A micromechanical approach to effective elastic properties of fiber-reinforced fractured cementitious materials

Marcos B. Guimarães[1], Cássio B. Aguiar[2], Samir Maghous [1]

[1]*Graduate Program in Civil Engineering, Federal University of Rio Grande do Sul*
Av. Osvaldo Aranha, 99, Porto Alegre/RS, Brazil
bressan.marcos@hotmail.com, samir.maghous@ufrgs.br
[2]*Department of Civil Engineering, Federal Technological University of Paraná*
R. Marcílio Dias, 635, Apucarana/PR, Brazil
cassio.barros.aguiar@gmail.com

Abstract. A micromechanical model is formulated in this work to predict the effective elastic properties of fiber-reinforced fractured cementitious materials. Fractures are discontinuities able to transfer stress and, therefore, can be regarded from a mechanical viewpoint as interfaces endowed with a specific behavior under normal and shear loading. Fibers correspond to three-dimensional materials widely used to reinforce cementitious materials, and their presence minimizes the emergence and propagation of microfractures. This work employs a micromechanical approach to formulate the homogenized elastic behavior of fiber-reinforced fractured cementitious materials. In the context of Eshelby's equivalent inclusion theory, the approach makes use of the Mori-Tanaka scheme to estimate the homogenized elastic moduli. Fractures and fibers are modeled as oblate and prolate spheroids endowed with appropriate elastic properties. Particular emphasis is given to the situation of a cementitious matrix reinforced by aligned or randomly oriented fibers with randomly oriented microfractures.

Keywords: Micromechanics, Homogenization, Fiber-reinforced cementitious materials, Microfractures

1 Introduction

Most well-known solid materials exhibit heterogeneities at different scales with various sizes and orientations. Fiber-reinforced fractured cementitious materials represent a typical example of these materials. Fractures correspond to zones of small thickness along which the mechanical and physical properties of the material are degraded. Their presence is a fundamental weak component for the stability and safety of many engineering structures. Fiber-reinforced cementitious materials are cement-based composites with improved mechanical properties in terms of tensile strength and fracture toughness. The presence of fibers enhances the post-cracking strength of cementitious materials due to their bridging effect.

The assessment of the macroscopic properties of heterogeneous media, such as fiber-reinforced fractured cementitious materials, is a complex task from a mathematical viewpoint due to their morphological complexity, which makes them one of the major concerns in material and structural engineering. The homogenization theory has proven to be an efficient tool for simplifying the mathematical treatment of heterogeneous media. Classical homogenization techniques aim to replace a complex heterogeneous material by a fictitious equivalent homogeneous material that behaves globally in the same way [1].

A significant number of studies have demonstrated success in estimating the equivalent behavior of fractured materials (see references [2–5]) and fiber-reinforced materials (see references [6–8]) using different homogenization approaches. This work will address the particular situation of fiber-reinforced fractured cementitious materials. The applications extend mainly to cementitious matrix materials reinforced by any fiber type. For that, we must quote the relevant contributions of Maghous et al. [2, 3] and Dutra et al. [7], which proposed general micromechanics-based approaches for the poroelastic behavior of jointed rock and the fiber-reinforced concrete, respectively. The main contribution of the present work is to extend the above formulations to the situation of a cementitious matrix reinforced by aligned or randomly oriented fibers with randomly oriented fractures.

The micromechanical approach is briefly presented in section 2, followed by the formulation of the homogenized elastic behavior of fiber-reinforced fractured cementitious materials in section 3. Section 4 deals with illustrative examples of a cementitious matrix reinforced by aligned or isotropic distributed fibers with isotropic distributed microfractures. Conclusions and recommendations are given in section 5.

2 Micromechanics

In this work, we addressed the mechanical behavior of fiber-reinforced cementitious materials with a isotropic fracture distribution. In order to achieve mathematical simplicity in the following formulations, we will apply the homogenization theory to assess the effective behavior of fiber-reinforced fractured materials through a continuous formulation. For this purpose, we assume the existence of a volume with a sufficiently high number of fibers and fractures that can represent the average behavior of the material at any point in the structure. Based on homogenization principles (Zaoui [1]), this volume is referred as the Representative Elementary Volume (REV), which must meet the scale separation conditions: $(d \ll \ell \ll L)$. The typical length scale ℓ of the REV should be small enough as compared to the characteristic dimension L of the whole structure, to enable to use of the differential tools of continuum mechanics. In addition, ℓ should also be large enough as compared to the characteristic length d of the heterogeneities to ensure statistical representativeness.

