Nonlinear analysis of static and dynamic stability of space truss structures using the positional Finite Element Method

Mateus N. H. M. Dias[1], Arthur A. A. Albuquerque[1], Kaio C. B. Benedetti[1]

[1]*PPGGECON, Postgraduate Program in Geotechnics, Structures and Civil Construction, University of Goiás (UFG)*
Av. Universitária, 74605-220, Goiânia, Brazil
mateus.nunes@discente.ufg.br, arthuralax@ufg.br, kaiobenedetti@ufg.br

Abstract. In order to ensure structural safety, in addition to checking for strength, structures must also be analyzed for stability, as structural collapse can occur due to material rupture or instability. The study of local or global stability loss in a structure is conducted considering geometric nonlinearity, where the structure's equilibrium is always satisfied in its deformed configuration. One way to study structural instability is by using numerical methods such as the Finite Element Method (FEM) based on positions. Positional FEM relies on the change in the body's configuration through an initial and current mapping system. This characteristic is considered geometric nonlinearity. In this context, this work aims to apply positional FEM with a total Lagrangian reference system for the analysis of instability in space truss structures. The equilibrium equation system will be solved using the Newton-Raphson method, and dynamic behavior will be considered using the Newmark method. The expected results include nonlinear equilibrium curves, structure responses over time, and resonance curves.

Keywords: truss, stability, finite elements, geometric nonlinearity, static and dynamic analysis.

1 Introduction

Trusses are structural systems widely used in engineering, consisting of bars that have only uniaxial stiffness, interconnected by hinged nodes or flexible connections, allowing relative rotation between the elements. They can be classified as plane (two-dimensional) or space (three-dimensional), depending on the number of planes in which the acting forces and bars are arranged [1].

Being a practical and economical structural system, trusses are commonly used in various types of construction, such as bridges, offshore platforms, stadium roofs, gymnasiums, parking structures, transmission towers, cranes, among other applications. In truss systems, loads are applied at the nodes, not on the bar elements that comprise them. As for the weight of each element, it is assumed that this load is equally distributed between the two nodes [2].

In some cases, linear structural analysis may be considered satisfactory. However, trusses can be subjected to loads that cause large displacements. This significantly impacts the geometry of the structure and requires that the equilibrium equations be formulated based on the displaced configuration, highlighting the need for an investigation that considers the geometric nonlinearity of the system [3].

A numerical method capable of accounting for the geometric nonlinearity of a structure is the Finite Element Method (FEM) with a position-based formulation. The positional formulation of FEM was initially described by Bonet [4] and Coda [5]. Its main difference, compared to the classical FEM formulation, is the use of nodal positions as the unknowns of the problem (instead of displacements). In addition, in positional FEM, there is no need to calculate rotation matrices since the local positions of the elements coincide with the global positions.

This position-based methodology offers high accuracy in results and is capable of evaluating the structure according to its displaced state. Moreover, FEM can be widely used in the analysis of structures, considering both static and dynamic behaviors. Its application can provide various structural analyses, such as equilibrium curves, time response, and resonance curves.

The present paper will utilize the positional FEM formulation, applied to a computational code developed in Python by the authors, to perform static and dynamic analyses of a space truss, considering only geometric nonlinearity. To solve the system of nonlinear equations, the Newton-Raphson method will be used, and the dynamic behavior will be addressed using the Newmark method. The resonance response will be addressed by backward and forward sweep over a frequency range close to the first linear frequency of vibration. The system response will be demonstrated through time and phase-space of the permanent phase.

2 Mathematical formulation

The truss element is described by the formulation presented in Coda [5] and Coda [6]. The principle of mechanical energy conservation states that the total mechanical energy $\mathbb{E}$ is constant and is the sum of the potential energy Π and the kinetic energy $\mathbb{K}$. Mathematically, this can be expressed as

$$\mathbb{E} = \Pi + \mathbb{K}. \tag{1}$$

The total potential energy consists of two components: the internal deformation energy U and the potential of external loads P. Therefore,

$$\mathbb{E} = U + P + \mathbb{K}. \tag{2}$$

According to the principle of stationary energy, the equilibrium of the truss occurs when the variation in the mechanical energy of the system is zero. Thus,

$$\delta\mathbb{E} = \frac{\partial \mathbb{E}}{\partial Y_i}\delta Y_i = 0, \tag{3}$$

where Y is the current position of the structure and i represents the degree of freedom. Therefore, for a system with m degrees of freedom, the equilibrium of the structure will be given by m equilibrium equations resulting from

$$\frac{\partial \mathbb{E}}{\partial Y_i} = \frac{\partial P}{\partial Y_i} + \frac{\partial U}{\partial Y_i} + \frac{\partial \mathbb{K}}{\partial Y_i} = 0. \tag{4}$$

