Parametric study of combined tensile and shear-frictional damage models on the simulation of mesoscopic compressive failure in concrete

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Abstract. The mechanical behavior of concrete can be represented more realistically by adopting a mesoscale modeling, where its different phases are considered explicitly. To represent non-linear material behavior, a suitable constitutive damage model can be used along with the Mesh Fragmentation Technique. Thus, high aspect ratio interface elements are placed between regular mesh elements to delineate potential crack paths in quasi-brittle materials like concrete. This technique was extended with a two-layer condensed interface element to describe the compressive failure as a combination of debonding (mode-I) and sliding (mode-II) cracking. One advantage of the proposed model is the reduced number of required parameters: tensile strength, fracture energy, cohesion, friction coefficient, and shear softening. This work evaluates the influence of each parameter on concrete's mechanical behavior. The influence of each parameter is assessed in terms of the resulting stress-strain curve and cracking patterns for a series of uniaxial compression tests. The sensitivity analysis showed that the shear model parameters had more influence than the tensile damage model parameters. The friction coefficient was the most influential parameter, followed by the cohesion. The results are consistent with experimental information and aid in calibrating the numerical model, providing insights for structural member simulations in the future.

Keywords: Concrete; Mesoscale; Tensile damage model; Shear-frictional damage model; High aspect ratio interface elements.

1 Introduction

Concrete is a complex composite material whose mechanical behavior is influenced by its heterogeneous microstructure [1]. Traditional macroscopic models often neglect this intrinsic heterogeneity, leading to the need for a large number of parameters to accurately predict the mechanical behavior. However, advancements in computational resources have made the simulation of more refined scales, such as the mesoscale and microscale, more feasible in material research. A mesoscopic scale approach, which considers the different phases of the composite (namely, aggregate, mortar matrix, and interfacial transition zones), provides a more realistic representation of the problem. The random aggregate distribution can be obtained either by algorithmic generation or by digital image processing of a real specimen cross-section, providing a more realistic representation of the problem [2]. The employment of a mesoscale also allows for the study of the role played by input parameters such as grading curve, aggregate ratio and aggregate mechanical properties on the mechanical behavior of the composite.

In addition to a consistent representation of the material mesostructure, a precise prediction of the mechanical behavior with the Finite Element Method depends on the use of appropriate constitutive models that capture the non-linear behavior of the material, such as non-linear elasticity, damage, plasticity, smeared and discrete cracking models [1][2]. Specifically in a quasi-brittle material, such as concrete, the non-linear behavior is highly related to the crack opening and propagation process, which has been approached by the Extended Finite Element Method (XFEM) [3], phase-field method [4] and zero-thickness interface elements [5]. An effective technique for modeling this critical process in concrete is the Mesh Fragmentation Technique [6], which involves placing high aspect ratio interface elements between regular mesh elements, delineating potential crack paths. This technique has been successfully applied to the study of normal, high strength and recycled aggregate concretes for predominant tensile stress states. More recently, Gimenes et al. [2] have extended the technique in order to account for compressive failure as well. A two-layer condensed interface element was introduced with each layer governed by tensile and shear-frictional constitutive damage models. This element allows for the description of compressive failure as a combination of debonding (mode-I) and sliding (mode-II) cracking, respectively.

A significant advantage of the proposed model is its reliance on a minimal number of parameters (besides the elastic properties): tensile strength, fracture energy, cohesion, friction angle, and shear softening. These parameters are primarily material properties, which ensures a simplified modeling process while maintaining accuracy. To understand the influence of these parameters on the mechanical behavior of concrete, a comprehensive parametric study is conducted. This study demonstrates the effects of parametric variation through a series of mesoscale uniaxial compression tests, comparing the results with experimental data. The varied resulting stress-strain curves are analyzed, comparing the effect of the input parameters on aspects such as peak stress, peak strain, and energy dissipation in addition to specimen dilatancy.

A well-known uniaxial compressive test carried out by van Vliet and van Mier [7] is simulated with the proposed technique. The reference response is compared against the responses varying the material parameters one by one. In order to perform a more consistent parametric analysis, it is crucial to assess the sensitivity of the parameters influencing the material response. In this study, the sensitivity analysis is performed using the method of directional cosines [8]. The method involves calculating the gradients of the response with respect to each parameter and normalizing these gradients to determine their relative contributions.