2.1 Hill's lemma for the fractured media

Let Ω denote the REV of a cementitious matrix Ω^m reinforced by fibers $\Omega^r = \cup_i \Omega_i^r$ cut by a discrete distribution of microfractures $\omega = \cup_i \omega_i$. At the microscopic scale, each fiber Ω_i^r is modeled as an three-dimensional material, and each microfracture is modeled as an interface ω_i, geometrically described by a surface whose orientation is defined by a normal unit vector $\underline{n}_i$, as depicted in Fig. 1.

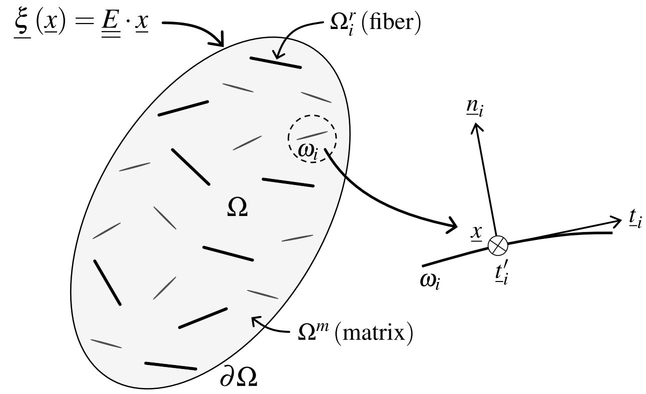

Figure 1. Representative elementary volume and loading mode

The cementitious matrix reinforced by fibers fills the domain $\Omega^m \cup \Omega^r = \Omega \backslash \omega$, where the symbol "\" stands for the set difference. Note that strains and stresses within the heterogeneous medium are defined on the fiber-reinforced cementitious matrix domain $\Omega \backslash \omega$ only, and not on the whole REV. Throughout the paper, the symbol $\langle \cdot \rangle$ denotes the volume average over the fiber-reinfoced cementitious matrix:

$$\langle \cdot \rangle = \frac{1}{|\Omega|} \int_{\Omega \backslash \omega} \cdot \ d\Omega \tag{1}$$

To define a boundary value problem in REV a homogeneous deformation condition approach is used (Fig. 1) which consists of prescribing the displacement field $\underline{\xi}$ at the boundary $\partial\Omega$ as follows:

$$\underline{\xi}(\underline{x}) = \underline{\underline{E}} \cdot \underline{x} \quad \forall \underline{x} \in \partial\Omega \tag{2}$$

where $\underline{\underline{E}}$ represents the macroscopic strain and $\underline{x}$ denotes the position vector. Hill's lemma extended to the fractured medium, established by Maghous et al. [2], for any statically admissible stress field $\underline{\underline{\sigma}}$ and any kinematically admissible displacement field $\underline{\xi}$ takes the form:

$$\langle \underline{\underline{\sigma}} \rangle : \underline{\underline{E}} = \langle \underline{\underline{\sigma}} : \underline{\underline{\varepsilon}} \rangle + \frac{1}{|\Omega|} \int_{\omega} \underline{T} \cdot [\![\underline{\xi}]\!] \, dS \tag{3}$$

in which, the term $\underline{T} = \underline{\underline{\sigma}} \cdot \underline{n}_i$ is the stress vector acting upon the fracture and $[\![\underline{\xi}]\!]$ is the displacement jump observed at the fracture interface. It is noted that $\underline{\underline{\Sigma}} = \langle \underline{\underline{\sigma}} \rangle$ represents the macroscopic stress equilibrated by the microscopic stress field $\underline{\underline{\sigma}}$ defined in the REV Ω. Since Hill's Lemma is valid for any stress and strain fields, not necessarily correlated, taking a symmetric and uniform tensor $\underline{\underline{\sigma}}$, the macroscopic strain can be written as:

$$\underline{\underline{E}} = \langle \underline{\underline{\varepsilon}} \rangle + \frac{1}{|\Omega|} \int_{\omega} [\![\underline{\xi}]\!] \overset{s}{\otimes} \underline{n} \, dS \tag{4}$$

where the symbol $\overset{s}{\otimes}$ refers to the symmetric part of the tensorial product: $(\underline{u} \overset{s}{\otimes} \underline{v})_{ij} = (u_i v_j + v_i u_j)/2$ and $\underline{n} = \underline{n}_i$ along ω_i. Therefore, the above equation shows that macroscopic strain contains two contributions, one from the cementitious matrix reinforced by fibers and the other from the microfractures.