The three components represent, respectively, the external forces vector, the internal forces vector and the inertial forces vector, given by

$$F_i^{ext} = \frac{\partial P}{\partial Y_i} = -F_i \ , \tag{5}$$

$$F_i^{int} = \frac{\partial U}{\partial Y_i} = \sum_{e=1}^{nel} \frac{\partial U^e(Y_k)}{\partial Y_i}, \tag{6}$$

$$F_i^{iner} = \frac{\partial \mathbb{K}}{\partial Y_i} = M_\alpha \ddot{Y}_i \ , \tag{7}$$

in which M is the mass related to the node α and $\ddot{Y}$ is the acceleration related to the degree of freedom. Using eq. (5), eq. (6) and eq. (7), it is possible to rewrite eq. (4) as

$$g_i = F_i^{int}(\vec{Y}) + F_i^{ext}(t) + F_i^{iner} = F_i^{int}(\vec{Y}) + F_i^{ext}(t) + \mathbf{M}\ddot{\vec{Y}} = 0_i, \tag{8}$$

where g is the residual vector and $\mathbf{M}$ is the mass matrix. By adding the damping term (proportional to mass and stiffness) to the system, it is possible to obtain the equation of motion described by

$$g_i = F_i^{int}(\vec{Y}) + F_i^{ext}(t) + \mathbf{M}\ddot{\vec{Y}} + \mathbf{C}\dot{\vec{Y}} = 0_i, \tag{9}$$

in which $\mathbf{C}$ is the daming Matrix. This system of nonlinear equations will be solved using the Newton-Raphson method and the results will be considered satisfactory when a previously defined tolerance is achieved. The time integration will be performed using the Newmark method.

3 Results

Considering the shallow space truss, formed by three bars, with its geometric and physical characteristics shown in Figure 1.

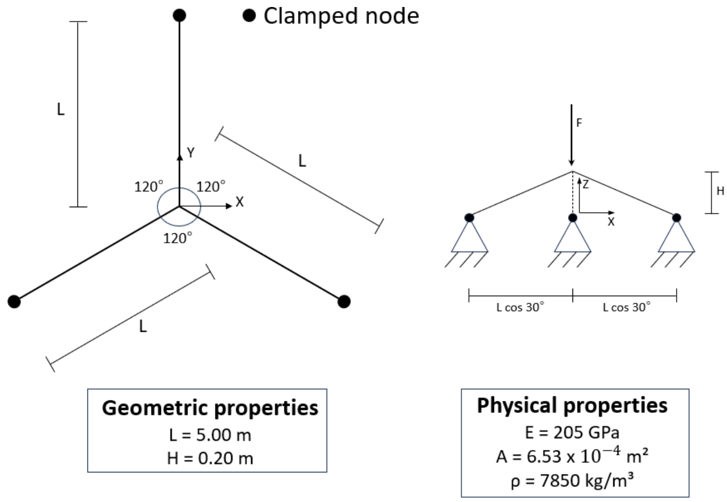


Figure 1. Three bar space truss

3.1 Static analysis

To obtain the equilibrium curve of the structure position control was used by applying an increment $\Delta Y =$ 0.01 m and adopting a tolerance $tol = 10^{-6}$ for equilibrium verification. The obtained result is shown in Figure 2.

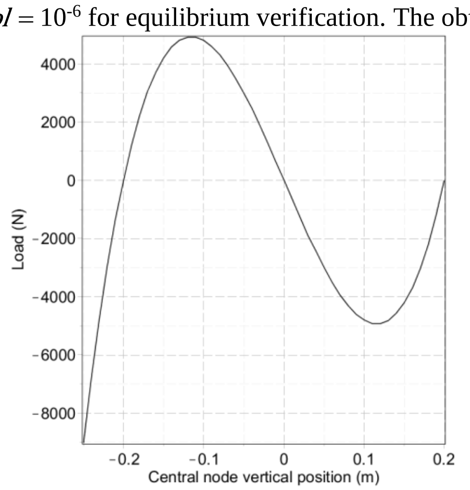


Figure 2. Equilibrium curve of the structure

It is observed that there are two limit points at $P_{lim} = \pm$ 4934.52 N, which, when exceeded, cause a snap-through of the truss. The region defined in between these two points is characterized as unstable. This is a typical shallow truss response curve, with two stable configurations with an unstable position in between. For the considered truss, this implies that $L \gg H$.