By combining a mesoscale model with the employment of directional cosines, it is possible to identify the most influential parameters in the combined material failure, providing valuable insights for future simulations of structural members using the proposed technique.

2 Mesoscale model with parametric study

A mesoscale representation of a concrete specimen is generated using the Take-and-Place algorithm proposed by Wriggers and Moftah [9]. This method randomly places particles across the domain, resulting in random aggregate particle inclusions in the mortar matrix. Once the problem geometry and boundary conditions are defined, the finite element mesh is generated. This regular element mesh is then modified and prepared for the numerical analysis. The elastic and inelastic mechanical properties of the aggregate, mortar matrix and interfacial transition zones (ITZs) can be experimentally given or theoretically estimated from the homogenized properties using well-established relations, such as the Mix theory [10][11]. The more relevant ingredients of the proposed technique are explained in the following sections.

2.1 Mesh fragmentation technique

The mesh fragmentation technique (MFT) consists in reducing the sizes of regular elements in order to accommodate high aspect ratio interface elements (HAR-IEs) between them. These interface elements represent the potential crack paths in the material. Previous works have demonstrated that the kinematics of the HAR-IE is similar to the Continuous Strong Discontinuity Approach (CSDA) [6], meaning this type of element is suitable to model the discontinuity behavior. Therefore, in this framework, the regular elements of the mesh present a linear behavior, whereas the HAR-IEs are ruled by damage models. More recently, Gimenes et al. [2] have extended the technique in order to account for combined mode fracture propagation. The modified HAR-IE is a two-layer element with one layer ruled by a tensile damage model and a second layer ruled by a shear frictional damage model. Figure 1 illustrates the main stages of MFT with the condensed tow-layer HAR-IE for a two-dimensional context. It is worth noting that, globally, the condensed HAR-IE is represented by a four-nodded quadrilateral element with thickness $2h \rightarrow 0$. Locally, this element is divided in two pairs of triangular elements with thickness $h \rightarrow 0$. The internal degrees of freedom resulting from the subdivision are condensed out using a local Newton-

Raphson procedure, hence, the nomenclature of condensed HAR-IE.