2.2 Formulation of the homogenized elastic behavior

This section deals with the formulation of the macroscopic elastic behavior of a fiber-reinforced fractured cementitious materials. The cementitious matrix and fibers are assumed to be linearly elastic with the fourth-order stiffness tensors $\mathbb{C}^m$ and $\mathbb{C}^r$, respectively. While the displacement jump $[\![\underline{\xi}]\!]$ is related in the local coordinate system $(\underline{t}_i, \underline{t}'_i, \underline{n}_i)$ of each fracture ω_i (illustrated in Fig. 1) to the stress vector $\underline{T}$ through the elastic stiffness of the fractures $\underline{\underline{k}}^i$. These relationships are summarized as follows:

$$\begin{cases} \underline{\underline{\sigma}} = \mathbb{C}^m : \underline{\underline{\varepsilon}} & \text{in } \Omega^m \\ \underline{\underline{\sigma}} = \mathbb{C}^r : \underline{\underline{\varepsilon}} & \text{in } \Omega^r \\ \underline{T} = \underline{\underline{\sigma}} \cdot \underline{n}_i = \underline{\underline{k}}^i \cdot [\![\underline{\xi}]\!] & \text{along } \omega = \bigcup_i \omega_i \end{cases} \tag{5}$$

According to Maghous et al. [9], considering isotropy for the fracture shear response $\left(k_t^i = k_{t'}^i\right)$, the fracture stiffness can be written as $\underline{\underline{k}}^i = k_n^i (\underline{n}_i \otimes \underline{n}_i) + k_t^i (\underline{t}_i \otimes \underline{t}_i + \underline{t}'_i \otimes \underline{t}'_i)$, where k_n^i is the normal stiffness and k_t^i represents the shear stiffness components of the ith fracture family, expressed in Pa/m. These parameters are usually obtained from laboratory tests performed on specimens containing a single fracture. Goodman [10] and Bandis et al. [11] present further details related to the physical interpretation and identification procedures of k_n^i and k_t^i. Note that the particular case of cracks (discontinuities that do not transfer stresses) can addressed in this formulation by considering a null value for the stiffness components $\left(k_n^i = k_t^i = 0\right)$.

The loading condition (2) combined with the state equations (5) constitute the elastic concentration problem, which solution is given by the pair $\left(\underline{\underline{\sigma}}, \underline{\xi}\right)$. The local strains $\underline{\underline{\varepsilon}}(\underline{x})$ linearly depend on the loading parameter $\underline{\underline{E}}$ through the concept of strain concentration tensor [1]:

$$\underline{\underline{\varepsilon}}(\underline{x}) = \mathbb{A}(\underline{x}) : \underline{\underline{E}} \quad \forall \underline{x} \in \Omega \backslash \omega \tag{6}$$

where $\mathbb{A}$ characterizes the fourth-order strain concentration tensor. In other words, tensor $\mathbb{A}(\underline{x})$ is the link between the local strain $\underline{\underline{\varepsilon}}(\underline{x})$ in the $\Omega^m \cup \Omega^r = \Omega \backslash \omega$ domain to the macroscopic strain $\underline{\underline{E}}$ applied to the REV. In contrast to the continuum framework in which the volume average of over the REV is equal to fourth-order identity tensor $\mathbb{I}$, the analysis involving discontinuities leads to $\langle \mathbb{A} \rangle \neq \mathbb{I}$. The discrepancy between $\langle \mathbb{A} \rangle$ and $\mathbb{I}$ is due to the fraction of macroscopic strain that localizes in the fractures (see Maghous et al. [2]). A classical reasoning in linear elastic homogenization (Zaoui [1]) shows the link between the stress average $\underline{\underline{\Sigma}} = \langle \underline{\underline{\sigma}} \rangle$ and the macroscopic strain:

$$\underline{\underline{\Sigma}} = \mathbb{C}^{hom} : \underline{\underline{E}} \quad \text{with} \quad \mathbb{C}^{hom} = \langle \mathbb{C} : \mathbb{A} \rangle \tag{7}$$

where $\mathbb{C}^{hom}$ denotes the homogenized (macroscopic) elastic stiffness tensor. For a multi-phase composite, $\mathbb{C}^{hom}$ takes the following expression:

$$\mathbb{C}^{hom} = \sum_{\alpha=1}^{n} f^{\alpha} \mathbb{C}^{\alpha} : \langle \mathbb{A} \rangle^{\alpha} \tag{8}$$

