Nonlinear Vibrations and Control of Wind Turbines with Variable Cross-Section and Elastic Foundation

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Abstract. This work the dynamic instability of a coupled wind tower-blade system, considering the blade rotation, variation of the tower cross-section, the elastic foundation and subjected to both wind forces and seismic loads is studied. The non-linear Euler-Bernoulli beam theory is used to model the tower and blade and both the Rayleigh-Ritz method and Hamilton's principle, are used to obtain the set of ordinary nonlinear differential equations of motion which are in turn, solved by the Runge-Kutta method. These are transformed into algebraic equations and solved in matrix form to obtain frequencies and vibration modes. Dynamic instability is analyzed by observing the variation of the natural frequency as a function of blade rotation speed. The results indicate that the coupled tower-blade system becomes unstable when certain vibration modes of the tower and blade are joined. Varying the cross-section of the tower increases the natural frequencies, while considering the elastic base, for constant or variable cross-sections, leads to a reduction of the natural frequencies. Nonlinear resonance curves were studied for a non-linear formulation, considering parameters such as the wind force and harmonic base acceleration. Dynamic instabilities are observed for some of the cases considered, such as increasing wind force or harmonic load amplitude.

Keywords: Structural control. Dynamic instability. Wind tower. Vibration analysis.

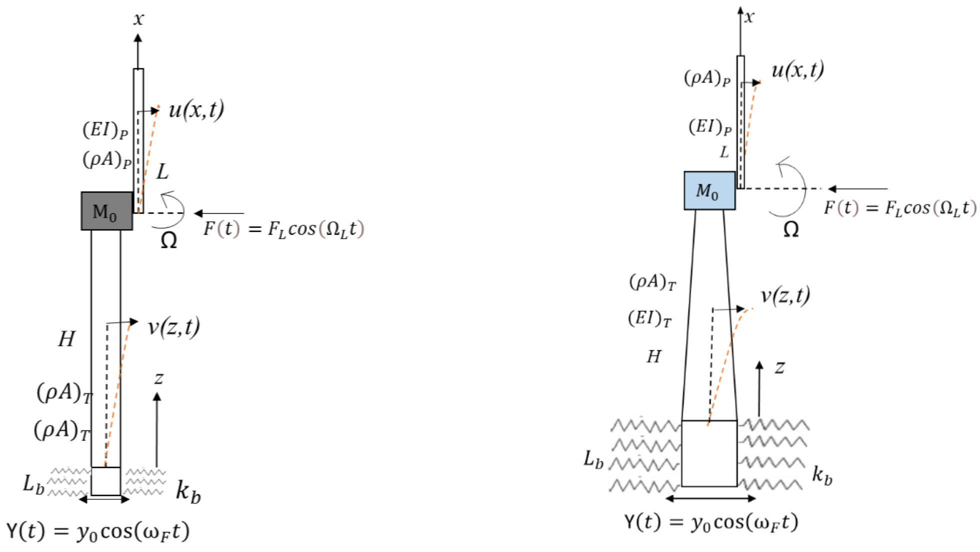
1 Introduction

The wind energy sector is rapidly growing, posing significant challenges for engineers. Wind farms, with their large turbines, face threats like excessive vibrations from wind loads, turbine operation, and seismic activity. Since the Tokyo Protocol of 2005 [1], many countries have increased efforts to cut greenhouse gas emissions through wind energy, a clean, renewable source of power. Johnson et al. (2013) noted that wind energy production requires fewer resources compared to solar or coal-based energy [2]. Larger wind generators have also lowered operating and maintenance costs while boosting efficiency and production (Manwell et al., 2003) [3]. However, blade or tower failures can cause disasters (Ciang et al., 2008) [4]. To prevent such failures, analyzing the dynamic characteristics of wind systems is crucial. Towers, exposed to wind and seismic loads, face repeated bending and torsional stresses that can lead to fatigue, cracks, and failures. Excessive vibrations also affect performance and safety. Studying the dynamic response and control of wind turbines to wind and seismic excitations is vital for enhancing efficiency and safety. Understanding how these factors influence structural responses is essential. Additionally, examining active and passive control strategies can help minimize vibrations and optimize energy conversion [5]. Chen et al. (2009) investigated wind-induced responses in turbine towers and found that tower-blade coupling effects could increase displacement by about 300% [6]. Thus, ensuring wind tower safety requires effective control strategies to reduce vibrations and enhance efficiency.

