Analytical Models for Railway Track Structural Analysis Based on Numerical and Experimental Data

Maria R. B. M. Ribeiro[1], Manoel. R. S. Filho[2], José A. F. Santiago[1]

[1]*Civil Engineering Program, Federal University of Rio de Janeiro*
Centro de Tecnologia, Ilha do Fundão, Zip-Code 21941- 972, Rio de Janeiro/Rio de Janeiro, Brazil
maria.ribeiro@coc.ufrj.br, santiago@coc.ufrj.br
[2]*Faculty Railway Engineering and Logistics, Federal University of Pará*
Rua Augusto Corrêa, Zip-Code 66075 - 900, Pará/Belém, Brazil
ricardoamabilio@hotmail.com

Abstract. Since the 19th century, mechanistic models based on the theory of elasticity have been developed to structurally analyze the permanent way. In this context, the Winkler model considers the rail as a beam infinitely supported on an elastic base, from which the analytical solutions derived and presented in this research were compared with numerical and experimental results, based on operational data recorded in the Carajás Railway. The software Matlab and SAP 2000 were used to develop the analytical and numerical models, respectively. The comparison considered the loading scenarios for a wheel, with the bogie and the coupling region between wagons, and evaluating the influence of the superposition of effects. The results obtained in both the analytical and numerical models proved to be reliable in the real structural longitudinal representation of the permanent way.

Keywords: Railways, Analytical Models, Structural Analysis.

1 Introduction

Various analytical models play a crucial role in structural design, offering an accurate and technical approach to assessing the integrity and load capacity of infrastructure projects. In railroads, these models employ principles of structural mechanics and stress analysis to determine the forces and pressures exerted on rail track components such as rails, sleepers, and subgrade. By considering factors such as traffic load, train speed, environmental conditions, and soil characteristics, analytical models can make it possible to accurately estimate the useful life of structural elements and identify potential points of failure. In this sense, the approach given for this article will be to carry out the longitudinal analysis of a permanent track based on beam theory under an elastic base considering the efforts that occur on the rails such as displacements, moment, tension in the lower fiber of the rail base and shear.

2 Theoretical Background

2.1 Winkler Model

The Beam on Elastic Foundation (BOEF) model by Winkler [1] is a widely used mathematical expression that relates pavement stiffness to deflection. This represents the railway as an infinitely long beam (rail) on top of a uniform, linear, and elastic foundation. Figure 1 indicates the schematization of a load P referring to a wheelset under a floor that can be defined by several springs of stiffness k under the beam. The contact pressure between the rail base and the track foundation increases linearly with vertical deflection. Thus the differential equation of the problem can be defined as:

$$E.I\,\frac{d^4y(x)}{dx^4} + k.y(x) = p(x) \tag{1}$$

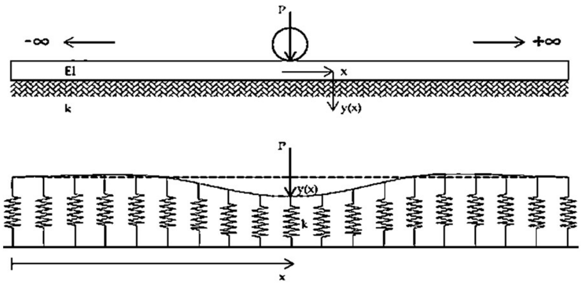


Figure 1. Physical Representation of a Beam on Elastic Foundation

By solving this differential problem, the displacement and other forces acting on the railway track are obtained, such as:

$$y(x) = -\frac{P.\lambda}{2.k}.e^{-\lambda.x}.[\cos(\lambda.x) + \sin(\lambda.x)] \tag{2}$$

$$M(x) = \frac{P}{4.\lambda}.e^{-\lambda.x}.[\cos(\lambda.x) - \sin(\lambda.x)] \tag{3}$$

$$Q(x) = \frac{P}{2}.e^{-\lambda.x}.\cos(\lambda.x) \tag{4}$$

With λ being the inverse of the elastic length of the rail, defined as:

$$\lambda = \sqrt[4]{\frac{k}{4.E.I}} \tag{5}$$

In Figure 2, a graph is presented with the equation of the form of the longitudinal vertical displacements on the track, where its maximum is equal to 1 for the point x = 0. For reasons of symmetry, we can assume the deflection of the track as being equal in both directions. By solving the other derivatives, the following equations can be obtained, which will be related to moment and shear, respectively.

