PARAMETRIC STUDY OF THE APPARENT MODAL PARAMETERS OF CROWD-FOOTBRIDGE SYSTEMS

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Abstract. Low damping and slender footbridges are susceptible to human-structure interaction (HSI) effects, in which a new mechanical system composed of the walking persons and the footbridge-structure is formed, with modal parameters different from those of the isolated structure. Given the importance of HSI in footbridge behavior, numerical methods for estimating these apparent modal parameters have been proposed on the literature. This paper presents an alternative algorithm for determining the modal parameters of the coupled system through the numerical solution of the crowd-structure system's equations in a time domain analysis. A parametric study is then conducted to show the variation of the modal parameters of the coupled crowd-footbridge system relative to those of the isolated footbridge, as the pedestrian-structure mass ratio increases. The study focuses on simply supported structures under unrestricted pedestrian flow and in resonant vibration with the first harmonic of the pace frequency (1.7 Hz - 2.3 Hz). The results reported in this work show that the differences between the apparent modal parameters of the coupled system and those of the isolated system are particularly significant for high values of the pedestrian-structure mass ratio (up to 0.5), low damping ratios (down to 0.5%) and high natural frequency of the isolated structure, for the ranges of values evaluated herein.

Keywords: footbridges, human-structure interaction, biodynamic models, coupled crowd-structure system.

1 Introduction

Lightweight and low damping footbridges are susceptible to human-structure interaction (HSI) effects, where a new mechanical system, composed of the structure plus the walking persons is formed, with apparent modal properties different from those of the isolated structure. Several studies [1-5] have experimentally demonstrated the effects of the HSI in footbridges, particularly the increase in the damping ratio. For instance, Gallegos-Calderón [1] conducted experimental measurements on an ultralightweight footbridge with varying pedestrian densities and observed a significant increase in the modal damping ratio of the coupled system, even with just one walking person. The apparent natural frequency of the coupled system can increase or decrease, depending on the natural frequencies of both the structure and the pedestrians, which depend on the activity and posture performed [2].

When compared to models proposed by codes and guidelines [6], which neglect HSI effects, the predicted dynamic response of the footbridge appears unrealistic when contrasted with experimental measurements. The studies by Sachse *et al*. [7], Zall *et al*. [8], Shahabpoor *et al*. [9], and Van Nimmen *et al*. [10] introduce numerical methods to estimate the apparent modal parameters of the crowd-structure system. These parameters can be incorporated into a single degree of freedom model (SDoFM) representing both the pedestrians and the structure. By integrating the models suggested by codes and guidelines into the SDoFM of the coupled system, a more accurate prediction of the structure's dynamic response can be achieved.

This study aims to contribute to an ongoing effort to develop a straightforward approach for estimating the modal parameters of the coupled system through a set of simple equations that can be easily applied by engineers.

The apparent modal parameters of the coupled system, namely the damping ratio and the natural frequency, are determined herein by numerically solving, in a time domain analysis, the differential equations representing the mathematical model of the crowd-foobridge system. The step by step of the algorithm of this procedure is detailed herein. The validation of the methodology and its numerical implementation is also presented by comparing the theoretical apparent modal parameters with those experimentally measured on a laboratory footbridge from the literature. A parametric study is performed to evaluate the importance of the parameters of the isolated structure (natural frequencies, damping ratios and modal mass) and the pedestrian-structure mass ratio on the normalized modal parameters of the coupled system.

The results presented herein are consistent with those reported in the literature and show that the normalized apparent modal parameters of the coupled system do not depend on the modal mass of the structure, only on the pedestrian-structure mass ratio, and that the normalized apparent natural frequency is minimally affected by the damping ratio of the unoccupied structure. Furthermore, this study contributes to an ongoing effort aimed at developing a straightforward approach for estimating the modal parameters of the coupled system.

2 Human-structure interaction mathematical model

2.1 Biodynamic model (BM)

To address the effects of human-structure interaction (HSI), each pedestrian may be simulated as a dynamic system coupled to the model of the footbridge [1,2,4,5,7-10]. A single degree of freedom biodynamic model (SDoFBM) proposed by Pfeil *et al*. [11] and Varela *et al*. [12], whose modal parameters are presented in Eq. (1), is selected herein to represent each walking person. This model was derived under the assumption that the walking person can be represented by a SDoF system subjected to base excitation, simulating the vertical displacement imposed by the movement of his/her heels. This model was selected because its formulation is consisted with the development of the HSI equations [15], as opposed to some models presented in the literature, and presented good correlations with experimental results [11].

$$\begin{aligned} m_p &= 0.874M - 9.142f_p + 12.940 \\ k_p &= 360.300m_p - 1282.500 \\ \xi_p &= -20.818f_{md} + 87.513 \end{aligned} \tag{1}$$

where M and f_p are the mass (in kg) and the walking frequency (in Hz) of the pedestrian, respectively; m_p (kg), k_p (N/m) and ξ_p (%) are the modal mass, stiffness and damping ratio of the BM, respectively; f_{md} is the damped natural frequency of the human-model, that is iteratively determined by: $f_{md} = f_m\sqrt{1-\xi_p^2}$, whereas $f_m = (1/2\pi)\sqrt{k_p/m_p}$ is the natural frequency of the BM.