in which n is the number of different phases of the REV, f^{α} and $\mathbb{C}^{\alpha}$ are respectively the volume fraction and the elastic stiffness tensor of phase α, whereas $\langle\mathbb{A}\rangle^{\alpha} = \frac{1}{\Omega^{\alpha}}\int_{\Omega^{\alpha}}\mathbb{A}(\underline{x})\,dV$ represents the average of the strain concentration tensor in the phase α. Therefore, the determination of $\mathbb{C}^{hom}$ reduces to that of the average concentration tensor $\langle\mathbb{A}\rangle^{\alpha}$. In this work, the evaluation of $\langle\mathbb{A}\rangle^{\alpha}$ is achieved within the framework of Eshelby's equivalent inclusion theory [12] by resorting to the Mori-Tanaka scheme [13] to evaluate the average tensor $\langle\mathbb{A}\rangle^{\alpha}$, and them $\mathbb{C}^{hom}$.

3 Homogenized elastic properties of fiber-reinforced fractured cementitious materials

The determination of the effective stiffness tensor $\mathbb{C}^{hom}$ will treated in the sequel by considering fiber-reinforced fractured cementitious materials. It is recalled that the REV is divided into three parts, namely the cementitious matrix (superscript m, volume fraction f^m), the fibers (superscript r, volume fraction f^r), and the fractures (superscript f, volume fraction f^f). It should be noted that the volume fraction of the cementitious matrix (which appears in the subsequent expressions) is directly represented by $f^m = 1 - f^f - f^r$, where $f^f = \sum_i f_i^f$ and $f^r = \sum_i f_i^r$. From a geometrical viewpoint, the fibers are represented by a set of prolate spheroids. The shape of each fiber is defined by the aspect ratio $X^r = a^r/b^r$ of the prolate spheroid, where the scalars a^r and b^r refer respectively to the equatorial radius (semi-major axis) and the polar radius (semi-minor axis). While the fractures are represented by a set of oblate spheroids with attached orthonormal frame $(\underline{t}_i, \underline{t}'_i, \underline{n}_i)$. The radius of the oblate is a^f, and the half opening is c^f. The aspect ratio $X^f = c^f/a^f$ of such a penny-shaped fracture is subjected to the condition $X^f \ll 1$. In the continuum micromechanics approach employed herein, the fractures and fibers represent inhomogeneities embedded within the cementitious matrix.

The cementitious matrix and fibers are assumed elastically isotropic and the associated stiffness tensors take the following form:

$$\mathbb{C}^m = 3k^m\mathbb{J} + 2\mu^m\mathbb{K}, \quad \text{and} \quad \mathbb{C}_i^r = 3k_i^r\mathbb{J} + 2\mu_i^r\mathbb{K} \tag{9}$$

where k^m and μ^m define the bulk and shear moduli of the cementitious matrix, k_i^r and μ_i^r represent the bulk and shear moduli of the ith fiber family. The fourth-order spherical tensor $\mathbb{J}$ and deviatoric tensor $\mathbb{K}$ are defined as $\mathbb{J} = \frac{1}{3}\underline{\underline{1}} \otimes \underline{\underline{1}}$, and $\mathbb{K} = \mathbb{I} - \mathbb{J}$.

The volume fraction relative to the ith fracture family ω_i is defined by $f_i^f = \frac{4}{3}\pi\varepsilon_i^f X_i^f$, in which $\varepsilon_i^f = \mathcal{N}_i^f\left(a_i^f\right)^3$ represents the fracture density parameter, introduced by Budiansky and O'connell [14], $\mathcal{N}_i^f$ is the number of fractures per unit volume and a_i^f is the average radius relative to the fracture family ω_i. Based in interface layer modeling, Maghous et al. [2] introduced the isotropic fracture fourth-order tensor behavior $\mathbb{C}_i^f$ related to the fracture stiffness $\underline{\underline{k}}^i$ by means of:

$$\mathbb{C}_i^f = 3X_i^f a_i^f\left(k_n^i - \frac{4}{3}k_t^i\right)\mathbb{J} + 2X_i^f a_i^f k_t^i\mathbb{K} \quad \text{with} \quad X_i^f \ll 1 \tag{10}$$

