Influence of the unilateral elastic base on the backbone curves of an imperfect cylindrical panel

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Abstract. This work evaluates the influence of a discontinuous unilateral elastic base and an initial geometrical imperfection on the nonlinear free vibration of cylindrical panels. The Donnell's nonlinear shallow shell theory is considered to describe the cylindrical panel, then the equations are discretized by the Galerkin method. The unilateral elastic base is represented by the Signum function, and the Heaviside function describes the discontinuity of the elastic base. The results show the analysis of the nonlinear free vibrations of the cylindrical panel through the backbone curves, investigating the influence of the hypothesis of contact of the unilateral elastic base and the initial geometrical imperfection of the cylindrical panel. The modal solution employed has with five degree-of-freedom, being sufficient to describe the nonlinear softening behavior of the imperfection cylindrical panel in contact with the discontinuous unilateral elastic base. The numerical results reveal that the imperfect cylindrical panel in contact with unilateral elastic base presents less structural stiffness than in contact with bilateral elastic base, with decreasing the natural frequencies. In conclusion, the backbone curves are strongly influenced by discontinuous unilateral elastic base and the imperfections of the cylindrical panel.

Keywords: cylindrical panel, unilateral elastic base, backbone curves.

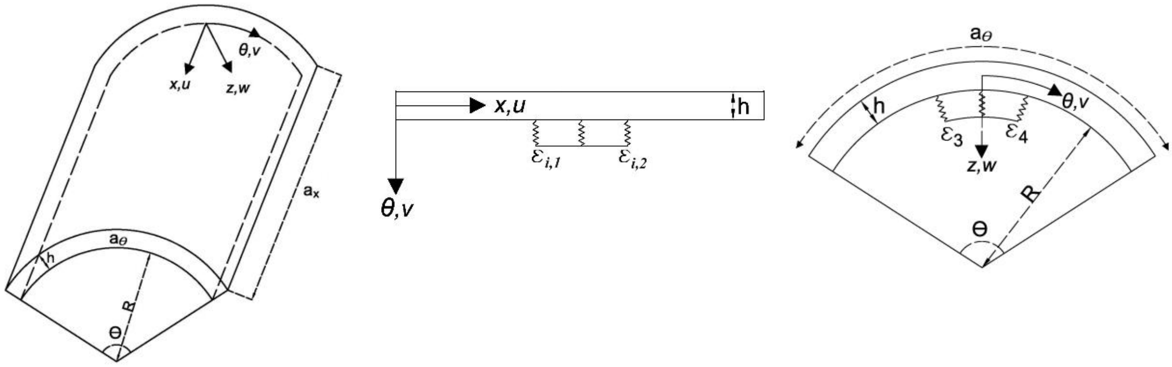
1 Introduction

Thin-walled cylindrical shells and panels are structural elements that can be applied in civil, aerospace, mechanical engineering, among others. To analyze the behavior and stability of these structures it is necessary to consider the physical and geometric non-linearities in the mathematical model. The literature presents several works on shells and cylindrical panels under different boundary conditions, loading conditions, materials, initial geometric imperfections, elastic bases, referenced in reviews by: Amabili and Paidoussis [1], Alijani and Amabili [2], Thai and Kim [3] and Martins et al. [4]. Studies of vibrations in structures supported on elastic foundations have motivated several researchers. Younesian et al. [5], Malekjafarian et al.[6], Lamprea-Pineda et al. [7] and Zhao et al. [8] present studies of beams, plates, cables, shells and cylindrical panels supported on an elastic base. Yechiel Weitsman [9] evaluated unilateral elastic bases in a Euler-Bernoulli beam subjected to a concentrated moving load, considering only a compression reaction of elastic bases. Subsequently, several authors focused on investigations of nonlinear structures supported on an elastic base, researching the influence of different types of foundation stiffness, type of contact and the contact area of the elastic base in the domain of the structural system: Amabili and Dalpiaz [10], Tj et al. [11], Silveira et al.[12], Kim[13], Bahadori and Najafizadeh [14], Bhattiprolu, Bajaj and Davies [15], Yang et al. [16], Babaei and Eslami [17], Song et al.[18], Morais and Silva [19].

