Vibration of extremely flexible beam axially tensioned by incremental force

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Abstract. Cables can be associated with extremely flexible beams. Necessarily, electrical system cables supported on transmission towers need to be tensioned for proper use. The tensile force modifies the stiffness of the system, causing an increase in natural vibration frequencies. In solving the vibration problem, numerical methods can be employed. To do this, the adopted model has an analogy with a simply supported beam, for which the successive integration of the differential equation of the bending moment takes to displacements of the axis, or the elastic line, for each stage of the iterative process. For a refined result, large number of iterations are required, increasing the computational effort. To solve the proposed problem, a programming routine was developed in the "Python Jupyter Notebook" code that allowed calculating the deformation of the structural system and the natural frequency of vibration for each iteration step. The process begins with the analytical solution of the approximate elastic line under the effect of gravity and the subsequent tension in the cable. After that, the definition of the natural frequency of vibration is determined by using the Rayleigh method. In the end, it was observed that the first natural vibration frequency of the cable was raised non-linearly from 0.358 Hz to 5.944 Hz.

Keywords: beam, flexibility, incremental force, numerical solution, natural frequency of vibration

1 Introduction

During their operational life, overhead cables used in power transmission and communication lines face climatic conditions that can mobilize their natural modes of vibration. Dias et al. [1] proposed the implementation of damper systems along the line in order to reduce the amplitude of vibrations induced by the wind. Dua et al. [2] highlight the complexity that exists when considering wind forces in relation to transmission line cables. This difficulty is mainly attributed to the non-linearities of these structural systems and the turbulence associated with the average wind speed. To mitigate these effects, they also suggest the implementation of vibration dampers (or attenuators). Golebiowska et al. [3] highlight the need to reduce the amplitudes of wind-induced vibrations, due to the possibility of interruption they represent to overhead transmission lines.

Onunka and Ojo [4] used a beam element compatible equation of motion to simulate the cable in order to calculate wind-induced vibration frequencies in high voltage transmission lines. Results considered satisfactory with other analysis methods were obtained, which validates the beam model in representing these structural elements. Dan et al. [5] presented an equation for determining the vibration frequency related to the transverse component of the wind flow, where specific properties of cables were incorporated, such as linear density, bending stiffness and tensile strength. Dan et al. [6] included damping in cable dynamics via a numerical solution and concluded that the damping coefficient and tension in the cable mainly influenced the damped natural frequencies of the system. Formica et al. [7] developed a code in "Python" that combined three-dimensional modeling and nonlinear dynamic analysis using the finite element method to analyze the dynamics of cable suspension bridges.

Cables, by their own nature, are extremely slender and flexible structural elements. Its equilibrium is only possible in the deformed configuration of the system. As a rule, overhead cables need to be tensioned for proper and correct use in transmission lines. The tensile force used during the line assembly operation changes its natural frequencies, due to the portion of geometric stiffness (Wahrhaftig et al. [8]) that is introduced into the total system stiffness (Wahrhaftig et al. [9]). Cable deformation is influenced by its geometric and material properties, including viscoelastic behavior, when considered, making these systems intrinsically nonlinear in both geometric and material terms.

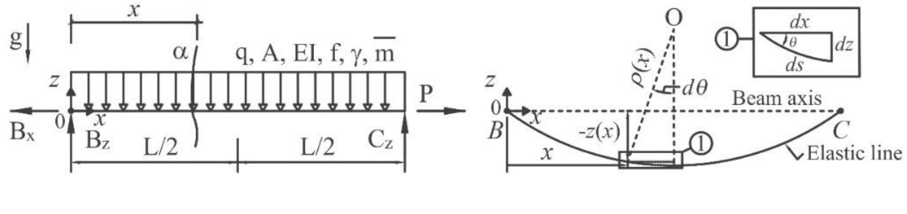
The model adopted, in this work, to represent the cable is analogous to that of a simply supported beam, for which the successive integration of the differential equation of the bending moment leads to displacements of the axis, or the elastic line, for each stage of the iterative process. These iterations are repeated until the sum of the parcels equals the total force to be applied. As smaller the force parcel in each iteration, a better result is obtained. However, the number of iterations required will be greater, which can make computational processing costly. To solve the proposed problem, a programming routine is developed in the "Python Jupyter Notebook" language that allows calculating the deformed shape of the structural system and the natural frequency of vibration, in each iteration. The process begins with the analytical solution of the approximate elastic line under the effect of gravity and the subsequent tension on the cable. The natural vibration frequency solution was based on the Rayleigh method.

