Nonlinear metabeam for vibration absorption and energy harvesting

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Abstract. Metamaterials are engineered materials that surpass conventional ones with their exceptional performance. These systems exhibit diverse properties, including excellent energy absorption, acoustic insulation, cloaking, negative Poisson's ratio, and more. In addition, they can possess the remarkable ability to serve multiple functions simultaneously, thereby achieving more than one objective. This study delves into the creation of a beam based on metamaterial engineered to attenuate vibrations while harvesting this kinetic energy for electricity generation. The system works by preventing elastic wave propagation along its length and focusing the incident wave into piezoelectric generators, responsible for the energy conversion. Consequently, we can use this energy to power microelectronic devices such as sensors and actuators. The proposed metabeam consists of a cantilever beam with nonlinear resonators equipped with piezoelectric layers. This arrangement establishes a wide local resonance bandgap on the frequency spectrum of the host beam forbidden the elastic wave propagation. The incident wave is transmitted to the resonators, where they convert kinetic energy into electricity. Nonlinearity is strategically introduced into the resonator to broaden the operational bandwidth. Mathematical modeling is developed, and numerical simulations are conducted to study and demonstrate the performance of the proposed system.

Keywords: Metastructure, Nonlinear dynamics, Vibration attenuation, Energy harvesting

1 Introduction

Metamaterials are specially engineered systems that possess unique properties not typically found in [1]. These systems are often made up of carefully arranged periodic or non-periodic configurations, allowing them to exhibit behavior and properties that are difficult to find in nature. Examples of such systems include photonic crystals [2] and locally resonant structures [3]. Other notable properties of metamaterials are discussed in [4] and [5]. At the same time, metastructure refers to a larger-scale system composed of arranged metamaterial unit cells or resonant elements. It is often used to describe a structural configuration that leverages the principles of metamaterials to achieve specific mechanical or physical responses.

Locally resonant metastructures are systems with an effective ability to store and transfer energy through resonant elements, whether mechanical or electromechanical. These structures are highly capable of absorbing vibrations and integrating energy harvesters. For instance, Sugino and Erturk [6] introduced an electromechanical modeling framework for two representative systems and analyzed their frequency response and energy harvesting potential. They demonstrated that significant energy could be harvested without compromising the vibration attenuation achieved in the bandgap. Nowadays, nonlinear metastructures are developed based on inserting nonlinearities into the resonators. In this way, Vasconcellos et al. [7] explored a discrete metastructure combining linear and nonlinear absorbers, highlighting the benefits of nonlinear oscillators for optimizing system performance. Similarly, Chen et al. [8] studied a plate metastructure with linear resonators, while Lu et al. [9] developed a plate model incorporating nonlinear resonators.

The present study focuses on developing a metastructure based on a continuous beam that can effectively reduce vibrations while simultaneously generating electrical energy. The main objective is to design a metastructure that utilizes nonlinear bistable resonators to create optimal bandgaps and address the challenges associated with vibrational energy harvesting. A cantilever beam is selected as the host structure and is modeled using the Euler-Bernoulli beam theory. Multiple local resonators, functioning as mechanical vibration absorbers, are distributed along the beam, each equipped with piezoelectric patches to transform the kinetic energy into electricity. The resonators are designed with a nonlinear restoring force with two-stable equilibrium points (bistable oscillator) to expand the operating frequency range, thereby enhancing the system's real-world performance. Numerical simulations are conducted to examine the benefits and potential challenges of utilizing bistability for both vibration reduction and energy harvesting. A comparison with linear resonators is also provided, demonstrating the superior performance of the nonlinear resonators in this context.

2 Modelling of the metasctructure with piezoelectric bistable resonators

The system under consideration consists of a cantilever beam with piezoelectric bistable resonators under a base motion excitation, arranged as depicted in Figure 1. The beam, with a length of L, is clamped at one end while the other end remains free to vibrate. It has a mass of m_b and a flexural stiffness of EI. Each resonator, characterized by a mass m_j, acts as a bistable oscillator ($j = 1, \ldots, S$, where S represents the number of resonators) and is connected to the beam via a linear stiffness $k_{l_j} < 0$, cubic stiffness $k_{nl_j} > 0$, and damping coefficient c_j. Additionally, each resonator incorporates a piezoelectric element with electromechanical coupling ϑ_j, integrated into a resistive circuit.