Using a Mori-Tanaka scheme [13], the general estimate of the tensor $\mathbb{C}^{hom}$ for the fiber-reinforced fractured cementitious material reads:

$$\begin{aligned}\mathbb{C}^{hom} = \lim_{X_1^f,\ldots,X_n^f \to 0}\Bigg\{&\left[f^m\mathbb{C}^m + \sum_{i=1}^{n} f_i^f\mathbb{C}_i^f : \left(\mathbb{I} + \mathbb{P}_i^{m,f} : \delta\mathbb{C}_i^f\right)^{-1} + \sum_{i=1}^{p} f_i^r\mathbb{C}_i^r : \left(\mathbb{I} + \mathbb{P}_i^{m,r} : \delta\mathbb{C}_i^r\right)^{-1}\right] : \\ &\left[f^m\mathbb{I} + \sum_{i=1}^{n} f_i^f\left(\mathbb{I} + \mathbb{P}_i^{m,f} : \delta\mathbb{C}_i^f\right)^{-1} + \sum_{i=1}^{p} f_i^r\left(\mathbb{I} + \mathbb{P}_i^{m,r} : \delta\mathbb{C}_i^r\right)^{-1}\right]^{-1}\Bigg\}\end{aligned} \tag{11}$$

where $\delta\mathbb{C}_i^f = \left(\mathbb{C}_i^f - \mathbb{C}^m\right)$ and $\delta\mathbb{C}_i^r = \left(\mathbb{C}_i^r - \mathbb{C}^m\right)$. While $\mathbb{P}_i^{m,f}$ is the Hill tensor relative to the ith fracture family (oblate spheroid) with aspect ratio X_i^f, and $\mathbb{P}_i^{m,r}$ is the Hill tensor relative to the ith fiber family (prolate spheroid) with aspect ratio X_i^r. Closed-form solutions of Hill tensors $\mathbb{P}_i^{m,f}$ and $\mathbb{P}_i^{m,r}$, or equivalently of the so-called Eshelby tensors $\mathbb{S}_i^{m,f} = \mathbb{P}_i^{m,f} : \mathbb{C}^m$ and $\mathbb{S}_i^{m,r} = \mathbb{P}_i^{m,r} : \mathbb{C}^m$ can be found in Handbooks (see for instance [15–17]).

The general estimate of $\mathbb{C}^{hom}$ from eq. (11) can be particularized for several configurations. The first case addressed refers to the situation of a cementitious matrix reinforced by randomly (isotropic) oriented fibers with randomly (isotropic) oriented microfractures (subscript "i" is omitted in the following expression):

$$\mathbb{C}^{hom} = \lim_{X^f \to 0} \left\{ \left[f^m \mathbb{C}^m + f^f \mathbb{C}^f : \overline{\left(\mathbb{I} + \mathbb{P}^{m,f} : \left(\mathbb{C}^f - \mathbb{C}^m\right)\right)^{-1}} + f^r \mathbb{C}^r : \overline{\left(\mathbb{I} + \mathbb{P}^{m,r} : \left(\mathbb{C}^r - \mathbb{C}^m\right)\right)^{-1}} \right] : \left[f^m \mathbb{I} + f^f \overline{\left(\mathbb{I} + \mathbb{P}^{m,f} : \left(\mathbb{C}^f - \mathbb{C}^m\right)\right)^{-1}} + f^r \overline{\left(\mathbb{I} + \mathbb{P}^{m,r} : \left(\mathbb{C}^r - \mathbb{C}^m\right)\right)^{-1}} \right]^{-1} \right\} \tag{12}$$

where the operator $\overline{\bullet}$ applied over a quantity $\mathcal{Q}$ denotes the integral over the spherical coordinates $\theta \in [0, \pi]$ and $\varphi \in [0, 2\pi]$ [2]:

$$\overline{\mathcal{Q}} = \int_0^{\pi} d\theta \int_0^{2\pi} \mathcal{Q}(\theta, \varphi) \frac{\sin\theta}{4\pi} d\varphi \tag{13}$$

The isotropic distribution of fractures and fibers induces an isotropic effective stiffness tensor, expressed by:

$$\mathbb{C}^{hom} = 3k^{hom} \mathbb{J} + 2\mu^{hom} \mathbb{K} \tag{14}$$

Therefore, the homogenized bulk modulus k^{hom} and shear modulus μ^{hom} can completely define the homogenized elasticity tensor $\mathbb{C}^{hom}$. The analytical expressions of k^{hom} and μ^{hom} are obtained from eqs. (12) and (13) through the evaluation of double integral terms, which can be achieved by making use of a formal software.