In this work, we investigate the nonlinear free vibrations in an imperfect cylindrical panel supported on a discontinuous unilateral elastic base. For that, it is considered the Donnell's nonlinear theory to describe the imperfect cylindrical panel, then it is discretized by the Galerkin method. Perturbation technique is applied to obtain a consistent transversal displacement field which, in its turn, it is used to discretize the nonlinear equilibrium equation of the imperfect cylindrical panel. To describe the unilateral and discontinuous elastic base, the Heaviside and Signum functions are used, respectively, in the mathematical model. The obtained backbones curves are strongly influenced by the initial geometric imperfection and, mainly, of the unilateral contact hypothesis of the elastic base.

2 Problem formulation

Consider an imperfect simply supported thin-walled circular cylindrical panel with radius R, thickness h, axial length a_x, circumferential length a_θ, and open-angle Θ ($=a_\theta/R$), as shown in Fig 1a. The displacement fields in the axial, u, circumferential, v, and transversal, w, directions are also represented in Fig. 1a in addition with their cylindrical coordinates x, θ, and z, respectively. The material of the cylindrical panel is assumed as linear elastic, isotropic and homogenous with Young's modulus E, Poisson's coefficient ν, and density ρ.



(a) (b) (c)

Figure 1. (a) Geometry and displacement field for a cylindrical panel. (b) Elastic foundation in the longitudinal direction in the region defined by $0\leq \varepsilon_1,\varepsilon_2\leq L$. (c) Elastic foundation in the circumferential direction in the region defined by $0\leq \varepsilon_3,\varepsilon_4\leq\Theta$.

The nonlinear equation of motion and the compatibility equation of the cylindrical panel are found from the Donnell's nonlinear shallow shell theory and are described by of the transversal displacement field w and the Airy's stress function f [19]:

$$
\begin{aligned}
&\rho h\ddot{w} + D\nabla^4 w - f_{,\theta\theta}\left(w_{,xx} + w_{0,xx}\right) + R\, f_{,xx} - f_{,xx}\left(w_{,\theta\theta} + w_{0,\theta\theta}\right) \\
&-2f_{,x\theta}\left(w_{,x\theta} + w_{0,x\theta}\right) + p_k = 0 \\
&\nabla^4 f = \frac{Eh}{R^4}\left(w_{,x\theta}^2 - w_{,xx}w_{,\theta\theta} + Rw_{,xx} + 2w_{,x\theta}w_{0,x\theta} - w_{,xx}w_{0,\theta\theta} - w_{,\theta\theta}w_{0,xx}\right) \\
&\nabla^4(\cdot) = (\cdot)_{,xxxx} + \frac{2}{R^2}(\cdot)_{,xx\theta\theta} + \frac{1}{R^4}(\cdot)_{,\theta\theta\theta\theta}
\end{aligned}
\tag{1}
$$

where ω_0 is the natural frequency of cylindrical panel, $D[=Eh^3/12(1-\nu^2)]$ is the flexural stiffness, w_0 is an initial geometrical imperfection, p_k are the reaction of the discontinuous unilateral elastic base described, respectively, by:

$$
\begin{aligned}
w_0 &= W_0^{imp} h \sin\left(\frac{m\pi x}{a_x}\right)\sin\left(\frac{n\pi\theta}{\Theta}\right) \\
p_k &= K_w w H_x H_\theta \frac{\left(1 - sgn(w + w_o)\right)}{2}
\end{aligned}
\tag{2}
$$

where K_w is the Winkler stiffness parameter and W_0^{imp} is magnitude of the initial geometric imperfection.

H_x and H_θ in eq. (2) are the Heaviside functions which describes the discontinuous elastic base in the longitudinal direction $H_x[=H(x-\varepsilon_1)-H(x-\varepsilon_2)]$ ($0\leq \varepsilon_1,\varepsilon_2\leq L$ in Fig. 1b), and in the circumferential direction $H_\theta[=H(\theta-\varepsilon_3)-H(\theta-\varepsilon_4)]$ ($0\leq \varepsilon_3,\varepsilon_4\leq\Theta$ in Fig. 1c). The term sgn is the Signum function that active the unilateral elastic base.

When ($w+ w_0$) takes positive values, the reaction p_k becomes zero. In the opposite way otherwise, when ($w+ w_0$) takes negative values, the reaction p_k acts on the cylindrical panel. To represent a bilateral contact of a discontinuous elastic base the term (1-sgn ($w+ w_0$))/2, in eq. (2), is considered equal to one.