When comparing the shape equations of the efforts that will be analyzed in this research, it is noted that the moment graph is narrower and at the maximum point it approaches a peak, while the displacement is more spaced and angular. This can be explained by the combination of sine and cosines. Since the deflection is given by the sum, and the moment is given by the difference between them. The shear equation is defined by the shape equation where there are only cosines, being intermediate between the two.

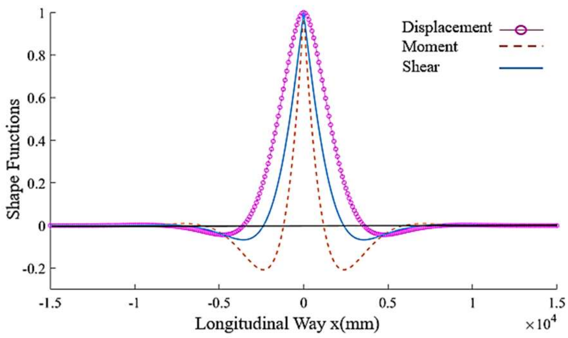


Figure 2. Comparison of Shape Functions

The limitations of the Winkler model are clear, given the widely accepted nonlinearity of the rail structure. According to Gonçalves *et al.* [4] there is no way to make a precise definition of the stiffness value "k" to be used, given that the springs do not represent the ground perfectly. But a value around the modulus of elasticity that could be attributed to the soil seems reasonable, considering the limitations of the model, for the case of a homogeneous, isotropic massif that can be considered elastic. However, according to Lu *et al*. [5] this model is often used because it provides an objective solution to the relationship between load and deflection in the track structure.

2.2 Superposition Effect

Applying this principle to beam models on an elastic basis, the phenomenon can be represented as a wave interference system, where two or more waves are crossing the same space, and the net amplitude of the effect of each load generates a new wave of greater amplitude. Details are described in more depth in Wheeler et al. [6].

When analyzing the railway composition, it is interesting to look for combinations of loads that generate the most critical effect of effort, that is, the greatest quantity of loads with the smallest distance between them. In this way, the Hitch (connection region between wagons that, in general terms, have 4 loads) can be defined, as shown in Figure 6 above. However, there are several works where analyses are developed that have 1 load (wheelset) and 2 loads (trick), such as Spada [7], Klincevicius [8] Miecoanski [9], Filho and Silva [10], with the use of impact coefficients to amplify these effects.

Kerr [11] presented a discrete formulation based on the Winkler model to determine the track module, which had multi-axis compositions of a locomotive, where "n" is the number of axles to be considered and "i" is the distance between axles from the origin considered. Next, the formulation is defined using the Kerr model for displacement, eq. (6 and 7), based on the Winkler model, as will be shown in the subsequent equations.

$$y(x) = -\frac{P_i.\lambda}{2.k}.\sum_{i=1}^{N} e^{-\lambda.x_i}.[\cos(\lambda.x_i) + \sin(\lambda.x_i)] \tag{6}$$

$$M(x) = \frac{P_i}{4.\lambda}.\sum_{i=1}^{N} e^{-\lambda.x_i}.[\cos(\lambda.x_i) - \sin(\lambda.x_i)] \tag{7}$$

Equation (8) of the tensile stress in the lower fiber of the rail runner follows the definition given by the resistance of the materials, which depends on the moment "M", the center of gravity of the section "cg" and the moment of inertia "I".

$$\sigma(x) = \frac{\sum_{i=1}^{N} M(x_i).cg}{I} \tag{8}$$

The stress in the rail runner can also be written as a function of the profile's modulus of resistance, being defined according to eq. (9).

$$\sigma(x) = \frac{\sum_{i=1}^{N} M(x_i)}{W} \tag{9}$$

The shear on the rail continues to follow what was initially defined by Winkler, being the adaptation for superposition, according to eq. (10).