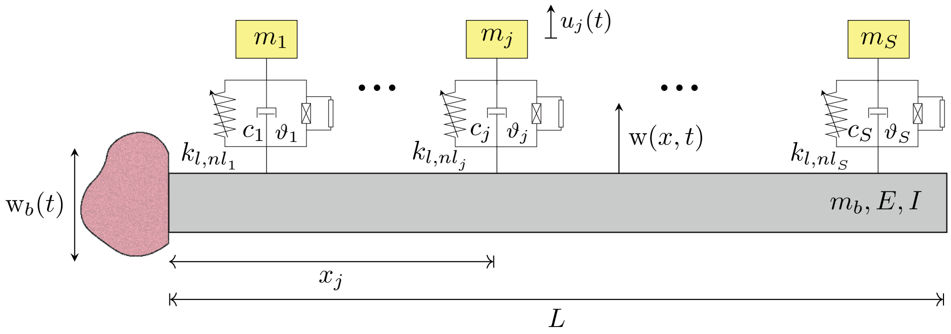


Figure 1. Schematic illustration of the metastructure based on a cantilever beam with bistable piezoelectric resonators inserted along its length.

The system dynamics is modeled using a continuous beam model based on Euler-Bernoulli beam theory with discrete masses coupled along its length. By utilizing the orthogonality properties of the modal shapes and applying the Galerkin method with the cantilever beam's modal functions, denoted as $\phi_r(x)$ with considering N number of modes, the resulting system of ordinary differential equations can be derived. Equation 1 represents the governing dynamics of the cantilever beam, where $\eta_r(t)$ corresponds to the modal displacement of the r-th mode, and ω_r and ζ_r are the natural frequency and the damping ratio associated with that mode. Equation 2 describes the governing of motion for each resonator. Equation 3 represents the electrical resistive circuit installed in each resonator, where $C_{p,j}$ is the effective piezoelectric capacitance, $\mathcal{Y}_j$ is the admittance of the resistive circuit with resistance of R_l.

$$\ddot{\eta}_r(t)+2\zeta_r\omega_r\dot{\eta}_r(t)+\omega_r^2\eta_r(t)-\sum_{j=1}^{S}\left(c_j\dot{u}_j(t)+k_{1_j}u_j(t)+k_{3_j}u_j^3(t)-\vartheta_j v_j(t)\right)\phi_r(x_j)=-m_b\ddot{\mathrm{w}}_b(t)\int_{x=0}^{L}\phi_r(x)dx\,, \tag{1}$$

$$m_j\left[\sum_{r=1}^{N}\ddot{\eta}_r(t)\phi_r+\ddot{u}_j(t)\right]+c_j\dot{u}_j(t)+k_{1_j}u_j(t)+k_{3_j}u_j^3(t)-\vartheta_j v_j(t)=-m_j\ddot{\mathrm{w}}_b(t),\quad j=1,2,\ldots,S\,, \tag{2}$$

$$C_{p,j}\dot{v}_j(t) + \mathcal{Y}_j v_j(t) + \vartheta_j \dot{u}_j(t) = 0\,, \quad j = 1, 2, \ldots, S\,, \tag{3}$$

Transmissibility in function of excitation frequency, Ω, is calculated as the root mean square (RMS) of the ratio between the total velocity at the beam's tip, $\mathrm{w}_t(L,t)$, and that at its base, $\mathrm{w}_t(0,t)$, as shown in Equation 4.

$$T(\Omega) = \text{RMS}\frac{\mathrm{w_t(L,t)}}{\mathrm{w_t(0,t)}} \tag{4}$$

To evaluate energy generation, it is calculated as the sum of the RMS voltage dissipated in each resistor, as given by Equation 5. Power can also be obtained as Equation 6.

