variables, then M will also be a random variable, and the probability of failure can be calculated from M as [10]:

$$P_f = \mathrm{P}[M \leq 0] = \int_{-\infty}^{0} f_M(m)dm. \tag{2}$$

Given that R and S are both normally distributed and uncorrelated, M will also be normally distributed, and its parameters are calculated as follows:

$$\mu_M = \mu_R - \mu_S \text{ and} \tag{3a}$$

$$\sigma_M = \sqrt{\sigma_R^2 - \sigma_S^2}. \tag{3b}$$

The reliability index, which is a measure of safety defined as the distance from the mean of M to the failure surface can be expressed as [10]:

$$\beta = \frac{\mu_M}{\sigma_M}. \tag{4}$$

By using the Hasofer-Lind transformation, one can convert M into a standard normal variable Y:

$$Y = \frac{M - \mu_M}{\sigma_M}. \tag{5}$$

This enables assessing probabilities related to M through the standard cumulative distribution funcition Φ. The probability of failure then becomes [10]:

$$P_f = \Phi(-\beta). \tag{6}$$

2.2 Limit states

Let $\mathbf{x} \in \mathbb{R}^n$ be a vector of all basic random variables. For each failure mode i, one can define a function of the design parameters $\mathbf{p}$, the random variables $\mathbf{x}$ and time t, so:

$$g_i = g_i(\mathbf{p}, \mathbf{x}, t), \tag{7}$$

called limit state function, which is positive only when a structure characterized by $\mathbf{p}$ is safe in mode i at time t.

The limit state functions establish, for each failure mode, the boundary between the failure and survival domains. It is convenient to consider $g_i(x) = 0$ in the failure domain. Therefore, the failure and survival domains are defined according to Eqs. (8a) and (8b), repectively [3]:

$$\Omega_f = \{\mathbf{x}|g_i(\mathbf{x}) \leq 0\} \text{ and} \tag{8a}$$

$$\Omega_s = \{\mathbf{x}|g_i(\mathbf{x}) > 0\}. \tag{8b}$$

3 Monte Carlo simulation

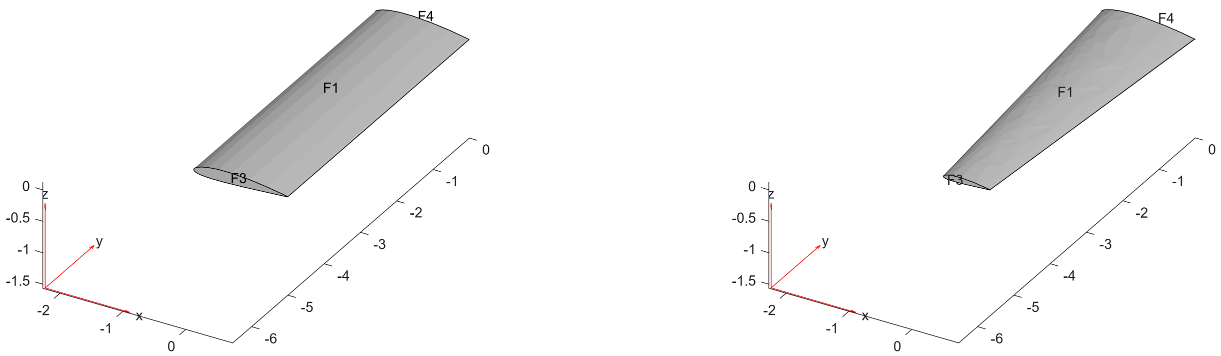
The Monte Carlo simulation consists of generating random samples to artificially simulate a large number of experiments and then observing the result. Applied to structural reliability, a sampling of each random variable is performed, and then the limit state function $g(x)$ is evaluated. If this function is violated (*i.e.* $g(x) \leq 0$), the structure has failed. The process is then repeated many times, and if N attempts are made, the probability of failure can be estimated as:

$$P_f \approx \hat{P}_f = \frac{n(g(x) \leq 0)}{N}, \tag{9}$$

where $n(g(x) \leq 0)$ is the number of attempts in which the limit state function was violeted [10].

