Coupling of discrete and smeared cracking models applied to reinforced concrete structures

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Abstract. The nonlinearity inherent to concrete comes from the cracking process and the inelastic phenomena in the fracture process zone (FPZ). It is understood that initially, microcracks appear diffusely in the material medium, even under low-intensity loading conditions. In these initial stages, cracking develops stably, causing the structure's nonlinear response, which can still absorb loads. With the advancement of the process, the crack coalescence causes the significant growth of cracks so that a discontinuity in the material medium is observed. At this stage, propagation occurs in an unstable manner, and the structure loses the ability to absorb loads. In reinforced concrete, the manifestation of cracks occurs through multiple fronts due to the reinforcement steel that absorbs tensile forces and locally contains the crack propagation, which initially appears in regions not covered by the reinforcement steel. In more advanced stages, cracks can be observed that surpass the reinforcement steel region and propagate throughout the structure until its collapse. Over the years, two main approaches have stood out in the representation of concrete fracturing: smeared and discrete crack models. Each model aims to represent the concrete response based on different hypotheses. In the smeared models, the medium is considered continuous, and the effects of cracking are represented by the elastic degradation of the material computed through constitutive models. In discrete models, the crack is represented by a material discontinuity inserted into the mesh. This work presents a strategy that combines smeared and discrete models applied to the modeling of flat reinforced concrete structures. The reinforcement steel will be represented by one-dimensional elements with elastoplastic behavior while maintaining the perfect bond between the steel rebar and concrete. In the representation of concrete fracturing, the FPZ will be represented by a scalar damage model for the continuous degradation of the material, and from a critical damage value, a discrete model, which considers multiple crack fronts, is activated by inserting a discontinuity into the mesh using a nodal duplication strategy, thus describing the final rupture process. Finally, numerical simulations will be presented to illustrate the model's performance.

Keywords: Reinforced concrete, Discrete crack model, Smeared crack model

1 Introduction

The mechanical behavior of concrete is highly complex to describe due to the heterogeneity of this material and its non-linear behavior caused by its bi-modularity. Furthermore, before being subjected to loading, concrete shows internal microcracks due to shrinkage. After loading, these microcracks propagate along the matrix-aggregate interfaces or through the cement matrix, developing cracks that amplify the material's non-linear behavior, resulting in a decrease in the structural integrity of the concrete.

For reinforced concrete, the description of mechanical behavior becomes even more complex due to bond mechanisms between the concrete and the reinforcement steel, which transmits the tensile stresses from the concrete to the steel bar. As a result, once the cracking process has started, there is a non-linear response from the concrete, and the reinforcement is responsible for absorbing the tensile stresses. In addition, with the presence of the steel bar, by distributing the shrinkage strains along the reinforcement through bond stresses, the cracks are distributed so that a more significant number of narrow cracks occur instead of a few wide cracks [1].

Over the years, several studies have been carried out to develop numerical models capable of simulating the cracking process in concrete, such as the discrete cracking model and the smeared cracking model. In the discrete crack model, the crack is treated as a geometric entity. Thus, within the framework of the Finite Element Method, cracks are represented from displacement discontinuities between elements. In this way, unless the crack propagation path is known in advance, cracks can be modeled by separating the edges of the elements and redefining the finite element mesh to accommodate crack propagation [2]. In smeared cracking models, the medium is considered continuous throughout the analysis process. In these models, cracks are represented by the constitutive description quantified as a function of strain measurements representative of the medium and constitutive laws (stress-strain relationship) appropriate for the material, such as models based on Continuous Damage Mechanics, which describe the process of deterioration of the integrity of the medium.

Discrete and smeared cracking models cannot fully represent the concrete cracking process, which includes the formation of microcracks, coalescence, nucleation, and propagation of cracks. Therefore, this paper presents a strategy for describing the fracturing process in concrete structures to analyze the behavior of reinforced concrete structures. The model was developed using the Finite Element Method, considering plane analysis with discrete reinforcement modeling and a continuous-discontinuous approach for representing the concrete. The damage model proposed by Mazars [3] was used to compose the continuous-discontinuous model and the crack description was based on the nodal duplication strategy.