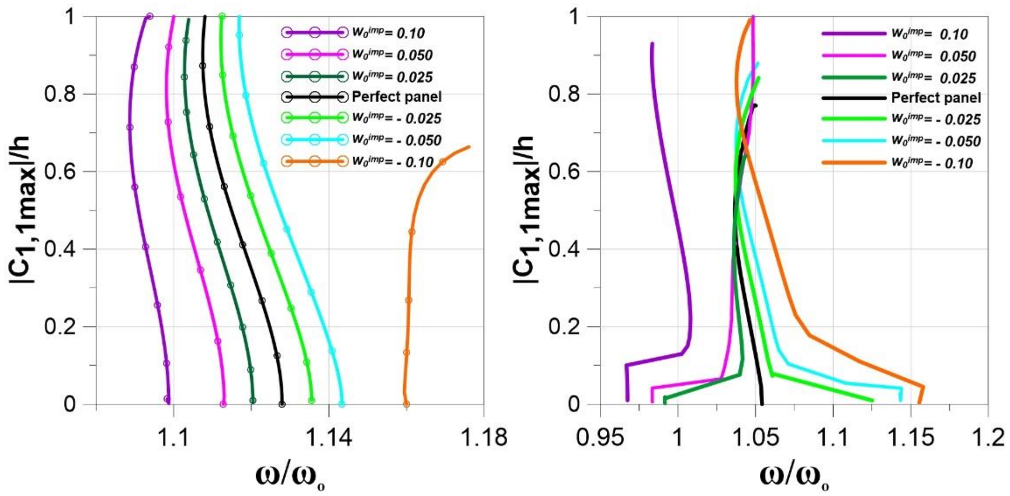
To discretize the nonlinear equilibrium equation of the imperfect cylindrical panel, the Airy's stress function f in eq. (1) is obtained analytically for a particular transversal displacement field w. According to Morais and Silva [20], a consistent transversal displacement field for a simply supported cylindrical panel is derived from a perturbation method, obtaining the following general transversal modal solution:

$$\begin{aligned} w = & \sum_{i=1,3,5}\sum_{j=1,3,5} C_{,ij}(t)\sin\left(\frac{im\pi x}{a_x}\right)\sin\left(\frac{jn\pi\theta}{\Theta}\right) \\ & + \sum_{\alpha=0,1,2,3\ldots}\sum_{\beta=0,1,2,3\ldots} \hat{C}_{,(2+6\alpha)(2+6\beta)}(t)\left\{\left[\frac{3+6\alpha}{4+12\alpha}\cos\left(\frac{6\alpha m\pi x}{a_x}\right) - \cos\left(\frac{(2+6\alpha)m\pi x}{a_x}\right)\right.\right. \\ & \left.+\frac{1+6\alpha}{4+12\alpha}\cos\left(\frac{(4+6\alpha)m\pi x}{a_x}\right)\right]\left[\frac{3+6\beta}{4+12\beta}\cos\left(\frac{6\beta n\pi\theta}{\Theta}\right)\right. \\ & \left.\left.-\cos\left(\frac{(2+6\beta)n\pi\theta}{\Theta}\right)+\frac{1+6\beta}{4+12\beta}\cos\left(\frac{(4+6\beta)n\pi\theta}{\Theta}\right)\right]\right\} \end{aligned} \tag{3}$$

Returning to the nonlinear cylindrical equilibrium equation, with the obtained f and the particular w, the Galerkin method is applied, obtaining a set of nonlinear second order differential equations in terms of modal amplitudes $C_{,ij}(t)$ and $\hat{C}_{,(2+6\alpha)(2+6\beta)}(t)$.