In this work, the dynamic instability of a coupled wind tower-blade system, considering the blade rotation, variation of the tower cross-section, the elastic foundation and subjected to both wind forces and seismic loads is studied. The non-linear Euler-Bernoulli beam theory is used to model the tower and blade and both the Rayleigh-Ritz method and Hamilton's principle, are used to obtain the set of ordinary nonlinear differential equations of motion which are in turn, solved by the Runge-Kutta method. First, the linear free vibrations are studied, observing the influence of blade rotation, variation of cross section and elastic foundation on the natural frequencies of the system and showing the veering phenomenon. The results indicate that the coupled tower-blade system becomes unstable when certain vibration modes of the tower and blade are coupled. Varying the cross-section of the tower increases the natural frequencies, while considering the elastic base, for constant or variable cross-sections, leads to a reduction of the natural frequencies. Nonlinear resonance curves were studied for a non-linear formulation, considering parameters such as the wind force and harmonic base acceleration. Dynamic instabilities are observed for some of the cases considered, such as increasing wind force or harmonic load amplitude.

2 Mathematical model

Consider a wind tower of height *H*, density ρ_T, cross-section area A_T, Young modulus E_T, and moment of inertia $I_T(z)$. The variable *z* corresponds to the spatial coordinate along the length of the tower, and $v(z,t)$ is its transverse displacement. The blade has length *L*, density ρ_P, cross-section area A_P, Young modulus E_P, moment of inertia I_P, and rotates with an angular velocity Ω. The spatial coordinate of the blade is represented by *x* and $u(x,t)$ is its transverse displacement. To represent the nacelle, a mass M_0 is placed at the top of the tower. The foundation is represented by a Winkler type with stiffness K_b and length L_b. The tower is subjected to a time-varying transverse force $F(t) = F_L cos(\Omega_L t)$ is applied at the top of the tower and to a base displacement $Y(t) = y_o cos(\Omega_F t)$ as seen in Figure 1.



(a) (b)

Figure 1: (a) Constant cross-section tower and elastic base; (b) Variable cross-section and elastic base

The mathematical formulation used to model the tower-nacelle-blade-foundation system is described by determining the energy function [7, 8] of the system through a nonlinear formulation that accounts for large displacements and moderate rotations.

Both, the tower and blade are modelled as clamped-free beams, then the strain energy of the tower (U_T), the blade (U_P) and elastic base (U_b) are given by:

$$U_T = \int_0^H \frac{1}{2} E_T I_T(z) \left(v_{,zz}{}^2 + v^2{}_{,zz} \, v^2{}_{,z} + \frac{1}{4} v^2{}_{,zz} \, v^4{}_{,z} \right) \tag{1}$$

$$U_P = \int_0^L \frac{1}{2} E_P I_P \left(u^2{}_{,xx} + u^2{}_{,xx} \, u^2{}_{,x} + \frac{1}{4} u^2{}_{,xx} \, u^4{}_{,x} \right) dx \tag{2}$$

$$U_b = \int_0^{L_b} \frac{1}{2} k_b v^2 dz \tag{3}$$

Next, the kinetic energy of the tower and the blade is calculated.

$$T_T = \int_0^H \frac{1}{2} \rho_T A_T(z) \left(\frac{\partial v(z,t)}{\partial t} + \frac{\partial y(t)}{\partial t} \right)^2 dz + \frac{1}{2} M_0 \left(\frac{\partial v(z,t)}{\partial t} + \frac{\partial y(t)}{\partial t} \right)^2 \Bigg|_{z=H} \tag{4}$$