2.2 Crowd-structure mathematical model

Pfeil *et al*. [15] present a mathematical model to account for the HSI effects on the dynamic response of footbridges subjected to the passage of a single pedestrian. This model is adapted herein to incorporate multiple pedestrians, as illustrated in Fig. 1, resulting in a system of coupled differential equations that combines the SDoF model of the isolated footbridge with the SDoF models representing each walking person. Eq. (2) presents the system of motion equations which is numerically solved using the Runge-Kutta method, effective for initial-value problems and providing high accuracy without requiring high-order derivatives [16].

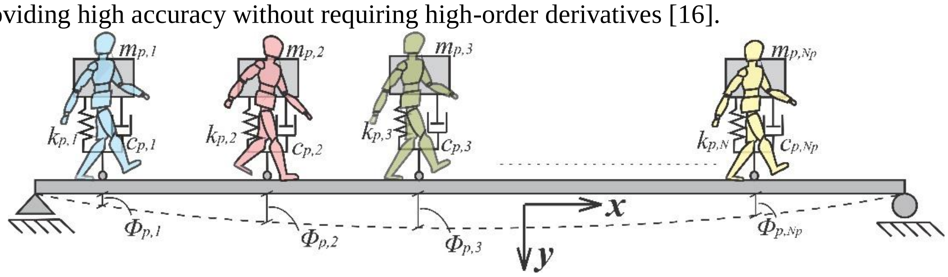

Fig. 1: Illustration of persons walking along a simply supported footbridge represented as SDoF biodynamic models.

$$\begin{bmatrix} m_s & 0 & \cdots & 0 \\ 0 & m_{p_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & m_{p_{N_p}} \end{bmatrix} \ddot{\boldsymbol{Y}} + \begin{bmatrix} \left(c_s + \sum_{i=1}^{N_p} \Phi_{p_i}^2 c_{p_i}\right) & -\Phi_{p_1} c_{p_1} & \cdots & -\Phi_{p_N} c_{p_{N_p}} \\ -\Phi_{p_1} c_{p_1} & c_{p_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\Phi_{p_N} c_{p_{N_p}} & 0 & \cdots & c_{p_{N_p}} \end{bmatrix} \dot{\boldsymbol{Y}} + \begin{bmatrix} \left(k_s + \sum_{i=1}^{N_p} \Phi_{p_i}^2 k_{p_i}\right) & -\Phi_{p_1} k_{p_1} & \cdots & -\Phi_{p_N} k_{p_{N_p}} \\ -\Phi_{p_1} k_{p_1} & k_{p_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\Phi_{p_N} k_{p_{N_p}} & 0 & \cdots & k_{p_{N_p}} \end{bmatrix} \boldsymbol{Y} = \begin{bmatrix} \sum_{i=1}^{N_p} \Phi_{p_i} F_{GRF}^i(t) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (2)$$

where m_s, c_s and k_s are, respectively, the modal mass, the damping coefficient and the stiffness of the SDoF model of the isolated footbridge, c_{p_i} is the damping coefficient of the BM of the i^th^ pedestrian, $F_{GRF}^i(t)$ is the ground reaction force (GRF) induced by the i^{th} pedestrian and $\boldsymbol{Y}$ is the vector of modal coordinates.

This mathematical model (Eq. 2) was numerically implemented. The validation was demonstrated by comparing the obtained acceleration responses with those reported on the work of Caprani and Ahmadi [17] for a simply supported beam structure ($f_s = 2.0$ Hz; $m_s = 12.5$ t; $\xi_s = 0.5\%$; $L = 50$ m). Differences of less than 3% were achieved, indicating the validity of the numerical implementation of the model.

3 Identification of the modal parameters of the coupled system

3.1 Proposed procedure

The purpose of this procedure is to obtain the modal parameters, namely the damping ratio (ξ_{os}) and the natural frequency (f_{os}), of the coupled system comprising the simply supported structure and the pedestrians represented by the BM described in Section 2.1, for which the average values of f_{md} and ξ_p are equal to 2.9Hz and 30%, respectively [11].