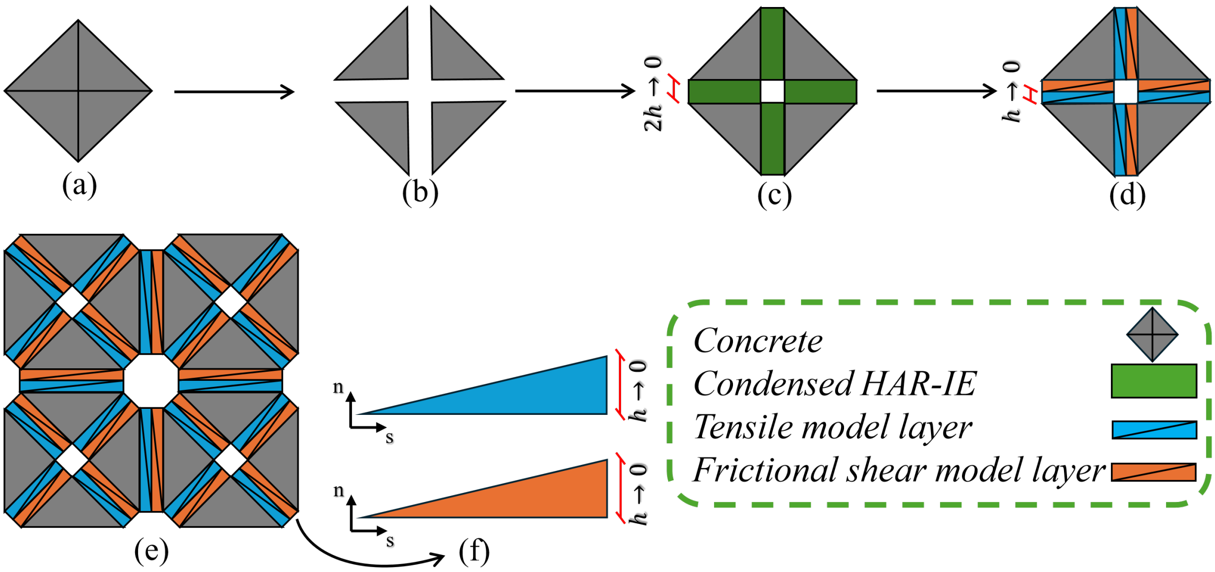


Figure 1. Stages of the MFT: (a) mesh with regular elements; (b) reduction in the size of regular elements; (c) insertion of condensed HAR-IEs; (d) condensed HAR-IEs are internally divided into two layers of triangular HAR-IEs; (e) fragmented mesh with the explicit representation of the condensed HAR-IE layers and (f) 2D triangular HAR-IE local coordinate system.

2.2 Constitutive damage models

For a local coordinate system (n, s) (illustrated in Fig.1(f)), the tensile and shear damage models in the triangular HAR-IEs describe the formation and propagation of fractures in Modes I and II, respectively.

Mode I propagation is described by a scalar damage variable $d_n \in [0,1]$, which affects all components of the effective stress tensor $(\bar{\boldsymbol{\sigma}})$, resulting in the nominal stress tensor $(\boldsymbol{\sigma})$ (Eq. 1).

$$\boldsymbol{\sigma} = (1 - d_n)\bar{\boldsymbol{\sigma}} \tag{1}$$

The effective stress tensor is computed from the current strain tensor $(\boldsymbol{\varepsilon})$ and the fourth order elastic tensor (C), such as:

$$\bar{\boldsymbol{\sigma}} = \mathbb{C}:\boldsymbol{\varepsilon} \tag{2}$$

The exponential softening damage evolution law is obtained from:

$$d_n = 1 - \frac{f_t}{\lambda} exp\left[A\left(1 - \frac{\lambda}{f_t}\right)\right] \tag{3}$$

$$\lambda = max[f_t, \bar{\sigma}_{nn}] \tag{4}$$

where the parameter $A = f_t^2/(EG_f)$ relates to material parameters, such as Young's modulus (E), tensile strength (f_t), and fracture energy (G_f).

For Mode II, the reduction of the shear components of $\bar{\boldsymbol{\sigma}}$ also depends on a scalar damage variable $d_s \in [0,1]$, as follows:

$$\begin{bmatrix} \sigma_{nn} & \sigma_{ns} \\ \sigma_{sn} & \sigma_{ss} \end{bmatrix} = \begin{bmatrix} \sigma_{nn} & 0 \\ 0 & \bar{\sigma}_{ss} \end{bmatrix} + (1 - d_s)\begin{bmatrix} 0 & \bar{\sigma}_{ns} \\ \bar{\sigma}_{sn} & 0 \end{bmatrix} \tag{5}$$

The damage evolution occurs according to a Coulomb-type model depending on the cohesion (β), friction coefficient (α) taken as the tangent of the friction angle, and softening parameter $\bar{H} < 0$. More details regarding the equations used, as well as the integration scheme adopted, can be found in the work of Gimenes et al. [2].

2.3 Sensitivity analysis

To understand the role played by each parameter on the mechanical behavior of concrete, a sensitivity analysis is applied [8], approaching each parameter as an input variable that influences the output data. For this problem, the input variables considered are: cohesion, friction coefficient and softening parameter. The considered output data of the compressive test analysis are: peak stress, peak strain, dissipated energy and dilatancy of the specimen.

The input variables are stored in a vector $\vec{x} = [x_1\ x_2 \dots x_n\ ]$ and each output data can be treated as a function of interest such as $f(x)$. For each variable x_i, the gradient $\partial f / \partial x_i$ is approximated using the difference between the function values at x_i and $x_i + \Delta x_i$:

$$\frac{\partial f}{\partial x_i} \approx \frac{f(x_i + \Delta x_i) - f(x_i)}{\partial x_i} \tag{6}$$

where Δx_i is a percentual increment of x_i.

These gradients are collected into a vector ∇f. Then, the directional cosines are computed as:

$$cos\ (\theta_i)\ = \frac{\frac{\partial f}{\partial x_i}}{\|\nabla f\|} \tag{7}$$

The square of the direction cosines indicates the sensitivity factors of each variable to the output function.