3 Numerical Results

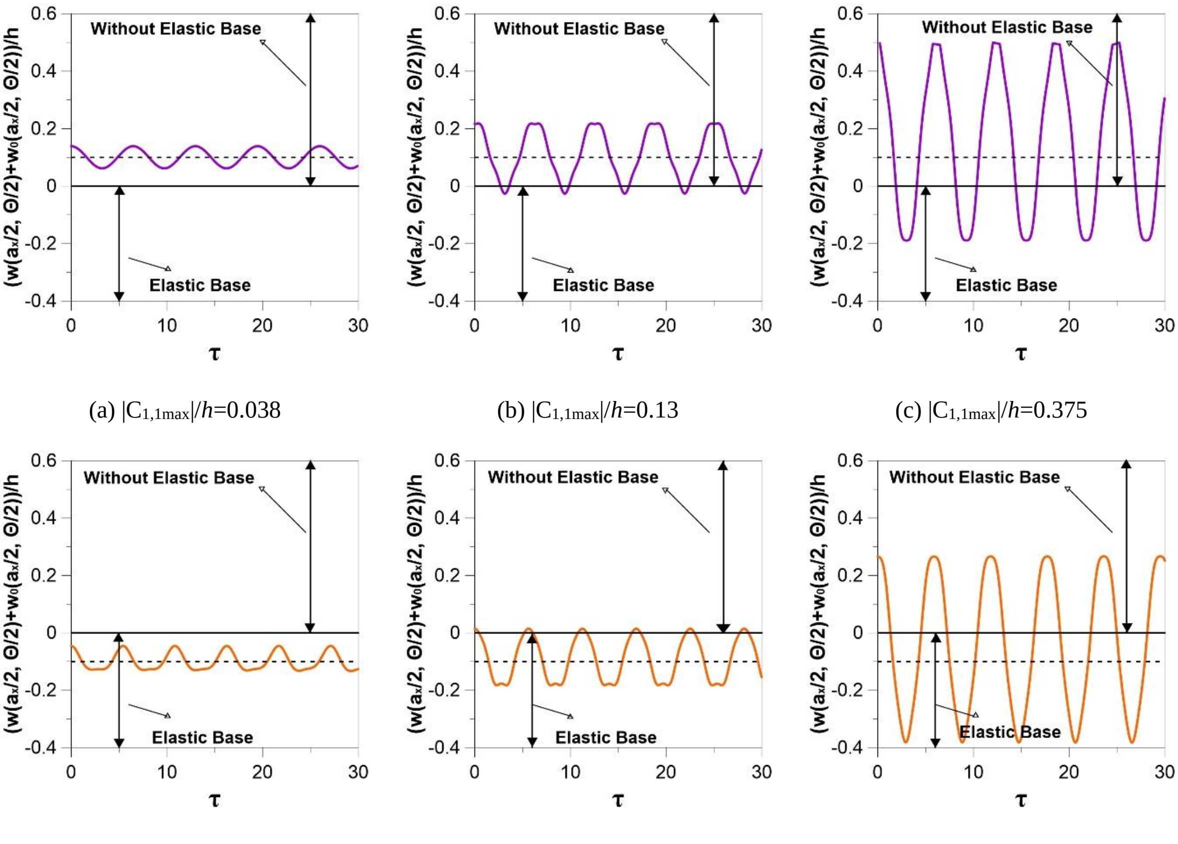
The cylindrical panel has the following geometrical and physical parameters: R=8.333 m, h=0.01 m, a_x =1 m, a_θ=1 m, E =210 GPa, ν = 0.3 and ρ=7850 kg/m³. This perfect cylindrical panel without elastic base presents the lowest natural frequency with ω_0=437.92 rad/s, for the wave numbers (m, n) = (1, 1). The presence of the elastic base increases the system stiffness and consequently the natural frequencies. Consider for this study, the cylindrical panel supported on an elastic base with Winkler base of K_w = 92.30 MN/m³. The contact area has 7.5% in relation to the area of the cylindrical panel and is centered in one of its quadrant (ε_1=0.1130, ε_2=0.3870, ε_3=0.01356 and ε_4=0.0464). Also, it is considered the presence of an initial geometrical imperfection in the shape of the fundamental vibration mode eq. (2), with magnitude W_0^{imp} equal to: ±0.025, ±0.05 and ±0.10. To discretize the nonlinear equilibrium equation a 5-DOF model ($C_{,11}(t)$, $C_{,12}(t)$, $C_{,21}(t)$, $C_{,22}(t)$ and $\hat{C}_{,22}(t)$) is considered for the transverse displacement field in eq. (3). This 5-DOF model can capture the lowest natural frequency of cylindrical panels with discontinuous elastic base and describe the nonlinear dynamic behavior until vibration's amplitude equal to shell's thickness.