$$Q(x) = \frac{P_i}{2}.\sum_{i=1}^{N} e^{-\lambda.x_i}.\cos(\lambda.x_i) \tag{10}$$

3 Methodology

Based on the theoretical framework listed above, the Matlab program was used to develop the formulations. To calibrate the analytical models carried out, there will be a comparison with the experimental data recorded on the Carajás Railway by Costa [12], as well as the numerical results using finite elements obtained by Ribeiro [13], considering the 1 mm mesh and 600 mm. The case study will refer to an unloaded hopper car with 40 kN per wheelset, and stiffness of approximately 6.3 MPa, in which a displacement of 2.1 mm was recorded. The characteristics of the rail in terms of modulus of elasticity and moment of inertia are $2,1.10^5$ N/mm^2 and 3920 mm^4.

4 Results

In Figure 3 shows the displacements considering the meshes adopted for the most discretized model (1 mm) and the one with equivalent sleeper spacing (600 mm), as well as the analytical one. In general terms, the analytical for both load cases was less conservative than the numerical one, however, it is worth highlighting that the values are in the tenths of millimeters, which in practice does not generate substantial differences.

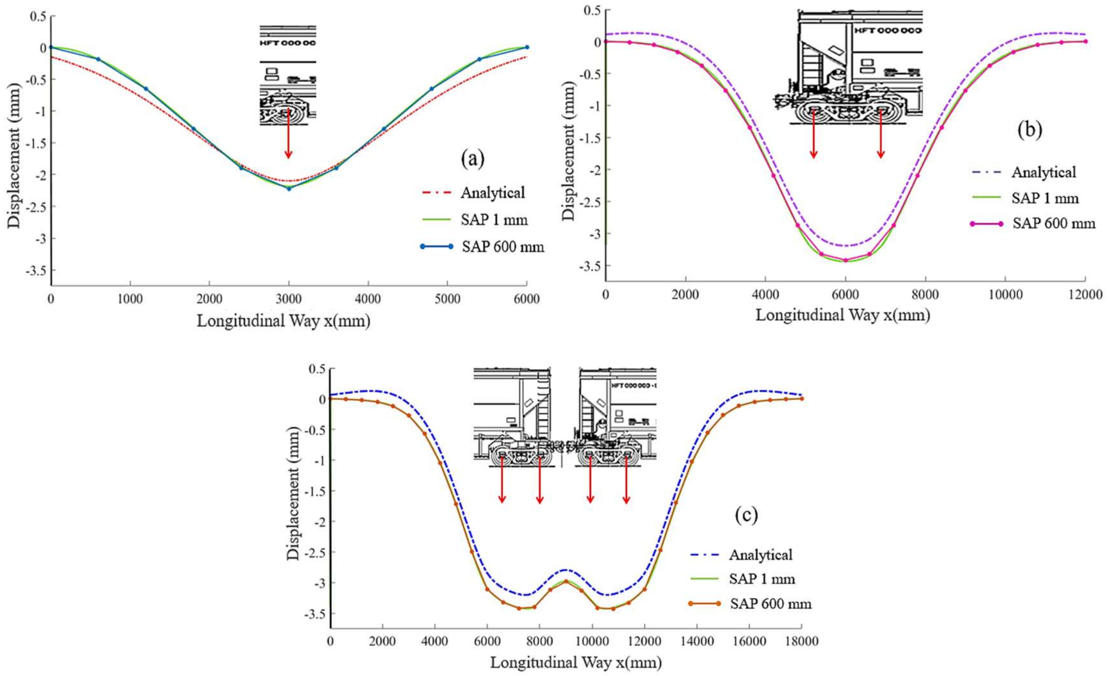
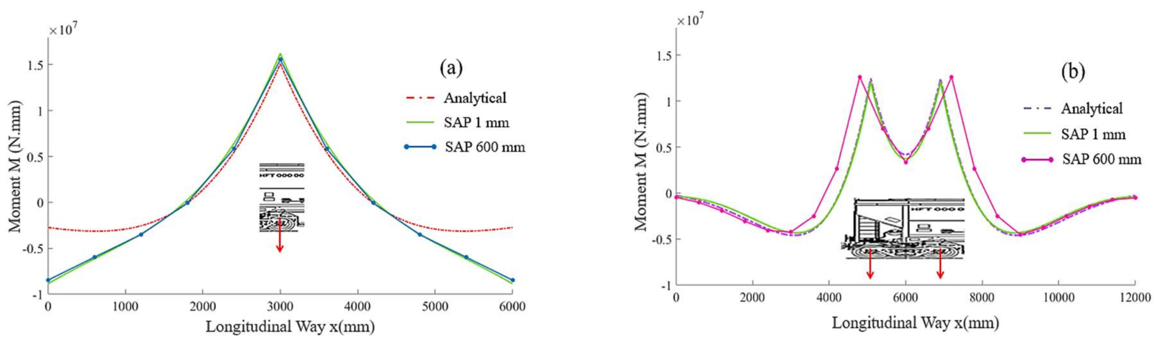