2 Numerical simulation

2.1 Elastic line

The approximate equation of the elastic line is the initial step in determining the first natural frequency of vibration. It is desired, in particular, to study the first mode since the shape assumed by the elastic line of the beam can be associated with this mode of vibration, aligning itself with the deformed configuration of the cable under gravitational influence. The elastic line equation describes the deformation of beams subjected to transverse loads, being an appropriate approximation to represent a cable simply supported at the ends.

Figure 1(a) illustrates the static model of a cable represented as a beam in balance, presenting the essential parameters for defining the elastic line equation, where f is the first natural frequency of vibration. The elastic line of a beam is the locus of points on its axis when deflected, Fig. 1(b), (see Wahrhaftig et al. [10]).



(a) (b)

Figure 1. Static diagram of the cable: (a) rigid body balance; (b) deformed configuration.

The model data shown in Fig. 1 are: apparent elastic modulus, *E*, of 2010820773.656 N/m^2; equivalent diameter, *d*, of 18.30 mm; area, *A*, of 2.63022 x 10^{-4} m^2; moment of inertia of the area, *I*, of 5.50521 x 10^{-9} m^4; span length, *L*, of 13.385 m; specific weight, *γ*, of 30301.13 N/m³ (acceleration of gravity, *g*, considered equal to 9.807 m/s^2), mass per unit of length, $\bar{m}$, equal to 0.813 kg/m and uniformly distributed loading, *q*, of 7, 97 N/m. The element identified by these parameters refers to an aerial cable used in transmission lines. The cable is subject to the action of gravity and, therefore, subject to its own weight. By integrating the bending moment equation with respect to *x* it is possible to find the equation for the rotation angle of a cross section at the position of interest, and by integrating the rotation angle equation, the equation for the vertical displacements of the axis is obtained, as described in eq. (1). Therefore,

$$\iint \frac{M(x)}{EI} dxdx = z(x) = \frac{1}{EI}\left(\frac{-qx^4}{24} + \frac{qLx^3}{12} + C_1 x + C_2\right) \rightarrow z(x) = -\frac{qx}{24EI}\left(x^3 - 2Lx^2 + L^3\right) \quad (1)$$

The successive integration of the bending moment and rotation equations provides two integration constants, C_1 and C_2, whose values are determined by the boundary conditions of the problem.

2.2 First natural frequency of vibration

When determining the first natural frequency using the Rayleigh method [11], the total stiffness of the system, *K*, is established by adding the flexural stiffness, k_0, eq. (2), with the geometric stiffness, k_G, eq. (3), which leads to eq. (4):

$$k_0 = \int_0^L EI\left(z''(x)\right)^2 dx \quad (2)$$

$$k_G = P\int_0^L \left(z'(x)\right)^2 dx \quad (3)$$

$$K = k_0 + k_G \quad (4)$$

with *P* representing the acting axial force. The mass distributed along the length of the cable is found according to the relationship $\bar{m}$ = q/g, which is used to determine the generalized mass, *Π*, according to eq. (5):

$$\Pi = \int_0^L \bar{m} z'(x)^2 dx \quad (5)$$

The angular frequency of interest, *ω*, is obtained by the square root of the ratio between the total stiffness, *K*, in eq. (4), by the generalized mass, *Π*, in eq. (5), as indicated in eq. (6):

$$\omega = \sqrt{\frac{K}{\Pi}} \quad (6)$$

Therefore, the first natural frequency, *f*, in Hertz, is given by eq. (7):

$$f = \frac{\omega}{2\pi} \quad (7)$$

2.3 Iterative process

The mathematical expression developed for the iterative solution of the elastic line under an incremental tensile force must incorporate the resulting changes in deformation caused by each increment *i*. Changing the shape of the cable involves an iterative numerical model, in which the current moment is decreased by the bending moment increment given by *ΔP*z(*x*)i in each iteration step, which can be represented by eq. (8).

$$M_i = M_0 + \Delta P \sum_{i=0}^{n-1} z(x)_i \quad (8)$$

The feasibility of calculating the frequency of the first mode of vibration of the cable using the Rayleigh method arises from the use of the elastic line equation, and its successive iteration stages, as a function of shape, until the desired level of force is reached. In the iterative process, the force *P*, predicted in eq. (3), is divided into steps of force ΔP that are added at each stage of the iteration, until the final value of *P* is reached. In the present case, the cable must be tensioned by a force of 16 N to be applied with successive increments of 0.2 N, in 81 iterations. ΔP is equal to zero when determining the first elastic line of the cable.