Finally, this article presents the initial results obtained from modeling two examples of reinforced concrete elements subjected to bending to validate the proposed model and its computational implementation.

2 Modeling strategy

2.1 Damage model for concrete

Damage models describe the gradual reduction in material integrity due to the propagation of microcracks, microvoids and similar defects in the medium, resulting in a reduction in the element's stiffness and, subsequently, failure [4]. Thus, after the linear regime:

$$D = 1 - \frac{E^S}{E^0}, \tag{1}$$

where D is the damage variable, which varies from zero (intact material) to 1.0 (maximum degradation), E^0 is the material elastic modulus and E^S is the secant modulus of the degrading. Over the years, several damage models have been developed, such as the damage model based on equivalent strain proposed by Mazars [3], in which damage is measured using a scalar variable calculated from the positive principal strains. Because it can simulate the different behavior of the material in tension and compression, this model is appropriate for modeling reinforced concrete.

In the Mazars [3] model, the equivalent strain $\tilde{\varepsilon}$ is given by:

$$\tilde{\varepsilon} = \sqrt{\sum_{i=1}^{3} (\langle \varepsilon_i \rangle_+)^2}, \tag{2}$$

with $\langle x \rangle_+ = \frac{|x|+x}{2}$, where $\varepsilon_{(i)}$ are the principal strains.

The degradation of the medium is obtained combining two damage variables, D_t and D_c, for tensile and compressive damage, respectively. In this way:

$$D = \alpha_t D_t + \alpha_c D_c, \tag{3}$$

where α_t and α_c are weight functions used to measure the material's behavior to tensile and compressive damage, respectively. The value of tensile damage, D_t, and compressive damage, D_c, are calculated by:

$$D_i(\tilde{\varepsilon}) = 1 - \frac{(1 - A_i)\kappa_0}{\tilde{\varepsilon}} - \frac{(1 - A_i)}{exp[B_i(\tilde{\varepsilon} - \kappa_0)]}, \tag{4}$$

where $i = t, c$ indicates tensile and compressive damage, respectively. The parameters A_i and B_i are obtained from compression and tension or three-point bending experimental tests. The weight functions α_t and α_c are given by:

$$\alpha^t = \sum_{i=1}^{3} H_i \frac{\varepsilon^t_{(i)}[\varepsilon^t_{(i)} + \varepsilon^c_{(i)}]}{\tilde{\varepsilon}^2} \quad \text{and} \quad \alpha^c = \sum_{i=1}^{3} H_i \frac{\varepsilon^c_{(i)}[\varepsilon^t_{(i)} + \varepsilon^c_{(i)}]}{\tilde{\varepsilon}^2}, \tag{5}$$

with $H_i = 1$ if $\varepsilon_{(i)} = \varepsilon^c_{(i)} + \varepsilon^t_{(i)} \geq 0$, otherwise $H_i = 0$. The values ε^t and ε^c, obtained from the positive and negative parts of the principal stress tensor σ^+ and $\sigma^-(\sigma = \sigma^+ + \sigma^-)$, respectively, are:

$$\varepsilon^c_{kl} = E^{-1}_{ijkl}\sigma^-_{kl} \quad \text{and} \quad \varepsilon^t_{kl} = E^{-1}_{ijkl}\sigma^+_{kl}. \tag{6}$$

From the damage parameter and assuming an isotropic degradation, the stress-strain relation is given by:

$$\sigma_{ij} = (1 - D)E^0_{ijkl}\varepsilon_{kl}. \tag{7}$$

2.2 Discrete crack model's for concrete

In the literature, the work by Ngo and Scordelis [5] was one of the first to approach the modeling of discrete cracks by finite elements. In this work, a discrete model was proposed for a reinforced concrete beam in which cracks were introduced into the mesh by separating elements using the nodal duplication process along the crack propagation path. Over the years, other authors have also used discrete crack models, such as the work by Tudjono et al. [6], which, similarly to Ngo and Scordelis [5], approaches the discrete crack model for reinforced concrete elements by finite elements, using the nodal duplication strategy to represent the crack propagation process and adopting springs to simulate the interaction between the concrete and the steel bars.