Figure 3. Analytical and Numerical Displacements. In (a) - Wheelset; In (b) - Bogie; In (c) - Hitch.

Observing the comparison of the moments, it was also found that analytical and numerical practically overlap when the 1 mm mesh is considered for the bogie and hitch region, and thus demonstrates that the numerical model is quite representative of the continuum, together with the analytical. In the case of the wheelset, the maximums coincide, but diverge in the tail elements, as shown in Figure 4.

In the case of stress, in both analytical and numerical situations, they follow the bending moment equation, but the results using SAP 2000 are much more conservative than the analytical one, as shown below in Figure 5.

To shear force on the rail, it is observed that the longitudinal shape functions diverge, however the maximums are very close, as seen in Figure 6. This can be justified by the fact that the deduction of the shear equation is different for the specific case of a beam on an elastic basis that follows a cosine function, while SAP 2000 starts from solving problems in classical beam theories. In general terms, although the shape presentation is different in both situations, the maximums converge, which are the main points of analysis in this case, especially when analyzing the model with of 1 mm mesh.



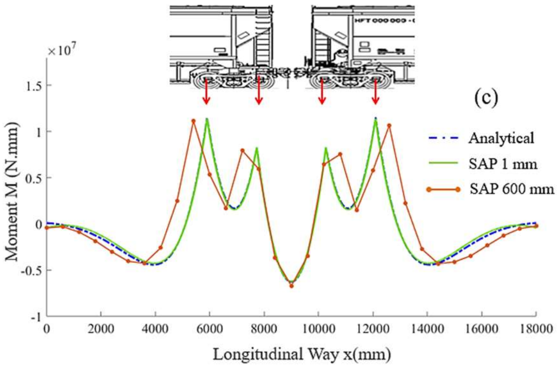


Figure 4. Analytical and Numerical Moments. In (a) - Wheelset; In (b) - Bogie; In (c) - Hitch.