4 Applications and results

Two NACA 2412 wing structural models are shown in Fig. 1, one straight and the other tapered. Face F4 is completed clamped to the aircraft fuselage, and a uniform load P is normally applied to face F1. The wings have a semi-span of 5 m and a chord of 1.5 m. The taper ratio of the tapered wing is 0.5. The wings are constructed from Al 2024-T3 aluminum alloy.



(a) Straight wing. (b) Tapered wing.

Figure 1. Wing structural models.

Table 1 presents the statistical parameters of the system's random variables. Additionally, the Poisson's ratio is assumed to be deterministic with value $\nu = 0.33$.

Table 1. Statistical parameters of the random variables

Random variable	Symbol	Distribution	Mean	Coefficient of variation
Young's modulus (GPa)	E	Normal	72	5%
Density (kg/m^3)	ρ	Normal	2800	5%
Load (kN)	P	Extreme Type II	40	25%
Yield strength (MPa)	σ_Y	Lognormal	345	5%

4.1 Static case

The first study involves assessing the reliability of the wings under a static load applied to the upper surface to simulate lift, where the wing stress caused by the load should not exceed its yield strength. Thus, the limit state function is defined as:

$$g(X) = \sigma_Y - \sigma_{max}. \tag{10}$$

By using MCS with 10000 samples, the reliability of the wings over the number of simulations and the 95% confidence interval (CI) are shown in Fig. 2. This shows that the curves tend to a stable value as the number of

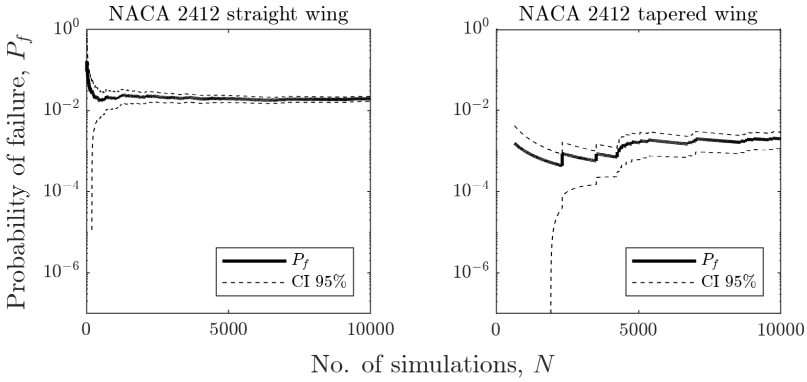


Figure 2. Probability of failure over the number of simulations.

simulations increases. It is also observed that the tapered wing presents a lower probability of failure compared to the straight one, with $\hat{P}_{f\,tap} \approx 10^{-3}$ versus $\hat{P}_{f\,str} \approx 10^{-2}$.

The calculated $\hat{P}_f$ and the related β are shown in Fig. 3, wich presents the relationship of both measures.