In smeared cracking models, the crack is not explicitly inserted into the mesh. Instead, degrading zones are used to represent the behavior of the cracked element. Rashid [7] pioneered the smeared cracking model, which became popular because it required fewer computational resources since it was not necessary to implement mesh redefinition strategies, unlike discrete cracking models. However, in this approach, the results present highly meshing dependency problems.

According to Galvez et al. [8], the propagation of a crack in quasi-brittle materials can be subdivided into three zones, as shown in Fig. 1: (1) the intact material, where the material has not reached the strength limit, (2) the fracture process zone, where the material has been loaded up to the strength and is partially broken, but with unbroken material bridges, and is able to transmit stresses through the interface, and (3) the *true crack*, with no stress transmission through the interface.

So, in order to describe the complete concrete cracking process, discrete and smeared models can be coupled. These approaches are known as continuous-discontinuous models, in which the smeared crack model is responsible for the continuous representation of the cracking process and the discrete model for inserting the discontinuity in the mesh that causes the *true crack* to appear. For the transition between these two approaches, the continuous-discontinuous models must include a criterion for crack nucleation and propagation. Initial approaches to the relationship between damage and fracture mechanics are presented in Legendre and Mazars [9] based on thermodynamics and equivalent cracking concepts. In more recent work, a continuous-discontinuous approach using volumetric damage to represent the initial stages of cracking and the nodal duplication technique to initialize the discrete process is proposed by Pereira and Penna [10] to evaluate of concrete structures.

In this work, a critical damage value must be pre-established to announce the crack initiation of the discrete cracking process in which the crack will occur using the nodal duplication strategy. When the discrete cracking process reaches the reinforcement region, the nodes of the concrete element and the steel bar are duplicates. Thus, a new element is added to the model between crack openings adjacent to the original steel bar with the same properties. This strategy is presented by Tudjono et al. [6], as illustrated in Fig. 2.

2.3 Reinforcement modeling

The longitudinal steel bars used in reinforced concrete elements mainly resist axial stresses and show similar behavior when subjected to compressive and tensile stresses. The stress-strain curve of steel is obtained from

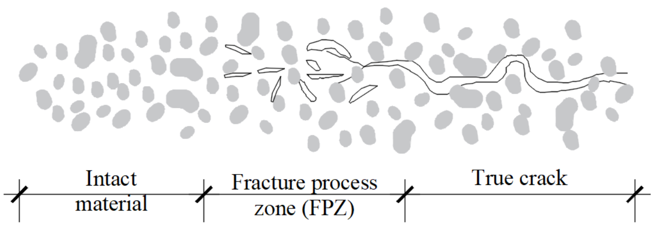

Figure 1. Cracking process in concrete (Adepted from Galvez et al. [8]).

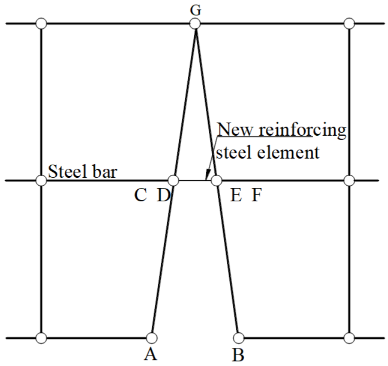

Figure 2. Crack propagation by nodal duplication (Adepted from Tudjono et al. [6]).

experimental tests. This work adopted an elastic, perfectly plastic behavior for the steel, considering the perfect bond between the concrete and the rebars.

The reinforcement was modeled by the discrete approach in which the steel rebars are represented by one-dimensional elements connected to the nodes of the concrete elements in the mesh. The advantage of this model is that it is easy to model and implement computationally. However, this model conditions the mesh to the geometry of the reinforcement, making it challenging to make mesh adjustments and requiring a more significant number of finite elements.

3 Numerical simulation

This section presents some numerical simulations to illustrate the cracking process in reinforced concrete elements using continuous-discontinuous models.

3.1 Reinforced Concrete Beam – Álvares (1993)

In this example, was modeled a beam subjected to bending at four points, as proposed by Alvares [11]. The details of the beams and the finite element mesh are shown in Fig. 3 and Fig. 4.