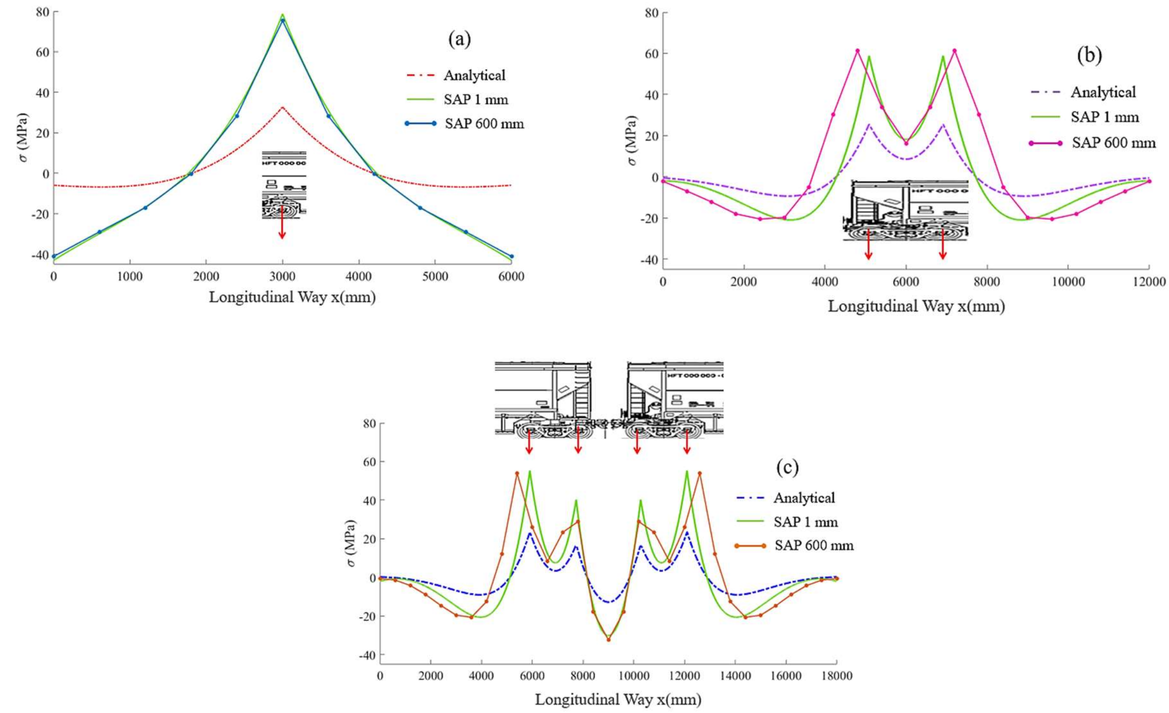
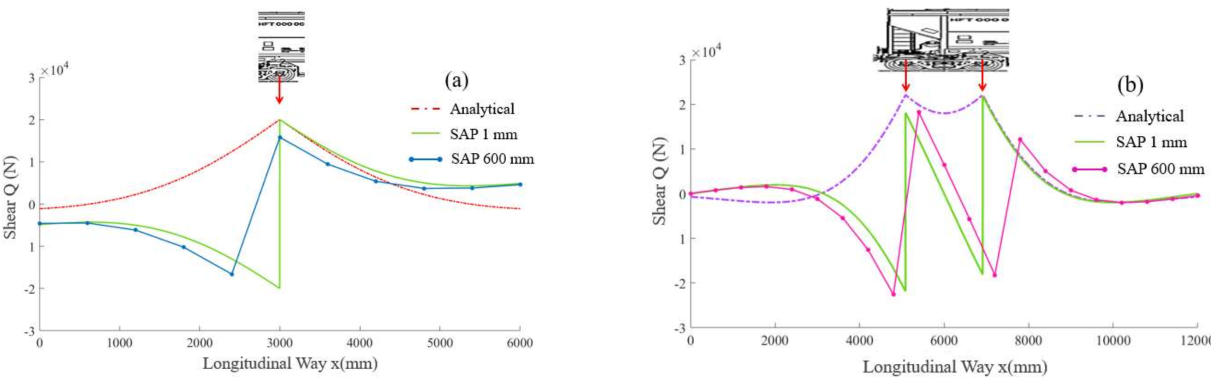


Figure 5. Analytical and Numerical Stress. In (a) - Wheelset; In (b) - Bogie; In (c) – Hitch



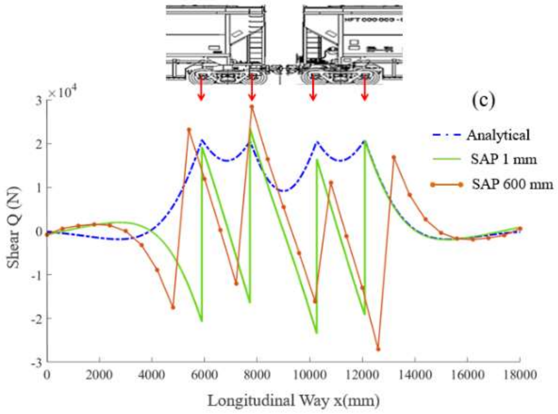


Figure 6. Analytical and Numerical Shear. In (a) - Wheelset; In (b) - Bogie; In (c) - Hitch

For the graphs presented below, the maximum points of each effort and load composition were considered, depending on the numerical meshes developed as well as the analytical one, in a way that represented a "zoom" of these regions, to carry out a more precise analysis, as can be seen in Figure 7.