(a) bilateral contact (b) unilateral contact

Figure 2. Backbone curves for the imperfect cylindrical panels in contact with (a) bilateral and (b) unilateral with different magnitude for the geometric imperfection.

Figure 2 shows the backbones curves for the imperfect cylindrical panels, obtained by the MatCont (Govaerts [21]), a Matlab software continuation package. It is observed in Fig. 2a the backbone curves for the panels in contact with the bilateral elastic base and in Fig. 2b in contact with the unilateral elastic base. The ratio between the nonlinear and the linear frequency, ω/ω_0, are lower for the panel in contact with unilateral than the panel in contact bilateral elastic base, because the natural frequencies for cylindrical panel with unilateral elastic base is lower than the natural frequencies of the cylindrical panel with bilateral elastic base. The influence of the initial geometric imperfection shifts the backbone curve to the left or right of the backbone of the perfect panel (black curve). If the initial geometric imperfection is positive, the backbone curves are moved to the left of the perfect panel backbone curve, and on the other hand, if the initial geometric imperfection is negative, the backbone curves are moved to the right if the initial geometric imperfection is positive. The cylindrical panel with a bilateral elastic base presents a well-behaved softening nonlinearity as depicted in Fig. 2a. The exception to this statement occurs for the imperfect cylindrical panel with W_0^{imp}=-0.10 where the backbone curve displays an almost linear behavior. On the other hand, the imperfect cylindrical panel with a unilateral elastic base shows a different behavior of its backbone curve, as displayed in Fig. 2b. Due to the presence of an initial geometric imperfection and the unilateral contact hypothesis, the nonlinear free vibration of the cylindrical is influenced by the partial contact of the elastic base.



(a) $|C_{1,1max}|/h$=0.038 (b) $|C_{1,1max}|/h$=0.13 (c) $|C_{1,1max}|/h$=0.375

(d) $|C_{1,1max}|/h$=0.0448 (e) $|C_{1,1max}|/h$=0.11 (f) $|C_{1,1max}|/h$=0.386

Figure 3. Time responses for imperfect cylindrical panel with unilateral elastic base and initial geometric imperfection with magnitude ±0.10 considering selected values of $|C_{1,1max}|/h$ in backbone curves of Fig. 2b.

Time responses for the cylindrical panel in contact with the unilateral base and initial geometric imperfection with magnitude ±0.10 are shown in Fig. 3 for the center point $(x, \theta)=(a_x/2, \Theta/2)$ of the cylindrical panel with certain vibration values chosen on the backbone curves of Fig. 2b. These time responses are obtained using the fourth-order Ruge-Kutta method with the initial conditions obtained in the backbone curves. The initial position of the imperfect cylindrical panel is represented by the horizontal dashed line. Cylindrical panels with a unilateral elastic base and positive imperfection have a gap between the elastic base and the imperfect panel and the vibration amplitude is not sufficient to cause this contact, Fig. 3a. As the vibration amplitudes increase, Fig. 3b, a small contact begins to be established and the backbone curve is shifted to the right in Fig. 2b. When the vibration amplitude increases again, Fig. 3c, the contact between the cylindrical panel and the elastic base increases the softening behavior of the backbone curve is well-defined. In imperfect panels with negative initial geometric imperfection and small vibration amplitude, the contact with the elastic base exists throughout the entire time, as shown in Fig. 3d, increasing the natural frequencies and moving the backbone curve of Fig. 2b to the right. Increasing the vibration amplitudes in Figs. 3e and 3f, the contact between the cylindrical panel and the elastic base is partial and the backbone curve is shifted to the left due to decrease of the natural frequency.

4 Conclusions

This work evaluated the influence of unilateral elastic base and an initial geometric imperfection on the nonlinear free vibration of cylindrical panel. The study presented the backbone curves and time response to analyze the nonlinear behavior of the cylindrical panel. The numerical results were obtained considering a modal solution with five degrees of freedom that was derived from a perturbation technique being able to capture the lowest natural frequency and the nonlinear modal coupling. It was observed that the initial geometric imperfection changes the natural frequencies of the cylindrical panel depending on its signal, leading to a shifting of the backbone curves. The unilateral elastic base changes the global stiffness of the cylindrical, strongly modifying the behavior of the backbone curves when the vibration amplitudes were increased.

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