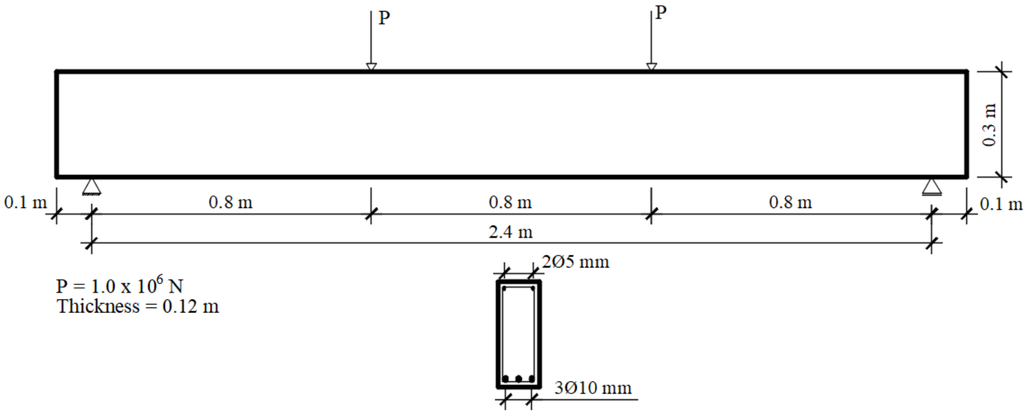

Figure 3. Geometric details and beam reinforcement.

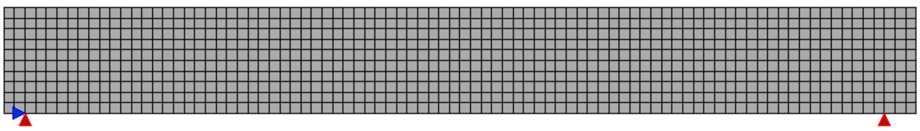

Figure 4. Finite element mesh.

In the computational modeling, four-node quadrilateral elements were used under plane stress conditions and reinforcement with one-dimensional two-node elements. Alvares [11] considered concrete with a modulus of elasticity of 29200 MPa and Poisson's ratio of 0.2 and steel with a modulus of elasticity of 196000 MPa and yield strength of 420 MPa. The Mazars [3] model was used with the following parameters: At = 0.995, Bt = 8000, Ac = 0.85, Bc = 1620 and $\kappa_0 = 0.00007$. A plasticity model was adopted for steel reinforcement based on a unidimensional elastoplastic constitutive law.

For non-linear analysis, the direct displacement control method was used, with an increment of 1x10^{-3} m, a tolerance for convergence of 1x10^{-4} and a reference load of P = 1x10^{6} N.

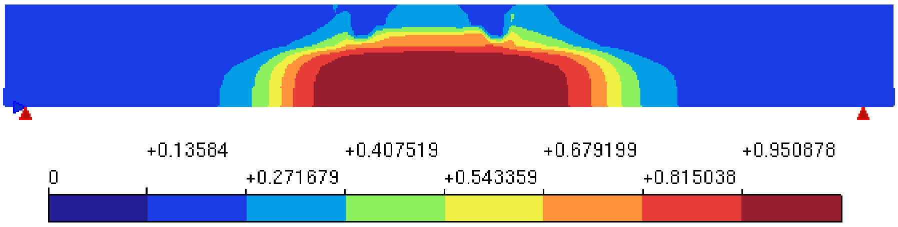

Figure 5. Damage propagation.

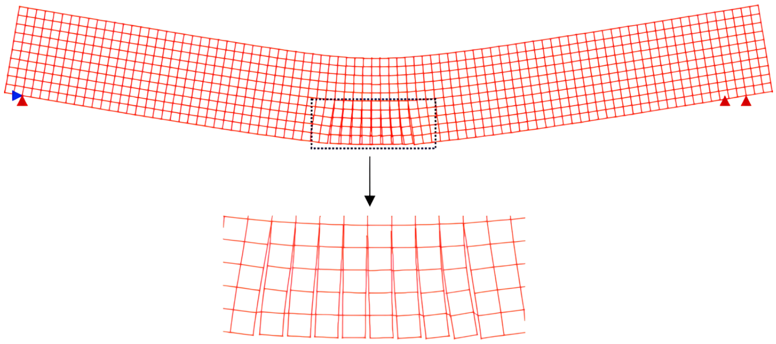

Figure 6. Multiple crack fronts.