In relation to displacements (a) in absolute terms, the case of a load was the closest to the experimental one with 2.18 mm (3.66 %). For two and four loads they were more conservative with approximately 3.18 and 3.36 mm for analytical and numerical, respectively, which represents a difference of 33.96 % and 37.5 %. The use of overlapping effects necessarily ends up being naturally more conservative, since the effect of several loads is superimposed. Now analyzing the variation of the numerical mesh, the displacement values were, in short, constant.

Analyzing the shear efforts (d), the values were very close in the analysis. In superposition, there is an amplification of these effects of 4.76 % and 9.09 % for the bogie and the coupler, on the wheelset. In numerical terms, it is 9.09 % and 15.96 %, respectively, considering 1 mm. The values are kept approximate in this analysis up to the 100 mm mesh, after which they disperse. Therefore, it is clear that to obtain greater precision in cutting, greater discretization of the mesh is necessary, for greater stability of the model.

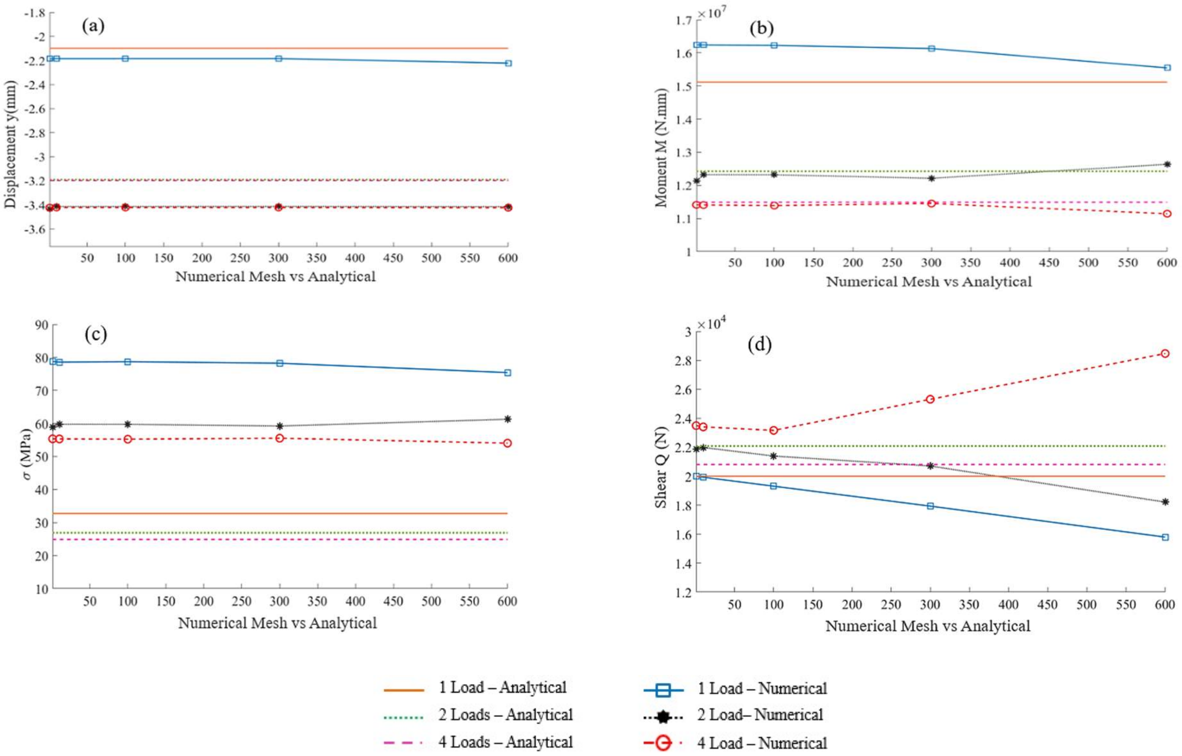


Figure 7. Comparison of maximum analytical and numerical Displacements (a), Moments (b), Stress (c) and Shears (d), depending on the number of loads and meshes.