$$\begin{aligned} T_P = \Bigg(\int_0^L \frac{1}{2}\, \rho_p A_p \left(\frac{\partial u(x,t)}{\partial t} \right)^2 + \frac{1}{2}\, \rho_p A_p \left(\frac{\partial y(t)}{\partial t} \right)^2 + \rho_p A_p \frac{\partial u(x,t)}{\partial t} \frac{\partial v(z,t)}{\partial t} \Bigg|_{z=H} \\ + \rho_p A_p \left(\frac{\partial y(t)}{\partial t} \right) \left(\frac{\partial v(z,t)}{\partial t} \Bigg|_{z=H} \right) + \rho_p A_p \left(\frac{\partial y(t)}{\partial t} \right) \left(\frac{\partial u(x,t)}{\partial t} \right) \\ + \frac{1}{2}\, \rho_p A_p \left(\frac{\partial v(z,t)}{\partial t} \right)^2 \Bigg|_{z=H} \Bigg) dx \end{aligned} \tag{5}$$

The work done by external force (W_T) and the work done by the centrifugal force due to blade rotation (W_C), can be written as:

$$W_T = -F_L \cos\,(\Omega_L t) v|_{z=H} \tag{6}$$

$$W_C = -\int_0^L \frac{1}{2} \rho_P A_P s \Omega^2 (L-x) \left(\frac{1}{2} u^2{}_{,x} + \frac{1}{8} u^4{}_{,x} \right) dx \tag{7}$$

The nonconservative forces of the tower (W_{ncT}) and the blade (W_{ncP}), are given by:

$$W_{ncT} = \frac{1}{2} c_T v_{,t}^2 \tag{8}$$

$$W_{ncP} = \frac{1}{2} c_P u_{,t}^2 \tag{9}$$

Finally, the Lagrangean functional of the system can be written as:

$$\mathcal{L} = T_T + T_P - U_T - U_P + W_T + W_C \tag{10}$$

The field displacements of the tower and blade, satisfying the geometric boundary conditions, can be written as:

$$u(x,t) = \sum_{j=1}^{n} b_j(t) \left[1 - \cos\left(\frac{(2j-1)\pi x}{2L} \right) \right] \tag{11}$$

$$v(z,t) = \sum_{i=1}^{m} a_j(t) \left[1 - \cos\left(\frac{(2i-1)\pi z}{2H} \right) \right] \tag{12}$$

The coefficients a_j and b_j represent displacement amplifications that vary over time, indicating the dynamic response of the system. The parameters n and m represent the number of modes considered in the analysis.

By substituting field displacements of Eq. (11 and (12) into Eq. (8), (9) and (10), applying the Rayleigh-Ritz method, the set of nonlinear dynamic equilibrium equations are obtained by using the Hamilton principle, given by:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{a}_i} \right) - \frac{\partial \mathcal{L}}{\partial a_i} = \frac{\partial W_{ncT}}{\partial \dot{a}_i} \quad i = 1..m \tag{13}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{b}_j} \right) - \frac{\partial \mathcal{L}}{\partial b_j} = \frac{\partial W_{ncT}}{\partial \dot{b}_j} \quad j = 1..n \tag{14}$$

3 Numerical results

To characterize the physical and geometric properties of the selected wind turbine tower model, the following data are used as reference. The tower has a cylindrical shape with a height of 107 m, an inner radius of 2.910 m, an outer radius of 2.940 m, a density of 8500 kg/m³, and a Young's modulus of 210 GPa. At the top of the tower there is the nacelle, which has a mass of 240000 kg. The blade, connected to the top of the tower, has a length of 60 m, a thickness of 0.60 m, a density of 1500 kg/m³, a Young's modulus of 69 GPa, and a width of 1 m. Considering the description of the mathematical formulation in the previous paragraph, it is possible to determine the natural frequencies and corresponding vibration modes. In the case of a variable-section tower, a 40% increase in the base radius of the constant-section tower is considered. A value of 1.34×10^8 N/m is used for the spring stiffness and, 1% of damping coefficient for blade and tower.

Figure 2 shows the variation of the natural frequencies of the system for increasing values of the blade rotation. Figures 2(a) and 2(b) represent the natural frequencies respectively of the constant-section tower and variable-section tower without elastic base. The analyses reveal a structural instability known as "veering", where two natural frequencies of distinct modes approach and abruptly exchange their directions. When the tower section varies, the natural frequencies increase with the rotation speed of the blades and regions where veering occurs, are shifted to the right, but also getting closer the frequencies at these points. In figures, dotted circles A, B, C and D marked on the figures correspond to the veering phenomenon.