3 Numerical example

A uniaxial compressive test carried out by van Vliet and van Mier [7] is numerically simulated using the proposed technique. The specimen dimensions are 100×100×100 mm^3 with steel plates on both ends. A vertical displacement of 0.001 mm is incrementally applied at each loading step on the top steel plate. The friction condition between the specimen and the steel plate is considered by a friction coefficient of 0.15 [2]. A 27% volume fraction of coarse aggregates is considered and the minimum and maximum aggregate sizes are 4 mm and 8 mm, respectively. The experimental cube compressive strength of the concrete is 50 MPa and the mechanical properties of each phase for the mesoscale model are described in Tab. 1. According to the study presented in Gimenes et al. [2], these parameters provided compatible results quantitatively and qualitatively when compared to the experimental results [7], as shown in Fig. 2 by the blue and yellow curves.

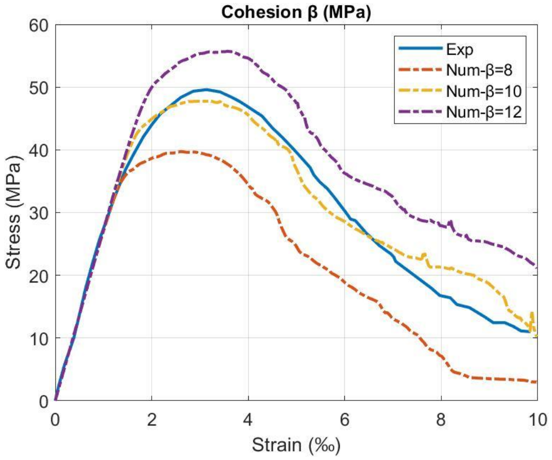
Table 1. Mechanical properties for each phase [2]

Interfacial Phase	Elastic Modulus E (GPa)	Tensile strength f_t (MPa)	Fracture energy G_f (N/mm)	Cohesion (MPa)	Friction coefficient	Softening parameter (mm^{-1})
Matrix	20	4.0	0.152	10	0.35	-0.003
ITZ	15	2.0	0.076	10	0.35	-0.003

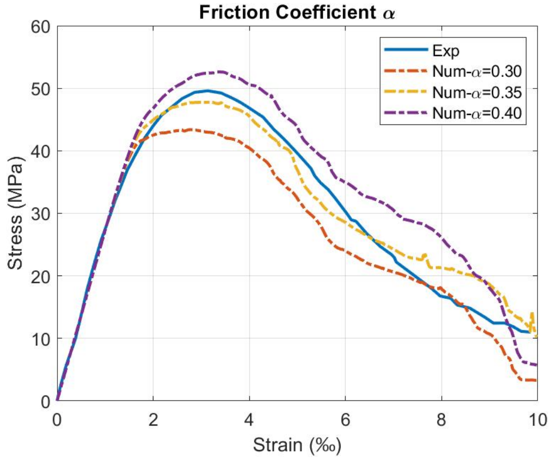
Table 2. Parameter variation

Parameter	Reference case	Cases -Δx_i	Cases +Δx_i
Cohesion (MPa)	10.0	8.0	12.0
Friction coefficient	0.35	0.30	0.40
Softening Parameter (mm^{-1})	-0.003	-0.0036	-0.0024
Matrix Fracture energy (N/mm)	0.152	0.132	0.172
Matrix Tensile strength (MPa)	4.0	3.0	5.0

For the parametric sensitivity analysis, all parameters were kept the same as those in the reference curve, except for the parameter of interest. Each parameter of interest was varied individually by adding or subtracting a Δx_i to the reference value, as shown in Table 2. Different Δx_i values were assumed for each variable within the range of 15% to 25%. This range was chosen to ensure a noticeable distinction between the resulting curves (Fig. 2) and to effectively demonstrate the influence of each parameter on the numerical model.