The general form of $\mathbb{C}^{hom}$ (11) also can be particularized to the case of a cementitious matrix reinforced by aligned fibers with randomly (isotropic) oriented microfractures (subscript "i" is omitted in following expression):

$$\mathbb{C}^{hom} = \lim_{X^f \to 0} \left\{ \left[f^m \mathbb{C}^m + f^f \mathbb{C}^f : \overline{\left(\mathbb{I} + \mathbb{P}^{m,f} : \left(\mathbb{C}^f - \mathbb{C}^m\right)\right)^{-1}} + f^r \mathbb{C}^r : \left(\mathbb{I} + \mathbb{P}^{m,r} : \left(\mathbb{C}^r - \mathbb{C}^m\right)\right)^{-1} \right] : \left[f^m \mathbb{I} + f^f \overline{\left(\mathbb{I} + \mathbb{P}^{m,f} : \left(\mathbb{C}^f - \mathbb{C}^m\right)\right)^{-1}} + f^r \left(\mathbb{I} + \mathbb{P}^{m,r} : \left(\mathbb{C}^r - \mathbb{C}^m\right)\right)^{-1} \right]^{-1} \right\} \tag{15}$$

It should be noted that a fiber family oriented in a specific direction induces an anisotropic effective stiffness tensor $\mathbb{C}^{hom}$, which cannot be completely defined only by two parameters, as in the previous case.

The particular cases of cracked cementitious materials reinforced by aligned or randomly distributed fibers are assessed by taking $\underline{\underline{k}}^i = 0$ in each one of the above estimates (i.e., eqs. (12) and (15)). It should be pointed that the fracture density parameter ε^f can be viewed as a damage parameter on the macroscopic scale, provided that the number of fractures per unit volume $\mathcal{N}^f$ remains constant.

4 Illustrative examples

This section deals with illustrative examples of the macroscopic behavior of fiber-reinforced fractured cementitious materials based on the estimates of $\mathbb{C}^{hom}$ presented in eqs. (12) and (15). The material properties of cementitious matrix are $k^m = 24.42$ GPa and $\mu^m = 13.27$ GPa. The bulk and shear moduli of fibers are $k^r = 175$ GPa and $\mu^r = 80.769$ GPa (these properties correspond to steel fibers). Dutra et al. [7] developed a parametric study of the aspect ratio and found that for volume fractions ranging within usual values ($f^r \leq 5\%$), the aspect ratio of fibers has a small influence on the elastic properties of the reinforced material. Therefore, for illustrative purposes in this study is adopted $X^r = 100$ for the aspect ratio of fibers. The normal and tangential stiffness of fractures are $k_n = 42.22$ GPa/m and $k_t = 16.89$ GPa/m, and the number of fractures per unit volume is $\mathcal{N}^f = 1$.

For illustrative purposes, are addressed two examples in the sequel. The first example corresponds to the cementitious matrix reinforced by randomly (isotropic) oriented fibers with randomly (isotropic) oriented microfractures, where the effective stiffness is given by the tensor $\mathbb{C}^{hom}$ of the eq. (12). Figure 2 shows the variations of normalized bulk modulus k^{hom}/k^m and normalized shear modulus μ^{hom}/μ^m ($k^m = 24.42$ GPa and $\mu^m = 13.27$ GPa) as a function of the damage parameter ε^f for several volume fractions of fibers ($f^r \leq 2.5\%$). The second example addressed refers to the cementitious matrix reinforced by aligned fibers ($\underline{e}_3 = \underline{e}_r$) with isotropic fracture distribution, where the effective stiffness is given by the tensor $\mathbb{C}^{hom}$ of the eq. (15). Figure 3 shows the variations of the