The procedure consists in a free vibration analysis of the coupled system model in which the BMs are located in fixed positions along the structure. The cited parameters are then obtained by applying the fast Fourier transform (FFT) and the logarithmic decrement technique to the structural response signals. A statistical approach using Monte Carlo simulations is employed to determine the cumulative distribution function of the goal parameters by sampling the position, physical and gait characteristics of the pedestrians. For a given footbridge with span length L, width B and modal parameters m_s, c_s and k_s, subjected to a crowd of pedestrians with density ρ, the procedure is described by the following steps:

(i) determine the number of pedestrians (N_p) distributed along the structure ($N_p = \rho BL$);

(ii) generate random samples for the mass (M) and natural frequency (f_p) of each person, following a probability distribution. In this study, a normal distribution is assumed for these parameters, with mean and standard deviation according to the recommendations made by Živanović [3]: M (kg) [$\boldsymbol{\mathcal{N}}$(700; 140)] and by Sétra Guideline [6]: f_p (Hz) [$\boldsymbol{\mathcal{N}}$(2.000; 0.175)];

(iii) calculate the dynamic properties of the BM by applying Eqs. (1), using the values sampled in step (ii);

(iv) generate the position (x_{p_i}) of each person along the footbridge by sampling it from a uniform distribution and calculate the modal coordinate amplitude (Φ_{p_i}) associated with each corresponding position: $\Phi_{p_i} = sin(\pi x_{p_i}/L)$;

(v) solve the system of nonlinear differential equations presented in Eq. (2), assuming the GRF (F_{GRF}^i) equal to zero and imposing an initial perturbation condition equal to the displacement resulting from the crowd weight;

(vi) apply the logarithmic free decay technique to calculate the apparent damping ratio of the occupied structure (ξ_{os_j}) of each simulation j, based on the acceleration time history of the footbridge obtained from step (v);

(vii) determine the apparent natural frequency of the occupied structure (f_{os_j}) of each simulation j by applying the Fast-Fourier Transform (FFT) algorithm to the signal obtained from step (v).

3.2 Validation

To address the validation of the implemented procedure, a set of numerical simulations are performed on a simply supported post tensioned concrete slab from the literature [4] in which the natural frequency (f_{os}) and damping ratio (ξ_{os}) of the footbridge was experimentally determined when occupied for a varying number (N_p) of walking and standing persons. The structure is 12 m long, 2 m large and the experimentally determined natural frequency (f_s), damping ratio (ξ_s) and modal mass (m_s) associated with the first bending mode are, respectively, 4.44 Hz, 0.65% and 7128 kg.

The numerical procedure proposed in Section 3.1 is applied to determine ξ_{os} and f_{os} of this footbridge. For comparison with experimental results, as the physical and gait characteristics of the few pedestrians participating in the tests were not made avaliable, the modal parameters of the occupied structure are assumed to be equal to the values for 50th percentile of the established cumulative distribution function (CDF), denoted herein as $\xi_{os,50\%}$ and $f_{os,50\%}$.

As can be seen in Fig. 2, in this case both experimental and numerical modal parameters increase with the number of pedestrians. The numerical-experimental differences are less than 4.5% for ξ_{os} and 0.5% for f_{os}, validating the procedure for this case. The behaviour of the apparent damping ratio is highly supported by results from literature [1-5, 7-10]. In relation to the behaviour of the apparent natural frequency of the structure it is seen in Fig. 3b that for standing persons f_{os} decreases with N_p, as expected. On the other hand, an increase in f_{os} with the number of pedestrians is observed, in agreement to Shahabpoor [2] results in which the variation of f_{os} with N_p depends on the natural frequency of the pedestrian biodynamic model: for $f_{md} < f_s$, f_{os} increases with N_p; for $f_{md} > f_s$, f_{os} decreases with N_p.

By comparing the numerical and experimental results for the persons walking presented in Fig. 2, it is confirmed that the proposed methodology for estimating the modal parameters of the coupled system, along with its numerical implementation, is valid, demonstrating a maximum percentage difference of less than 5% for this case example. This consistency with the expected results The good theoretical – experimental correlation obtained for the analyzed structure supports the accuracy of the methodology and enables allows for further simulations to conduct the parametric study outlined in this paper. However, these findings also indicate that while the methodology performs well within the tested parameters, any discrepancies may arise from the lack of available information regarding the physical and gait characteristics parameters of the pedestrians involved in the experimental tests.