$$V(\Omega) = \sum_{j=1}^{S} \text{RMS}\ v_j(t) \tag{5}$$

$$P(\Omega) = \sum_{j=1}^{S} \text{RMS}\ \frac{v_j(t)^2}{R_l} \tag{6}$$

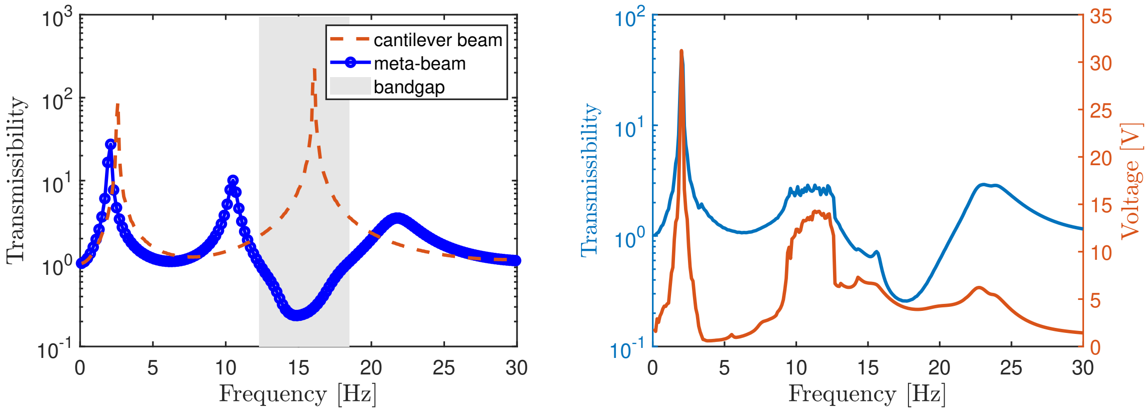
3 Numerical results and discussion

The model equations are numerically integrated using the Runge-Kutta method. All resonators ($S = 7$) are considered identical and equally spaced. The resonators were designed to absorb the vibration at the second natural frequency of the beam (around 16 Hz). These parameter values are presented in Tab. 1 according to Norenberg and Cunha Jr [10].

Table 1. Physical parameters of the metastructure beam with piezoelectric bistable resonators.

Beam length, L	889 mm
Beam width, b	31.75 mm
Thickness of the beam, h	2.6 mm
Young's modulus of the beam, E	69 GPa
Mass of the beam, m_b	218 g
Mass of the resonator, m_j	36 g
Linear stiffness, k_{l_j}	-63.451 N/m
Cubic stiffness, k_{nl_j}	634509 N/m^3
Beam modal damping, ζ_r	0.002
Resonator damping ratio, ζ_j	0.02
Effective piezoelectric capacitance, $C_{p,j}$	43 nF
Electromechanical coupling, ϑ_j	-4.57 mN/V
Electrical admittance, $\mathcal{Y}_j$	1/ 5kΩ

Figure 2a shows the transmissibility of the cantilever beam without the resonators and with the linear resonators. The results indicate that when the resonators are introduced, the transmissibility around the second mode is less than one, meaning that the velocity at the beam's tip is lower than the velocity of the base (excitation), indicating vibration absorption. The gray region on the graph represents the "bandgap", where this condition occurs. The bandgap has a bandwidth of 5.5 Hz. Figure 2b provides a detailed comparison of transmissibility and energy harvesting for bistable resonators. Transmissibility is shown on the left axis, while voltage is displayed on the right axis. The bandgap spans from 13.5 Hz to 21.2 Hz, with a width of 7.7 Hz (40% wider than the linear metastructure). Moreover, the nonlinear system can reduce the peak resonance before the bandgap, thereby improving its ability to dampen vibrations outside of the bandgap.