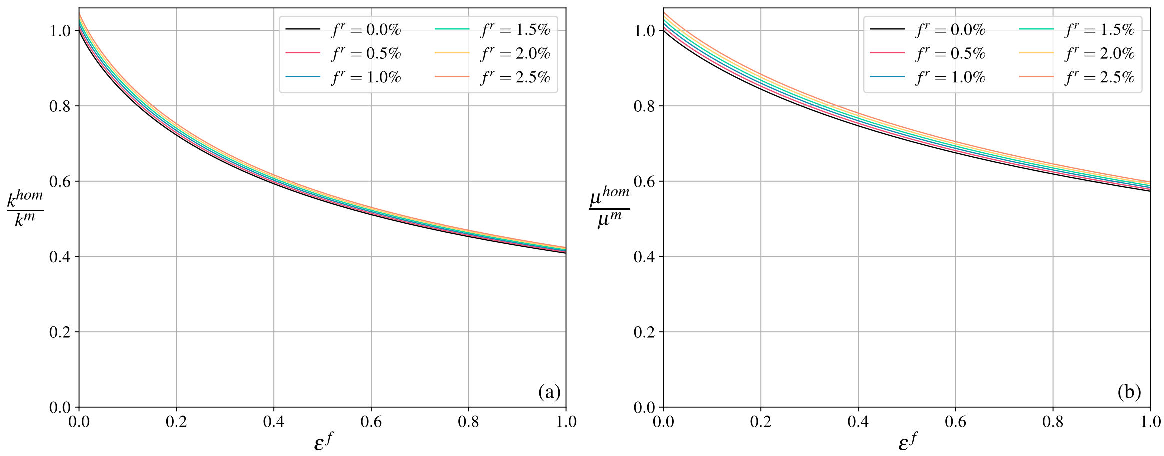

Figure 2. Normalized effective bulk and shear stiffness of the cementitious matrix reinforced by isotropic fiber distribution with isotropic fracture distribution versus damage parameter ε^f: (a) k^{hom} and (b) μ^{hom}.

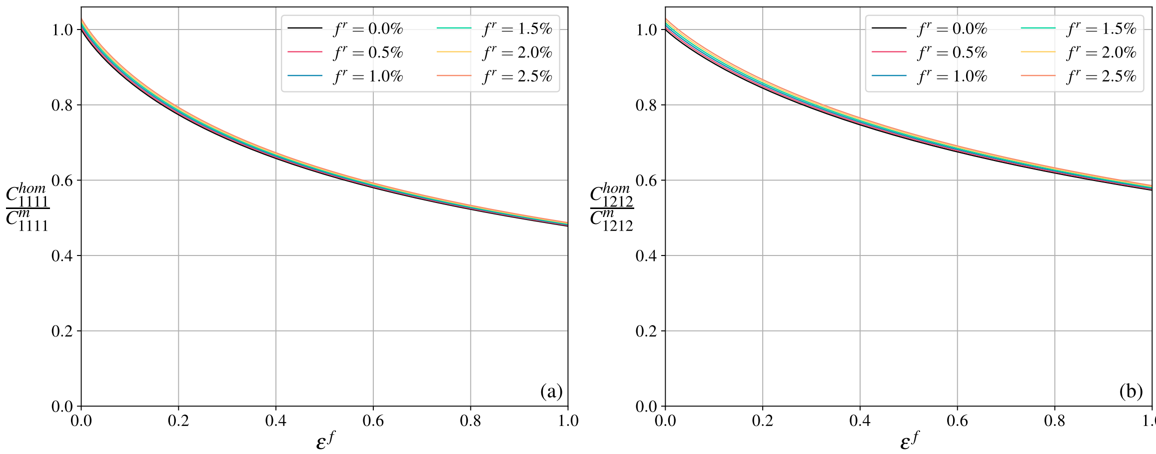

Figure 3. Normalized effective stiffness tensor components of the cementitious matrix reinforced by aligned fibers with isotropic fracture distribution versus damage parameter ε^f: (a) Component C^{hom}_{1111} and (b) Component C^{hom}_{1212}.

normalized components $C^{hom}_{1111}/C^m_{1111}$ and $C^{hom}_{1212}/C^m_{1212}$ ($C^m_{1111} = 42.11$ GPa and $C^m_{1212} = 13.27$ GPa) as a functions of damage parameter ε^f for several volume fractions of fibers ($f^r \leq 2.5\%$). The other components of $\mathbb{C}^{hom}$ show analogous behavior, but for the sake of shortness, they are not presented here.

In both examples, the stiffness degradation induced by the presence of fractures increases with the damage parameter ε^f, and the fibers slightly enhance the material stiffness. As widely reported in the literature [18, 19], the volume fraction of fibers typically remains low ($f^r \leq 2.5\%$). Consequently, as demonstrated in Figures 2 and 3, these fiber quantities do not significantly increase the effective elastic properties of fiber-reinforced fractured cementitious materials. It is well-known that the fibers enhance the post-cracking strength of cementitious materials due to the bridging effect of the fibers. It would be interesting to evaluate this phenomenon from the evolution of the damage parameter ε^f (i.e., directly through the fracture growth, considering the absence of nucleation).

5 Conclusions

In the present work, the effective properties of the fiber-reinforced fractured cementitious materials have been addressed. The macroscopic elastic properties of this class of materials have been determined in the framework of Eshelby's equivalent inclusion theory, employing the Mori-Tanaka scheme to formulate estimates for the homogenized elastic tensor $\mathbb{C}^{hom}$. The general form of $\mathbb{C}^{hom}$ is formulated and then particularized for two situations considering a cementitious matrix material reinforced by aligned or randomly distributed fibers with randomly distributed fractures. The illustrative examples show that the degradation of the stiffness components of the $\mathbb{C}^{hom}$ tensor induced by the presence of fractures increases as the damage parameter increases. The presence of fibers to the fractured cementitious material has little effect on the macroscopic elastic properties. As mentioned, it is necessary to formulate a damage evolution law for this class of materials and evaluate the influence of adding fibers on fracture growth (evolution of ε^f, considering the absence of nucleation).