Now, figures 2(c) and 2(d) represent respectively the variation of the natural frequency of the system for constant and variable cross-section of the tower when elastic base is considered. As can be seen, the elastic base affects strongly the natural frequencies of the system, increasing the values of the natural frequencies but also, shifting all regions of the veering phenomenon.

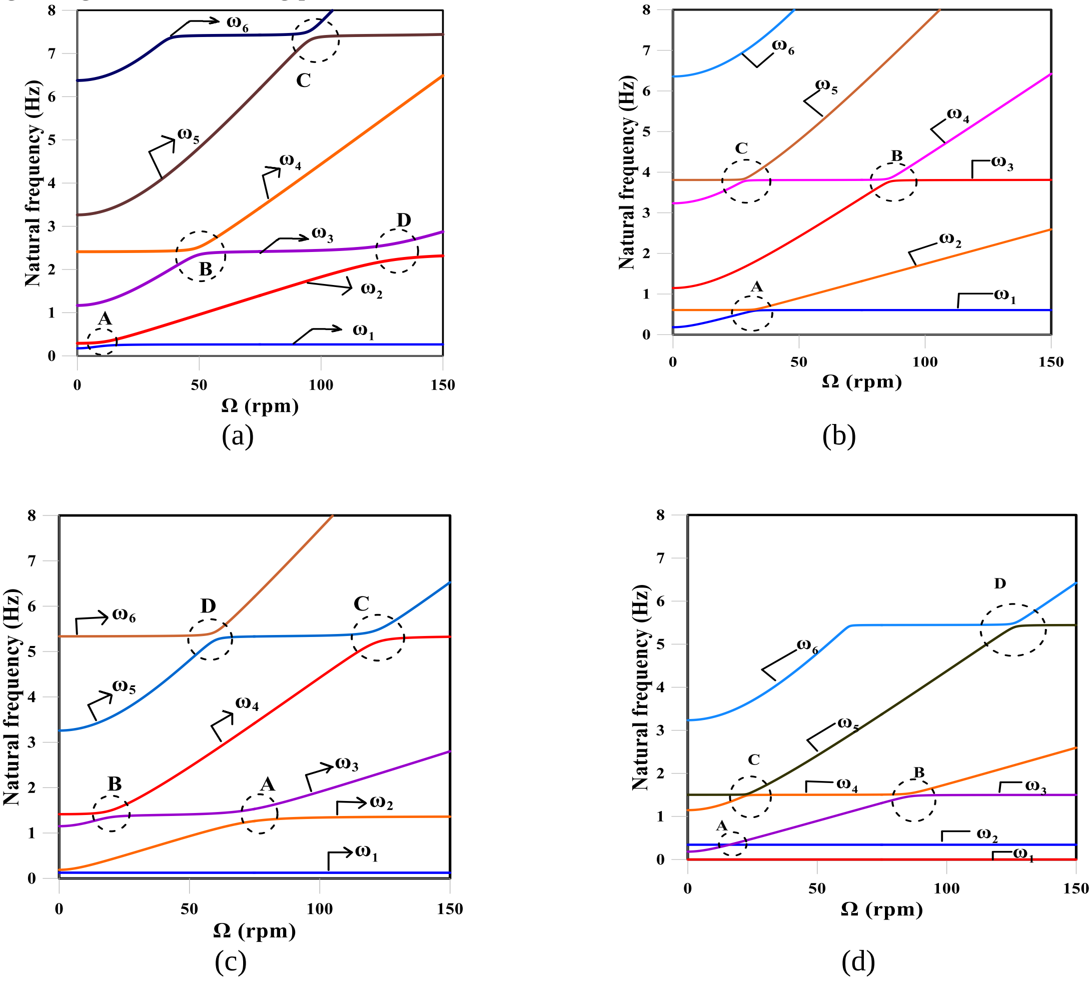


Figure 2. Variation of natural frequencies with increasing values of blade rotation and when elastic base is considered.