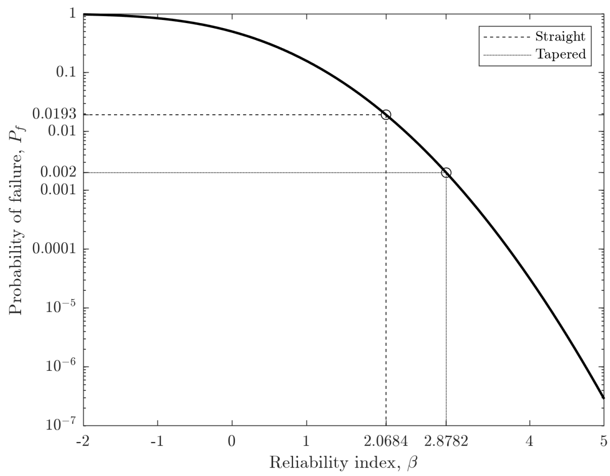
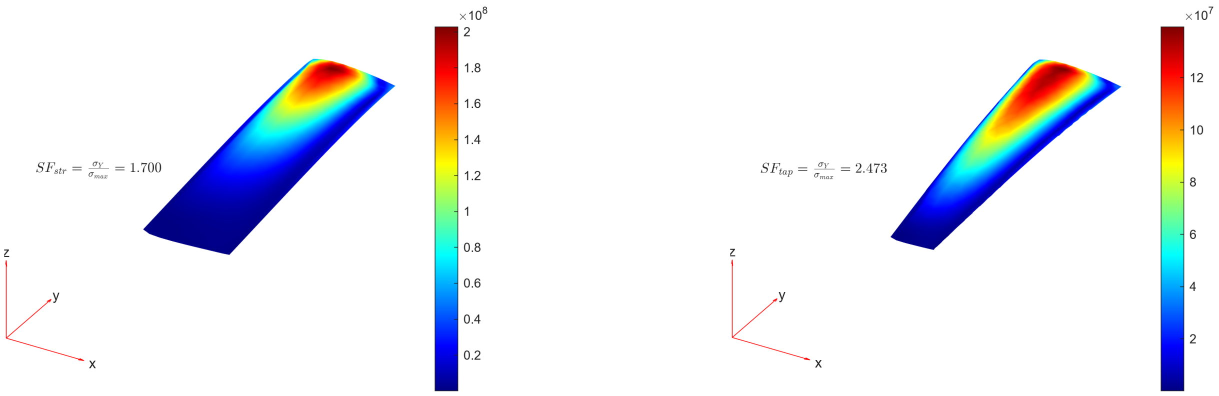


Figure 3. Relationship between probability of failure and reliability index.

A deterministc approach, using the mean values as input, shows that the safety factors of the wings are $SF_{str} = 1.70$ and $SF_{tap} = 2.47$ for the straight and tapered wing types, respectively, as shown in Fig. 4.



(a) Straight wing.

(b) Tapered wing.

Figure 4. Von Mises stress applied to the wings.

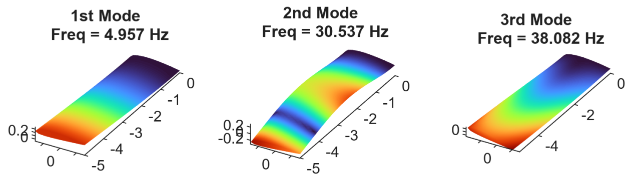
4.2 Modal case

The modal case study focuses on the first natural frequency, which should exceed $F_0 = 4.5$ Hz. The limit state function is defined as:

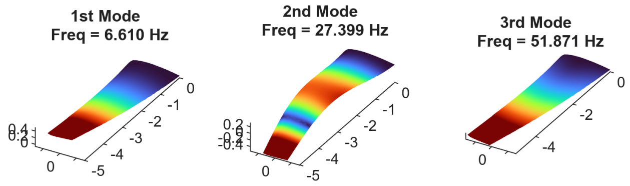
$$g(X) = F_1 - F_0, \tag{11}$$

where F_1 is the first natural frequency. The number of simulations is $N = 10000$ and the calculated reliability index of the straight wing is $\beta_{str} = 2.4949$, with a coefficient of variation $\text{cov}_{str} = 12.6\%$. No sample has fallen into the failure domain for the tapered wing, indicating a high level of reliability for the proposed limit state function. In cases like this, a higher number of simulations should be conducted to achieve a more accurate reliability assessment.

Using the mean values as input, the first three natural frequencies and their respective vibration modes are show in the Fig. 5:



(a) Straight wing.



(b) Tapered wing.

Figure 5. First three modes shapes and respectives vibrational frequencies of the wings.

It is observed in Fig. 5b that the first natural frequency of the tapered wing is $F_1 = 6.61$ Hz, significantly higher than F_0.

4.3 Transient case

To assess the impact of inertial forces on structural reliability, a transient analysis was performed using a sinusoidal force with amplitude P and a frequency $\Omega = 2$ Hz applied to the face F1 of the wings. The limit state function (Eq. (10)) was resolved using modal superposition, with a damping ratio $\zeta = 5\%$.