Acknowledgements. The authors are grateful for the financial support provided by CAPES and CNPq.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] A. Zaoui. Continuum micromechanics: survey. *Journal of Engineering Mechanics*, vol. 128, n. 8, pp. 808–816, 2002.
[2] S. Maghous, L. Dormieux, D. Kondo, and J.-F. Shao. Micromechanics approach to poroelastic behavior of a jointed rock. *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 37, n. 2, pp. 111–129, 2013.
[3] S. Maghous, G. Lorenci, and E. Bittencourt. Effective poroelastic behavior of a jointed rock. *Mechanics Research Communications*, vol. 59, pp. 64–69, 2014.
[4] C. B. Aguiar and S. Maghous. Micromechanical approach to effective viscoelastic properties of micro-fractured geomaterials. *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 42, n. 16, pp. 2018–2046, 2018.
[5] M. B. Guimarães, de C. B. Aguiar, and S. Maghous. Upscaling modeling of effective elastic properties and anisotropic damage propagation in fractured materials regarded as homogenized media. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, vol. 46, n. 1, pp. 1–21, 2024.
[6] F. Hild, A. Burr, and F. A. Leckie. Matrix cracking and debonding of ceramic-matrix composites. *International Journal of Solids and Structures*, vol. 33, n. 8, pp. 1209–1220, 1996.
[7] V. P. Dutra, S. Maghous, A. Campos Filho, and A. Pacheco. A micromechanical approach to elastic and viscoelastic properties of fiber reinforced concrete. *Cement and concrete research*, vol. 40, n. 3, pp. 460–472, 2010.
[8] Z.-g. Yan, Y. Zhang, J. Woody Ju, Q. Chen, and H.-h. Zhu. An equivalent elastoplastic damage model based on micromechanics for hybrid fiber-reinforced composites under uniaxial tension. *International Journal of Damage Mechanics*, vol. 28, n. 1, pp. 79–117, 2019.
[9] S. Maghous, D. Bernaud, J. Fréard, and D. Garnier. Elastoplastic behavior of jointed rock masses as homogenized media and finite element analysis. *International Journal of Rock Mechanics and Mining Sciences*, vol. 45, n. 8, pp. 1273–1286, 2008.
[10] R. E. Goodman. *Methods of geological engineering in discontinuous rocks.*, volume 13. West Publishing Company, 1976.
[11] S. C. Bandis, A. C. Lumsden, and N. R. Barton. Fundamentals of Rock Joint Deformation. *International Journal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts*, vol. 20, n. 6, pp. 249–268, 1983.
[12] J. D. Eshelby. The determination of the elastic field of an ellipsoidal inclusion, and related problems. *Proceedings of the Royal Society A.*, vol. 241, n. 1226, pp. 376–396, 1957.
[13] T. Mori and K. Tanaka. Average stress in matrix and average elastic energy of materials with misfitting inclusions. *Acta metallurgica*, vol. 21, n. 5, pp. 571–574, 1973.
[14] B. Budiansky and R. J. O'connell. Elastic moduli of a cracked solid. *International journal of Solids and structures*, vol. 12, n. 2, pp. 81–97, 1976.
[15] N. Laws. The determination of stress and strain concentrations at an ellipsoidal inclusion in an anisotropic material. *Journal of Elasticity*, vol. 7, n. 1, pp. 91–97, 1977.
[16] T. Mura. *Mechanics of Defects in Solids*. Martinus Nijhoff, 2 edition, 1987.
[17] S. Nemat-Nasser and M. Hori. *Micromechanics: Overall Properties of Heterogeneous Materials*, volume 37. Elsevier Ltd, 1 edition, 1993.
[18] S. A. Ashour, F. F. Wafa, and M. I. Kamal. Effect of the concrete compressive strength and tensile reinforcement ratio on the flexural behavior of fibrous concrete beams. *Engineering Structures*, vol. 22, n. 9, pp. 1145–1158, 2000.
[19] J. Thomas and A. Ramaswamy. Mechanical properties of steel fiber-reinforced concrete. *Journal of materials in civil engineering*, vol. 19, n. 5, pp. 385–392, 2007.