(a)



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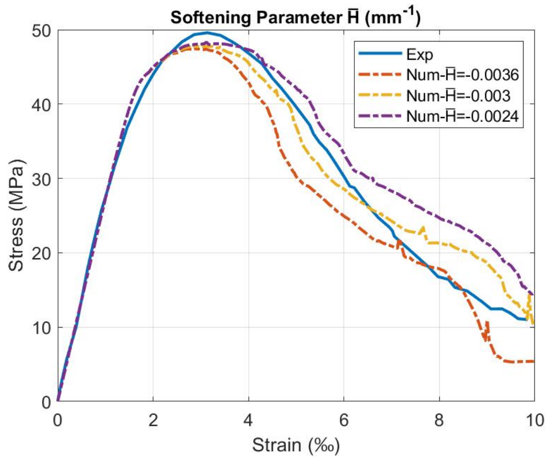

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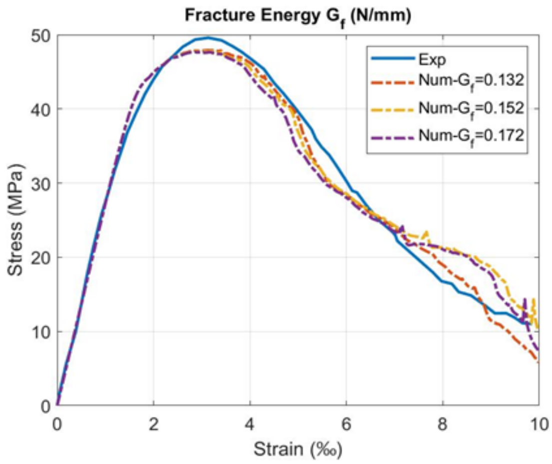
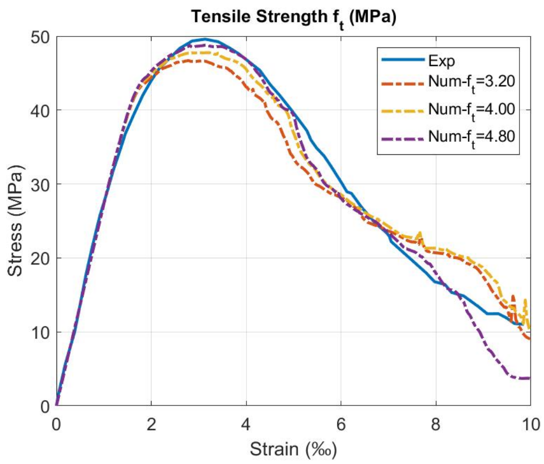


(d)



(e)

Figure 2. Experimental ("Exp") and numerical ("Num") stress-strain curves for cases with variation of: (a) Cohesion; (b) Friction coefficient; (c) Softening Parameter; (d) Fracture Energy and (e) Tensile Strength.

Figure 2 illustrates the stress-strain curves for the uniaxial compression test adopting parametric variations for all 5 parameters. In all the plots, the reference experimental and numerical curves are illustrated in solid blue and dashed yellow lines, respectively. By observing the curves, it is possible to visualize the effect of each parameter on different aspects of the stress-strain curve: for instance, the softening parameter illustrated in Fig.

2(c) clearly affects the slope of the curve in the post-peak regimen. The peak stress also increases with the increase in cohesion, as illustrated in Fig. 2(a).

Moreover, it can be observed that the influence of tensile parameters, illustrated in Fig. 2(d) and Fig. 2(e), was negligible when compared to the influence of shear-frictional parameters, illustrated in Fig. 2(a)-(c). Therefore, Fig. 3 illustrates the deformed configuration (2x amplified) of the specimen at the end of the loading process for the shear-friction parameters. All the illustrated crack patterns were affected to some extent by the parametric variation. Nevertheless, all the obtained deformations are consistent with typical experimental failures, presenting diagonal cracks that occur mostly at the ITZ, which is the weakest material phase.