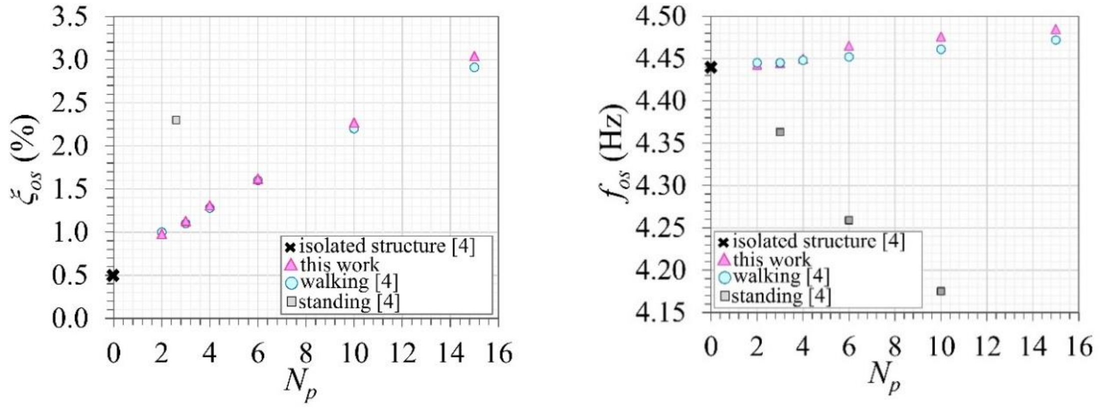

Fig. 2: Comparison between the apparent modal parameters of a simply supported footbridge obtained with this work and the experimental results from Shahabpoor *et al*. [4]: (a) damping ratio (ξ_{os}); (b) natural frequency (f_{os}).

4 Parametric study

The parametric study is conducted with the main purpose of investigating the role of the modal parameters of the isolated structure (ξ_s, f_s, m_s) and the crowd density in the behavior of the apparent modal properties of the coupled dynamic system. For the goal of this study, a simply supported footbridge displaying vibration in the first vertical bending mode is assumed as the reference-example (L=50m, B=2m, m_s=12500kg) [17], and the parameters of the coupled system are set to the 50th percentile of the CDF, denoted here as ξ_{os} and f_{os} for simplicity.

Figs. 3a-c present the variation of the normalized damping ratio of the coupled system (ξ_{os}/ξ_s) and Fig. 3d the variation of the normalized natural frequency (f_{os}/f_s) with the mass ratio ($\mu = 0.05 - 0.50$), defined as $\mu = \sum_i^{N_p} m_{p_i}/m_s$, for a range of values of the isolated structure parameters associated to the first vertical bending mode: damping ratio ($\xi_s = 0.5\% - 3.0\%$) and natural frequency ($f_s = 1.7$ Hz $- 2.3$ Hz). This range of values is selected to represent different footbridge systems, particularly covering lightweight and low-damping structures (with high mass ratios) and natural frequencies close to those induced by pedestrians walking.

From Fig. 3a-c, it is observed that the normalized damping ratio (ξ_{os}/ξ_s) increases nonlinearly with the mass ratio. Moreover, the lower the damping ratio ξ_s the greater the HSI effect, in agreement with observations made by other authors [2,7,9]. Fig. 3d shows that the normalized natural frequency (f_{os}/f_s) decreases linearly with the mass ratio (μ). Although this result is different from those presented in Fig. 3b, both examples are in agreement with the aforementioned Shahabpoor *et al*. [2] findings, as explained in Section 3.2.

When comparing Fig. 3a-c, it may be observed that the ratio between the natural frequency of the isolated structure (f_s) and the natural frequency of the human body (f_{md} = 2.9Hz [11]) plays a crucial role in the HSI effects leading to increasing values of the normalized damping ratio and decreasing values of the normalized frequency in the considered range of f_s (1.7Hz to 2.3Hz). Furthermore, it was verified in this study, but not showed in Fig. 3d, that the variation of (f_{os}/f_s) is not influenced by the damping ratio of the isolated structure (ξ_s), as expected for low damping ratios SDoF dynamic systems [18]. Additionaly, it is worth noting that the normalized damping ratio (ξ_{os}/ξ_s) and the normalized natural frequency (f_{os}/f_s) do not change for different values of m_s varying in the analyzed range (12.5t to 50t).

The graphs of Figs. 3 may be used in the design of footbridges to obtain the dynamic characteristics of the coupled system to be applied to the SDoF model of the structure under the harmonic load models provided by codes and guides [6]. This procedure would allow more realistic and less conservative results than those obtained with the properties of the isolated structure.