To implement the described solution, a programming routine was created in the "*Python*" language within the "*Jupyter Notebook*" environment, using three fundamental libraries: "*sympy*", "*numpy*" and "*matplotlib.pyplot*". The "*sympy*" library plays an essential role in defining symbolic variables and performing symbolic calculations, such as integration and solving equations. In turn, the "*numpy*" library works on numerical operations, being used to generate sequences of *x* values through "*np.linspace*", which is essential in preparing the functions to execute the plot. The "*matplotlib.Pyplot*" package assumes the crucial function of plotting the graphs. According to Bas and Mohamed [12], many studies highlight "*Python*" as an appropriate scientific programming language in computer simulation, due to its wide variety of structures and libraries.

In this "*script*", 81 iterations were carried out, on a notebook with the following characteristics: 3.2 GHz Intel core i5-8265U CPU; 16 GB of RAM; 2 TB HD; 1 TB of SSD; GPU 0 Intel(R) UHD Graphics 620; GPU 1 Radeon 520 2GB. The processing time for the total number of iterations, with the described configuration, was 40.5 hours. The input data requested from the user are: cable diameter (*d*), span length (*L*), specific weight (γ), modulus of elasticity (*E*), total applied force (*P*) and force increment (ΔP). With these variables defined, a report is then gradually generated containing the following output data, in each iteration *i*: accumulated force (*Pi*), bending moment equation (*Mi*), integration constant C_1i, constant of integration C_2i, angle equation ($z'i$), deformation equation (*zi*), conventional stiffness (k_0i), geometric stiffness (k_Gi), total stiffness (*Ki*), mass (Π_i) and natural frequency (*fi*).

The initial increment ΔP for the initial condition z_0 is equal to zero. The iterative process begins with the calculation of the first deformation z_0 on which the first force increment ΔP is applied until the final force value *P* is reached. This is done using the "*while*" command, which keeps the iterative process active in the condition $\Delta P(\text{i-1}) \leq P$. To perform the "*loop*", or loop of iterations, which symbolizes the product of the increment by the number of iterations minus one, empty lists were created, initially, to store all the program output data, because, at the end of each "*loop*", the variable *i*, which represents the current iteration, becomes $i = i + 1$, and the bending moment, rotation and cable deformation equations become the current iteration, as indicated in Fig. 2.

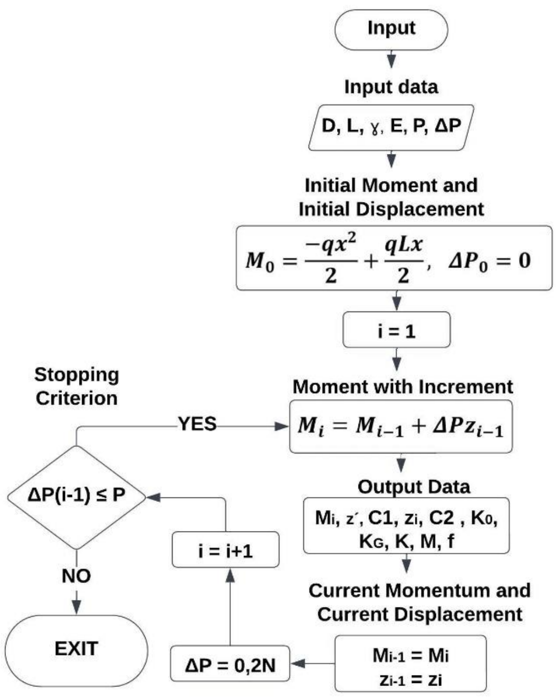
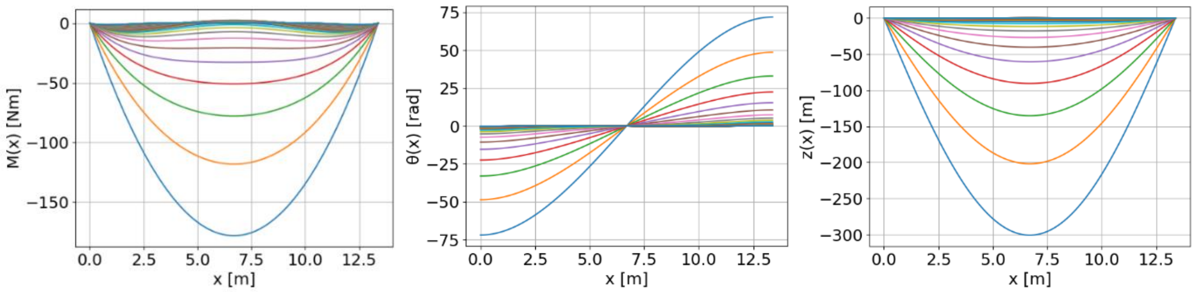