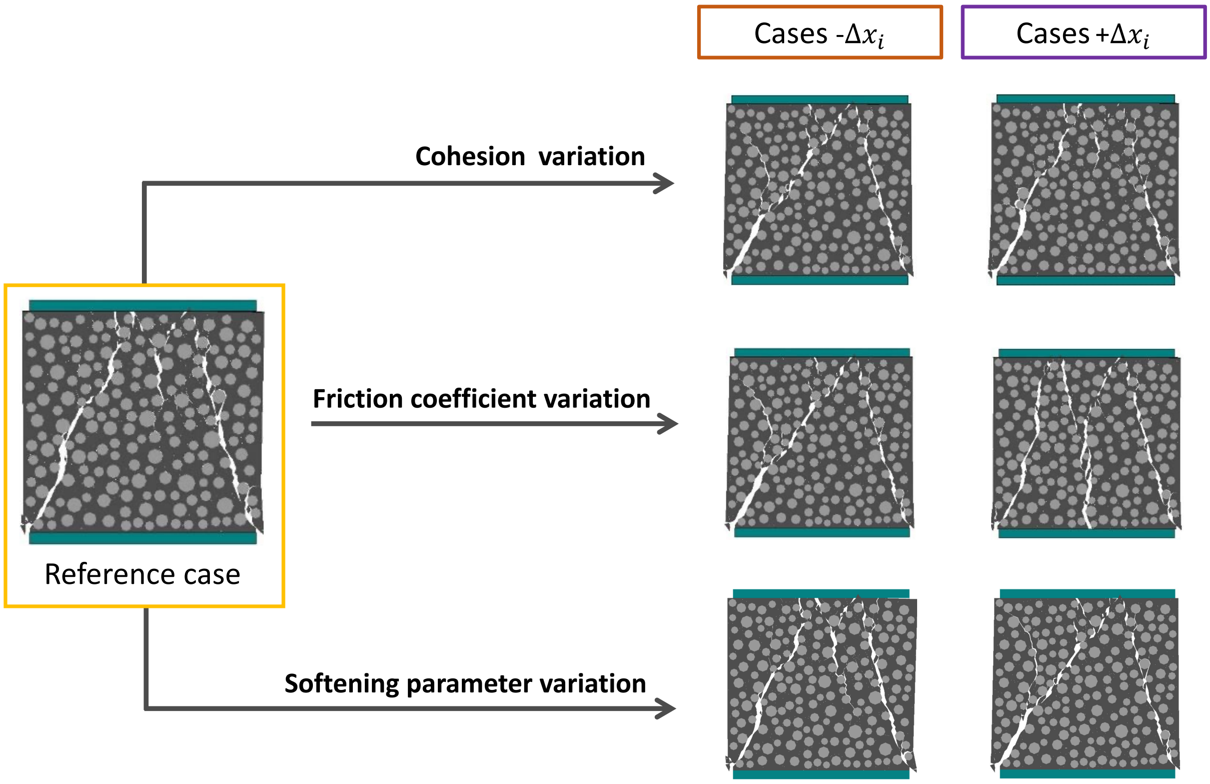


Figure 3. Deformed specimens at the end of the compressive loading process: (a) Cohesion; (b) Friction coefficient; (c) Softening parameter;

The numerical response was analyzed in terms of parametric sensitivity. From the curves in Fig. 2, the peak stresses and peak strains were determined. Additionally, the dissipated energy was computed as the area under the curve. The dilatancy was measured as the distance between the center of the left and right faces of the deformed specimens from Fig. 3. The sensitivity factors were obtained according to Eq.6 and Eq.7 and the data was organized in Tab. 3. It can be observed that the shear damage model parameters had a more predominant influence than the tensile damage model parameters.

Table 3. Sensitivity factors of the material parameters

Output data	Cohesion	Friction coefficient	Softening parameter	Tensile strength	Fracture energy
Peak stress	56.9%	41.9%	0.1%	1.0%	0.1%
Peak strain	39.2%	48.8%	11.0%	1.0%	0.0%
Dissipated energy	58.4%	29.2%	8.0%	3.0%	1.4%
Dilatancy	13.7%	81.4%	0.3%	3.6%	1.0%

4 Conclusions

This study numerically evaluated the influence of several material parameters on the characteristic stress-strain curve of a compressed concrete specimen, such as peak load, peak strain, and dissipated energy. Additionally, the specimen dilatancy given by the deformed finite element mesh was also measured and taken into account.

- The peak stress is mostly sensitive to the cohesion parameter, followed by the friction coefficient and tensile strength;
- The peak strain is mostly affected by the friction coefficient, cohesion and softening parameter;
- The dissipated energy is more sensitive to cohesion, friction coefficient and softening parameter;
- The dilatancy is mostly affected by the friction coefficient, followed by the cohesion;

The results demonstrate a clear predominance of Mode II fracture propagation over Mode I. From the sensitivity analysis, it was observed that among all the material parameters, the friction coefficient was the most influential, followed by the cohesion. The results are consistent with experimental information and aid in the calibration of the numerical model.

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