Tables 1 and 2 show the comparison of the first six natural frequencies for constant and variable cross-section of the tower with and without elastic base. As can be observed in Tab. 1 which considers constant cross-section of the tower, the elastic base affects strongly the two first natural frequencies of the system, but it has less effect on higher modes. On the other side, when variable cross section is considered, the elastic base affects strongly all natural frequencies. These results, show the importance of considering both, the elastic base and variable cross section of the tower.

Table 1. Natural frequencies of the constant-section tower with and without elastic base

Mode	Natural frequency (Hz)		
	Without elastic base	With elastic base	Difference (%)
ω_1	0.175	0.126	28.57
ω_2	0.291	0.188	35.39
ω_3	1.168	1.150	1.541
ω_4	2.411	1.414	41.35
ω_5	3.264	3.258	1.958
ω_6	6.375	5.335	16.31

Table 2. Natural frequencies of the variable-section tower with and without elastic base

Mode	Natural frequency (Hz)		
	Without elastic base	With elastic base	Difference (%)
ω_1	0.182	0.004	81.13
ω_2	0.605	0.343	43.30
ω_3	1.217	0.182	85.04
ω_4	3.234	1.146	64.56
ω_5	3.808	1.502	60.55
ω_6	6.556	3.235	50.65

4 Dynamic behavior of the non-linear turbine formulation

This investigation delves into the influence of the wind turbine's base displacement $Y(t)$ and applied load $F(t)$ on its nonlinear dynamic behavior for the constant cross-section. The analysis is conducted utilizing resonance curves derived from the application direct time integration and the brute force method. By simplifying the structure to a 4-degree-of-freedom nonlinear model (2 dof for the tower and blade), the resonance curves are obtained. These curves illustrate the system's dependence on a given parameter, enabling the deduction of the structure's qualitative behavior in terms of stability by observing the alterations in equilibrium trajectories and bifurcation points.

Figure 3, the resonance curves are displayed for increasing values of lateral load amplitude (F_L=10 KN, 30 KN, 100 KN, 300 KN and 600 KN) with blade rotation Ω = 12 rpm without considering elastic base. Figure 3(a) shows the resonance curve without base displacement. As can be seen, the observation highlights a gradual increase in tower displacement with the increase in load frequency, characterized by two distinct peak values which are located at the value of the first two natural frequencies of the system. As the value of the load amplitude is increased, the vibration amplitudes of the system are also increased, and for large values, there is a jump with typical hardening behavior. Now, in Figure 3(b) the resonance curves are displayed when harmonic base displacement is considered with amplitude y_o = 0.03 m. As can be observed, the base displacement has stronger effects on the curves for small force amplitudes and approaching each other. When the amplitude of the base displacement is increased to y_o = 0.06 m, the resonance curves depicted in Figure 3(c) are even closer each other.

In order to understand the effect of base displacement, Figures 3(d) and 3(e) display the resonance curves for the same load amplitude value and increasing base displacement amplitudes. In Figure 3(d), for small load amplitude as seen before, it is possible to observe that, the resonance curves are more affected and increased vibration amplitudes will be displayed for increasing base displacement amplitudes. In Figure 3(e), the resonance curves for large load amplitude are depicted. Now, it is possible to see that base displacement does not affect the resonance curves, and the dynamic instability phenomena are the same with and without base displacement.