Figure 2. Programming flowchart.

3 Results and discussions

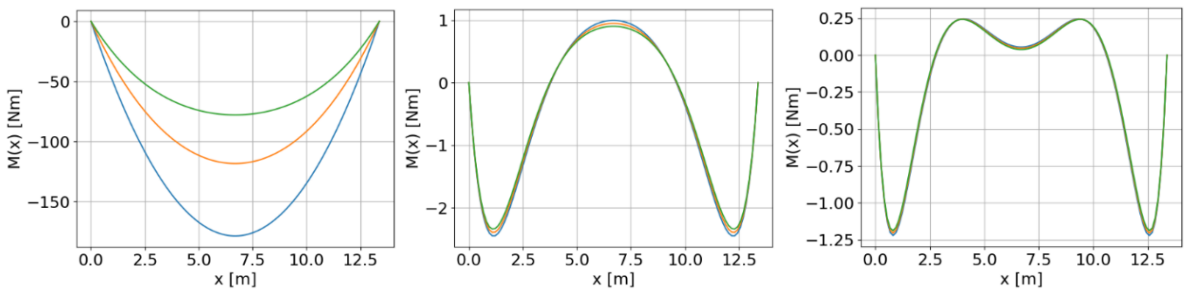
The bending moment values for each iteration can be seen in Fig. 3(a). The reduction in bending moment occurs with each applied increment due to the reduction in vertical displacements with each step of force, ΔP, since the incremental portion of bending moment has the opposite sign. As the tensile force accumulates, the cable tends to reduce the rotations of its cross sections, with these moving in the direction of horizontality, as shown in Fig. 3(b). The incremental deformation shape of the cable is shown in Fig. 3(c). In the shape change process, the cable displacement amplitudes decrease gradually and proportionally to the applied tensile force.



(a) (b) (c)

Figure 3: Diagrams for all iterations: (a) Bending moment; (b) Rotation; (c) Displacement.

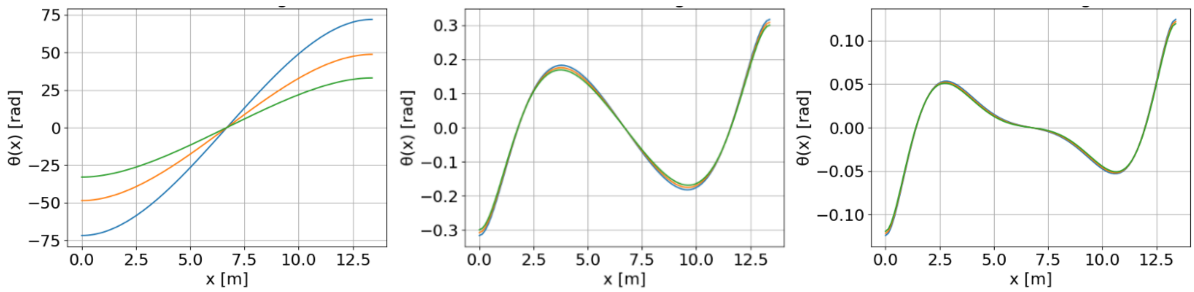
The detailed examination of the bending moments presented in Fig. 3(a), can be done by comparing the initial, intermediate and final iterations of the numerical process, Fig.4. The presence of bending moments with a downward concavity stands out, at the midpoint of the span. This aspect may indicate the need for an even greater reduction in the value of the tensile force increments. It is assumed that this measure would avoid inversion in the cable concavity.