3.2 Forced vibration analysis

For the dynamic analysis of the structure, a reference excitation frequency of $\omega = 57.72$ rad/s (equivalent to the first natural vibration frequency of the truss) was used, multiplied by an amplitude corresponding to 12% of the limit load. A total of 240 analyses were conducted (120 in forward sweep and 120 in backward sweep), covering an interval from $0.05\,\omega$ to $1.5\,\omega$. After that, it was possible to extract the resonance curve of the structure, defined by the maximum position amplitude reached by the central node of the truss when subjected to each frequency during the steady-state vibration phase. Following each analysis, the subsequent one was performed considering initial conditions (position and velocity) obtained from the equilibrium position that resulted in the highest amplitude value for the previously analyzed frequency. Each analysis was conducted with a time increment of 0.005 s, considering the steady-state vibration phase within the interval from $100\,T$ to $120\,T$, where T is the vibration period of the applied frequency. The resonance curve is illustrated in Figure 3, depicting a softening-type response of primary resonance for excitation frequencies between 43 rad/s and 51 rad/s. Secondary subharmonic resonance is also observed for excitation frequencies close to $\omega/2$.

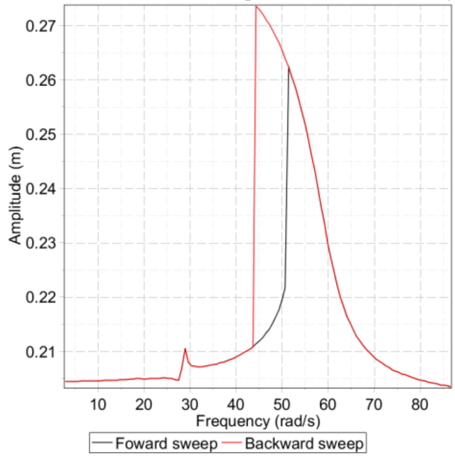
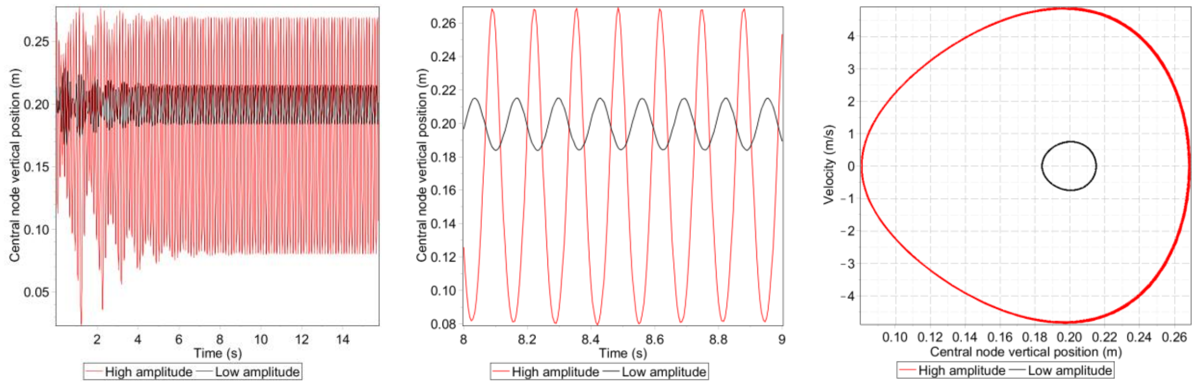


Figure 3. Resonance curve of the structure



(a) Time response (b) Time response (steady-state phase) (c) Phase plane

Figure 4. Results obtained with a frequency of 47.90 rad/s

To better understand the primary resonant behavior, time response, and the phase-plane were obtained for the frequency of 47.90 rad/s, as this frequency lies within the resonance zone, taking into account the high and low amplitude responses. These results are shown in Figure 4, with the small (large) amplitude in black (red). The small amplitude has a permanent phase-space response close to an ellipse, whereas the large amplitude has not. The large amplitude response is highly nonlinear, resulting from the closeness of the orbit to the limit point, with the vertical position as small as 0.08. Perturbations to this large amplitude response can trigger an escape to the inverted, post buckling position.

4 Conclusions

As expected, the implementation of the positional FEM was able to capture characteristics of the static behavior of the truss, such as the equilibrium curve, as well as characteristics of the nonlinear dynamic behavior, such as time responses, phase-planes, and resonance curves. The resonance curve depicts a softening-type response for frequencies close to the first natural frequency and a subharmonic resonance. Small and large amplitude responses were observed, with the large being too close to the limit point of the system. These results validate the implemented strategy based on the positional finite element method as a resource for the nonlinear dynamic analysis of space trusses.

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