The chart in Fig. 6 illustrates that both straight and tapered wings exhibit a lower reliability index under dynamic analysis compared to static analysis, with reductions of 13% and 16%, respectively.

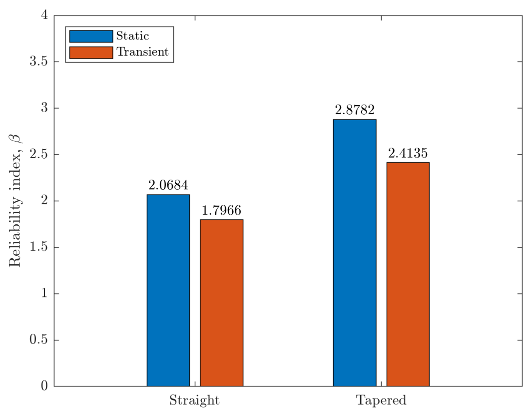


Figure 6. Comparison of reliability indexes for static and dynamic analysis.

5 Conclusions

This study assessed the structural reliability of two wing configurations, one straight and one tapered, using Monte Carlo simulation and finite element analysis. Applied forces and material properties were treated as random variables to account the uncertainties in the system. Three types of analyses were conducted: static, modal, and transient. The static and transient analyses focused on ensuring that the maximum stress within the wings did not exceed the material's yield strength, while the modal analysis verified that the first vibrational frequency of the wings was above a specified threshold. The results showed that the tapered wing demonstrated greater reliability than the straight wing. Additionally, inertial forces impacted wing reliability, as indicated by a decrease in the reliability index during dynamic analysis compared to the static analysis. This emphasizes the importance of considering dynamic forces in the design and evaluation of structural components to ensure their performance and reliability in real-world conditions.

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References

[1] F. H. Hooke. *Aircraft structural reliability and risk analysis*, pp. 131–170. Springer Netherlands, Dordrecht, 1987.

[2] H. Pradlwarter, M. Pellissetti, C. Schenk, G. Schuëller, A. Kreis, S. Fransen, A. Calvi, and M. Klein. Realistic and efficient reliability estimation for aerospace structures. *Computer Methods in Applied Mechanics and Engineering*, vol. 194, n. 12, pp. 1597–1617. Special Issue on Computational Methods in Stochastic Mechanics and Reliability Analysis, 2005.

[3] H. O. Madsen, S. Krenk, and N. C. Lind. *Methods of Structural Safety*. Dover, New York, 2006.

[4] S. M. C. Diniz. A confiabilidade estrutural e a evolução das normas técnicas. In *VI Simpósio EPUSP sobre Estruturas de Concreto*, São Paulo, 2006.

[5] X. Ning, Y. Yan, K. Qu, and Z. Li. Reliability analysis on wing structures under the gust load. In *Artificial Intelligence and Computational Intelligence*, pp. 31–37, Berlin, Heidelberg. Springer, 2010.

[6] R. El Maani, B. Radi, and A. ELHami. Vibratory reliability analysis of an aircraft's wing via fluid–structure interactions. *Aerospace*, vol. 4, pp. 40, 2017.

[7] A. Hraiba, A. Touil, and A. Mousrij. An efficient reduced model applied to the study of the mechanical reliability of an aircraft's wing. In *International Journal of Emerging Trends in Engineering Research*, volume 8, 2020.

[8] K. Wansaseub, S.Sleesongsom, N. Panagant, N. Pholdee, and S. Bureerat. Surrogate-assisted reliability optimisation of an aircraft wing with static and dynamic aeroelastic constraints. *International Journal of Aeronautical and Space Sciences*, vol. 21, n. 3, pp. 723–732, 2020.

[9] S. Kumar, A. K. Onkar, and M. Manjuprasad. Stochastic modeling and reliability analysis of wing flutter. *Journal of Aerospace Engineering*, vol. 33, n. 5, pp. 04020044, 2020.

[10] R. E. Melchers and A. T. Beck. *Structural Realiability Analisys and Prediction*. Wiley, New Jersey, 2018.