As a result, it can be seen that in the initial stages of the non-linear analysis, where the damage has not reached the limit value of 0.95, there is no insertion of geometric discontinuity in the mesh. Therefore, in these stages, only smeared cracking is computed by the damage model. When the critical damage value is reached, the formation of multiple discrete crack fronts is activated by nodal duplication in the region containing only concrete, and it progresses to the region covered by the steel bar. When the steel bar is duplicated, a new element is added to the model between crack openings. When higher damage values are reached, the crack propagates through the reinforcement, as observed in the more advanced stages of the non-linear analysis. Figure 5 illustrates the damage propagation, and the Fig. 6 illustrates the respective cracking pattern for the last step of the analysis.

3.2 Reinforced Concrete Corbel

In this example, a reinforced concrete corbel subjected to a concentrated load at the end and wich geometry is adapted from Araujo et al. [12] was modeled. The model was discretized with triangular elements with three nodes in plane stress condition and reinforcement with one-dimensional elements with two nodes. The corbel details and the finite element mesh are shown in Fig. 7. The materials used were concrete with a modulus of elasticity of 28000 MPa and a Poisson's ratio of 0.2 and steel with a modulus of elasticity of 20000 MPa and a yield strength of 500 MPa. The Mazars [3] model was used with the following parameters: At = 0.995, Bt = 8000, Ac = 0.85, Bc = 1620 and $\kappa_0 = 0.00007$. An unidimensional elastoplastic constitutive law was considered to describe the steel behavior. For non-linear analysis, the direct displacement control method was used, with an increment of 5x10^{-5} m, a tolerance for convergence of 1x10^{-4} and a reference load of P = 1x10^{6} N.

This simulation shows that the damage initially propagates in the support region of the column. After a few stages of the non-linear analysis process, the damage manifests in the corbel region. Therefore, only the nodes at this location were monitored to investigate the cracking process in the corbel. The discrete cracking process begins with nodal duplication in the corbel region after the damage reaches the pre-established critical value of 0.95. After this stage, multiple discrete crack fronts form and propagate through the reinforcement. Figure 8 illustrates the propagation of damage in the corbel and the multiple cracks fronts at the last step of the analysis.