In the maximum moment graph (b), the numerical value remained constant up to the 300 mm mesh, with a slight slope in the result for the 600 mm mesh. This variation is due to the number of nodes used being smaller compared to the other meshes, generating inaccuracies, but in any case, they are around the exact solution. The graph of maximum tensions (c) also remained constant up to the 300 mm mesh, and slight variation as in the 600 mm mesh of moments. However, as previously noted, the results were much more conservative than the analytical results. One of the hypotheses for this to have happened is because the convergence to voltages requires other types of elements other than the frame, such as shell or solid elements, for example. Now analyzing the number of loads, contrary to what happens in displacement, that the superposition increases the results, for the moment, and consequently also for stress, there is a decrease in these effects in 2 and 4 loads, compared to the effect of one wheelset.

5 Conclusions

Through the results, it can be concluded that the analytical models converged well, with small differences from the numerical ones, and very close to the experimental displacements. The results where superposition occurred were more conservative, as the effects of multiple loads were combined. When comparing analytical and numerical to analyze the three load cases, with or without superposition, the displacements remained practically constant for the mesh variations, as well as for the moment. In stress, the numerical value was much more conservative, due to the element used not being the most suitable for obtaining this type of effort. In cutting, the maxima were close, around the exact solution, but the form equations assumed by both methods are different. Therefore, it was realized that the analytical model also has its advantages, being quite representative of the global longitudinal structural behavior of railways, combined with its computational cost and practicality.

Acknowledgements. The authors acknowledge the financial support from CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico) for promoting research.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] E. Winkler, “Die Lehre von Elastizitat und Festigkeit (on Elasticity and Fixity)”, Czech Republic, 1867.
[2] S. Timoshenko. Strength of Materials II. New York, 1947.
[3] F. R. Zaragueta. Análisis de una Vía con Traviesas Cuadro. M.Sc. dissertation. Polytechnical University of Catalunya, 2010.
[4] H. Gonçalves and others. Estruturas Subterrâneas. Geothecnical Departament. University of São Paulo, 2018.
[5] S. Lu and others. “On the relationship between load and deflection in railroad track structure”, NASPA Journal, v. 42, n. 4, 2008.
[6] L. N. Wheeler, W. A. Take, N. A. Hoult. “Performance assessment of peat rail subgrade before and after mass stabilization”, Canadian Geotechnical Journal, v. 54, n. 5, p. 674–689, 2017.
[7] J. L. G. Spada. Uma Abordagem de Mecânica dos Pavimentos Aplicada ao Entendimento do Mecanismo de Comportamento Tensão - Deformação da Via Férrea. Phd. Thesis. Federal University of Rio de Janeiro, 2003.
[8] M. G. Y. Klincevicius. Estudo de propriedades, de tensões e do comportamento mecânico de lastros ferroviários. M.Sc. dissertation. University of São Paulo, 2011.
[9] J. F. Miecoanski. Avaliação do Comportamento Mecânico da Via Férrea Utilizando Elementos Finitos considerando Fadiga: Estudo Paramétrico. M.Sc. dissertation. Federal University of Paraná, 2019.
[10] M. R. S. Filho and S. F. Silva, “Análise de tensões e deformações no trilho: influência do módulo de via no processo”. Revista Eixo, n. 2, p. 25-36, 2021.
[11] A. D. Kerr, “A Method for Determining the Track Modulus Using a Locomotive or Car on Multi-axle Trucks”, Proc. AREA, v. 84, 1983.
[12] R. C. Costa. Proposição de dispositivo de medidas “in situ” para avaliação do comportamento mecânico de lastro ferroviário: estudo de caso na Estrada de Ferro Carajás. M.Sc. dissertation. University of São Paulo, 2016.
[13] M. R. B. M. Ribeiro. Análise Numérica - Analítica do Comportamento Estrutural de Pavimentos Ferroviários. M.Sc. dissertation. Federal University of Rio de Janeiro, 2023.