(a) (b) (c)

Figure 4. Bending moment: (a) Iterations 0, 1 and 2; (b) Iterations 40, 41, and 42; (c) Iterations 79, 80, and 81.

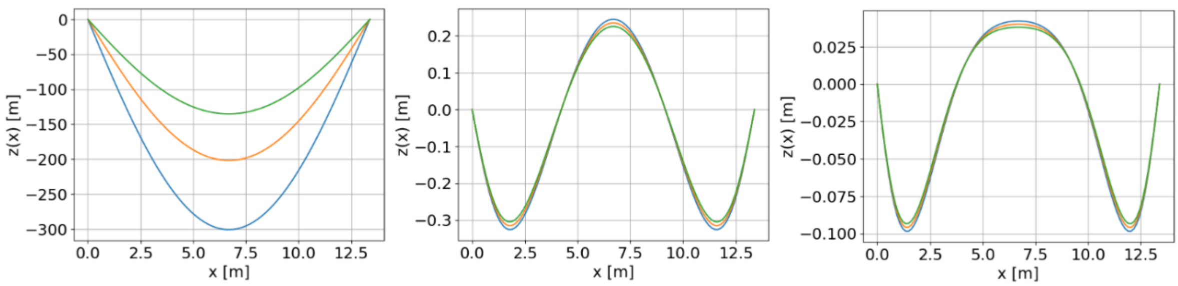
One aspect that deserves to be highlighted is the symmetry of results. Another is the fact that, as the tensile force approaches its final value, with the reduction of bending moments, it is possible to observe a reversal in the rotation of the sections in the last stages of the iterative process, as illustrated in Fig. 5.



(a) (b) (c)

Figure 5. Rotation: (a) Iterations 0, 1 and 2; (b) Iterations 40, 41, and 42; (c) Iterations 79, 80, and 81.

Confirming the existing relationships with the bending moment and the rotation of the sections, the elastic line of the cable inverts in the higher iterations, when the tensile force approaches its final value, as can be seen in Fig. 6.



(a) (b) (c)

Figure 6. Displacement: (a) Iterations 0, 1 and 2; (b) Iterations 40, 41, and 42; (c) Iterations 79, 80, and 81.

The first natural vibration frequency appears to be proportional to the applied force (Fig. 7). This arises from the increase in system stiffness related to the geometric stiffness portion. It is important to note that the results obtained by the Rayleigh method are strongly influenced by the shape function assumed to represent the vibration of the considered mode. In the present case, the elastic line of the beam fulfills this function. Therefore, at each iteration, the shape function changes, reflecting this change in both the stiffness and the generalized mass of the system.

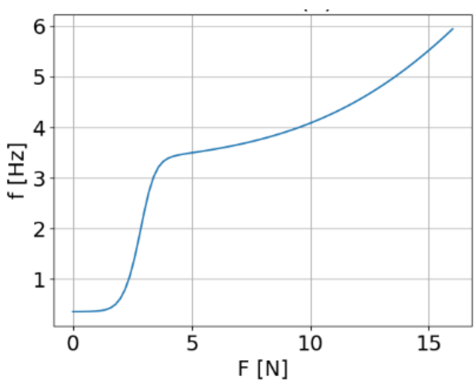


Figure 7. Variation of the cable's first natural frequency with the axial tensile force.

4 Conclusions

In view of what was accomplished, it can be concluded that the computational processing was costly, taking more than 40 hours to converge with a force step of 0.2 N. The initial displacement of the cable was high due to the low modulus of elasticity of the material, requiring small force steps to avoid inverting the curvature. The curvature of the cable was reversed with the increase in force, despite the reduction in absolute displacement, with the initial amplitude changing from -300.895 m to 0.038 m at the end of the iterative process. The vibration frequency increased non-linearly, from 0.358 Hz to 5.944 Hz, influenced by the shape function of the beam's elastic line and the increase in the tensile force, which changes the portion of the system's geometric stiffness. The iterative process allows monitoring both the assembly and the useful life of the transmission line, providing useful information for regressive analyses. The incremental method can be applied to other similar structural systems, such as prestressed beams, using the same principles of the study.

Acknowledgements. The authors would like to thank PIBIC/UFBA, Institutional Scientific Initiation Scholarship Program of the Federal University of Bahia, FAPESB, Bahia State Research Support Foundation, and CNPq (National Council for Scientific and Technological Development) for the scholarships provided.

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