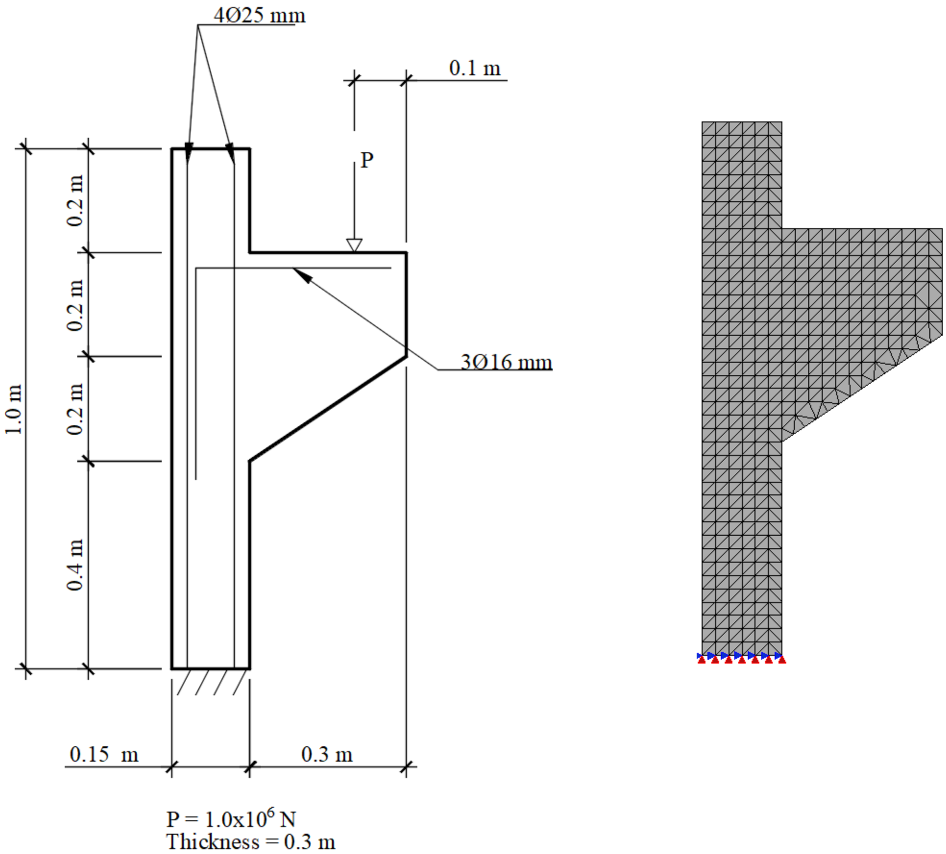

Figure 7. Geometric details and corbel reinforcement and finite element mesh.

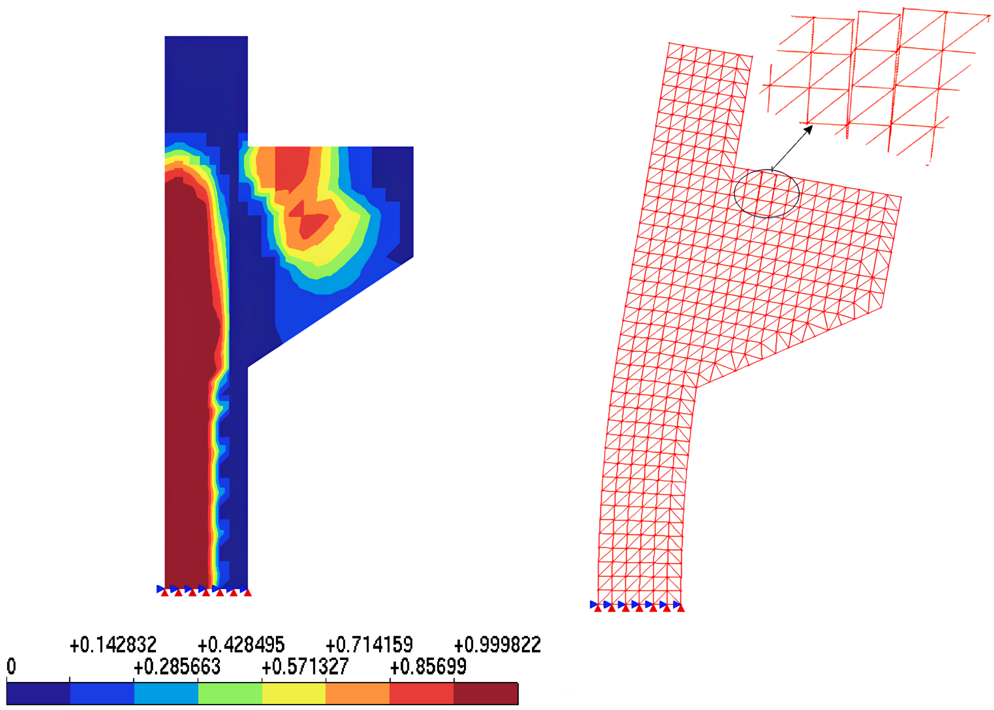

Figure 8. Damage propagation and multiples cracks fronts.

4 Final Remarks

The presented strategy adequately represented the concrete cracking process in the reinforced concrete finite element models. Based on this proposal, a damage model was used to conduct the continuum material's degradation process until a threshold was reached. The formation of multiple crack fronts was observed after the critical damage reached, which is a typical behavior of reinforced concrete structures. The nodal duplication technique proved suitable for application in the discrete model, as observed in the literature.

Future stages of this implementation consist of evaluating the interaction between concrete and reinforcement to obtain the complete non-linear structural response.

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