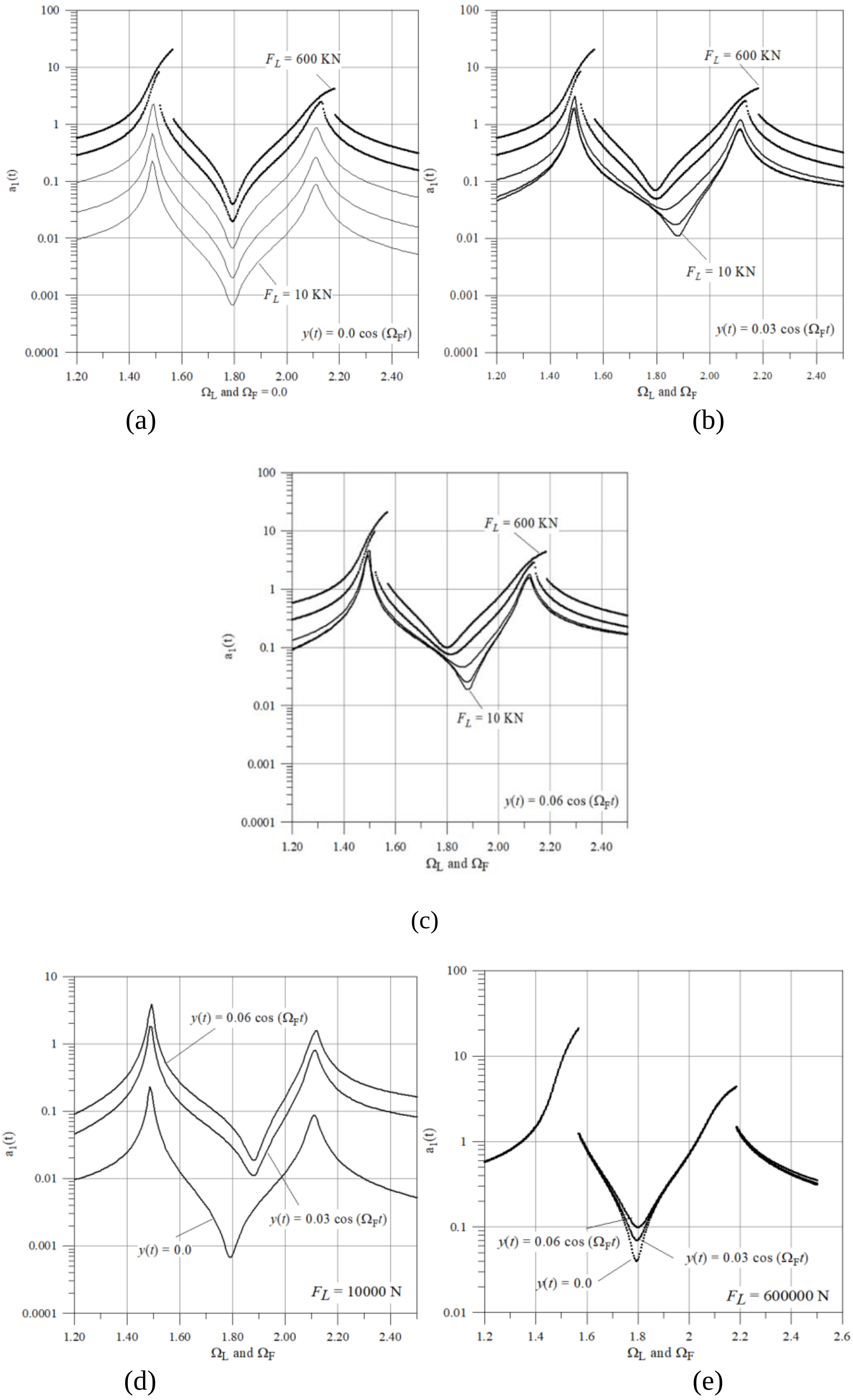


Figure 3. Resonance curves of nonlinear formulation for constant cross section tower

5 Concluding Remarks

In this study, the nonlinear dynamic behavior of a wind turbine considering the interaction between the wind tower and the rotating blades was studied. An efficient model of wind tower with an elastic base and variable cross-section was applied and better reflects the real behavior. The coupled motion equations were derived and solved to determine the natural vibration modes of the system. The results showed that the presence of an elastic base and a variable cross-section of the tower significantly influence the vibration modes. Furthermore, a phenomenon known as "veering" was observed. This phenomenon occurs when the vibration frequencies of the tower and the blades come close, which can lead to instabilities. The analysis was then extended to the nonlinear formulation by considering the application of wind load at the top of the tower and a harmonic load representing the base displacement. The resonance curves obtained indicate that the displacements increase with the frequency of the excitation force and that dynamic instability zones appear at higher forces and when the base displacement is taken into account. These results confirm the instability mechanism observed in previous studies and highlight the importance of considering the dynamic interaction between the tower and the blades for the safe and efficient design of wind turbines.

This in-depth study has provided a better understanding of the complex dynamic behavior of wind turbines, taking into account various factors such as tower flexibility, cross-section variation, and applied forces. The results obtained provide valuable information for the design and operation of more efficient and stable wind turbines.

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