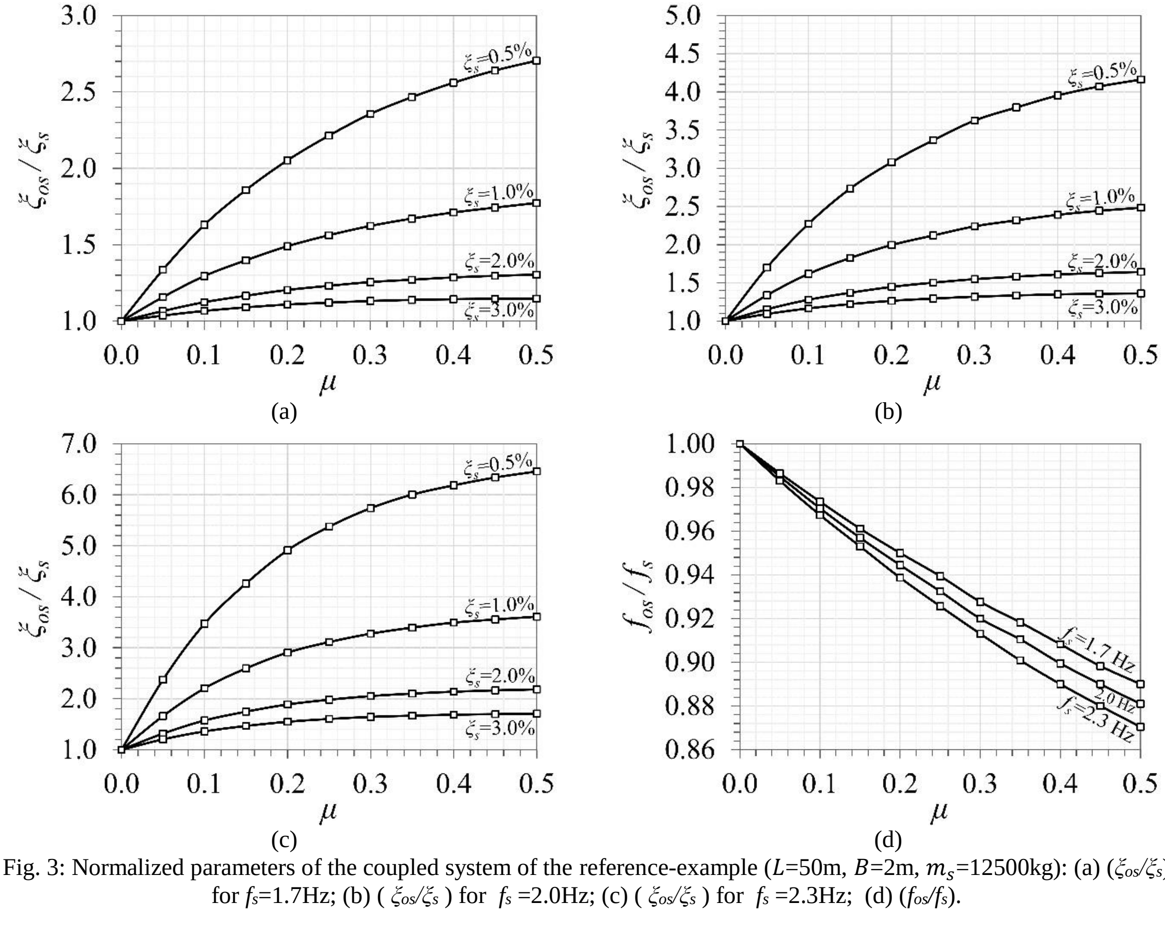

Fig. 3: Normalized parameters of the coupled system of the reference-example (L=50m, B=2m, m_s=12500kg): (a) (ξ_{os}/ξ_s) for f_s=1.7Hz; (b) (ξ_{os}/ξ_s) for f_s =2.0Hz; (c) (ξ_{os}/ξ_s) for f_s =2.3Hz; (d) (f_{os}/f_s).

5 Conclusions

This study investigates the impact of human-structure interaction (HSI) on the damping ratio and natural frequency of coupled crowd-footbridge systems. The results show that increasing the pedestrian-to-structure mass ratio significantly enhances the system's damping and decreases its natural frequency compared to the isolated structure. These effects are more pronounced in lightweigth and low damping footbridges. Therefore, neglecting HSI can lead to significantly conservative dynamic responses, particularly for these types of footbridges. While the developed algorithm provides a realistic estimation of the coupled system's parameters, when compared to experimental results, its high computational cost limits practical use when estimating the dynamic response of footbridges. Future research should explore simpler and more straightforward methods, such as simple equations derived from the presented parametric study.

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