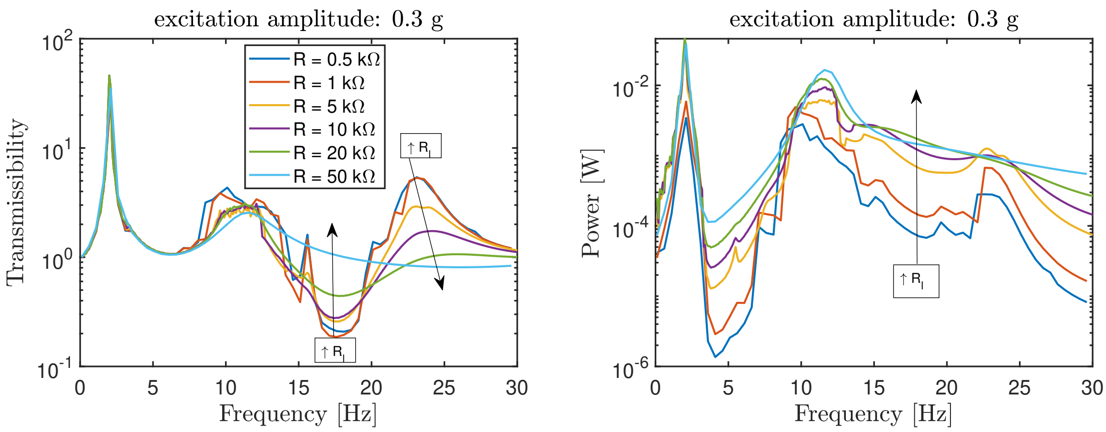


(a) linear metastructure (b) nonlinear metastructure

Figure 2. Vibration mittigation and energy harvesting performance: (a) linear metastructure and (b) nonlinear bistable metastructure.

Figure 3 shows the transmissibility for a bistable metastructure with different resistance values (500 Ω, 1 kΩ, 5 kΩ, 10 kΩ, 20 kΩ, and 50 kΩ) and an excitation amplitude of 0.3 g. The resistance has a slight influence on wave propagation and a more significant impact on the power harvested. When the resistance value is low (short-circuit $R_l \rightarrow 0$), the transmissibility response appears as if no energy harvesting system is connected to the resonator, resulting in a small power recovery. As the resistance increases, the attenuation factor on the bandgap decreases, making the gap shallower. When the resistance value is higher (open-circuit $R_l \rightarrow \infty$), there is no bandgap region. Additionally, the resonance peak after the bandgap region is attenuated as the resistance value increases.

In evaluating harvesting performance, the power curve in Fig. 3b is the most suitable metric for investigating the influence of resistance. As resistance values increase, the harvested voltage increases, albeit with a decrease in current. In the context of harvesting applications, low current is less desirable as it would result in extended charging times for batteries or devices. Consequently, the power response provides an essential method for finding a balance and determining the optimal resistance value. As depicted in Fig. 3, it becomes evident that when the resistance (R_l) is set to 50 kΩ, the power output is maximized. However, it is essential to consider that in this configuration, there is a compromise in terms of vibration attenuation observed in the transmissibility. This leads to the conclusion that a resistance value of 5 kΩ emerges as the most favorable choice. It not only delivers high power recovery but also ensures that the transmissibility remains uncompromised.



(a) Transmissibility (b) Power harvested

Figure 3. (a) Transmissibility and (b) power harvested under different resistance values for bistable metastructure.

4 Conclusions

This research demonstrates that piezoelectric metastructure offers a dual solution by attenuating incoming mechanical waves and generating low-power electrical energy. The design incorporates a cantilever beam equipped with resonators that utilize the piezoelectric effect to generate electricity. These resonators induce local resonance and create a bandgap, effectively blocking the propagation of elastic waves. The study evaluates bifunctional systems using metamastructure with linear and nonlinear resonators (bistable oscillations). Simulation results indicate that nonlinearities reduce resonance peaks in the attenuation band while expanding the bandgap. For energy generation, nonlinearity increases the operational bandwidth and enhances the total energy recovered. Therefore, the nonlinearity of resonators plays a vital role in improving both vibration suppression and energy harvesting. Additionally, the harvesting ability of the resonators can not compromise the creation of a local resonance phenomenon, demonstrating that it is possible to perform both objectives simultaneously.

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