Formulation of a nonlocal bimodular damage model based on mechanical properties parametrization

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Abstract. Material media representation by damage models presents difficulties in the parametrization process. The damage laws are generally written as functions of variables without physical meaning. This lack of connection between the evolution of degradation and the properties of materials requires a diversity of tests to numerically reproduce results obtained experimentally, which makes such a process slow, costly, and subjective. Based on this context, a constitutive model capable of describing the material response with damage evolution laws defined in terms of material parameters obtained from experimental tests is proposed in this paper to overcome parametrization adversities. Such a model is focused on reproducing the bimodular behavior of quasi-brittle materials, such as concrete, thar respond differently to tension and compression efforts. While these materials have a significant resistance under compression, they manifest cracks when subjected to tension, collapsing due to fracturing. The current model presents a nonlocal character as a regularization technique to avoid strain localization phenomena. Finally, numerical simulations are conducted to verify the constitutive model performance and to validate the possibility of parameterizing computation analyses using exclusively physical parameters.

Keywords: Nonlocal bimodular damage model, Constitutive laws, Physically nonlinear analysis, Concrete structures.

1 Introduction

Concrete is one of the most widespread materials applied in the construction industry. Models capable of describing this material behavior are essential to developing safe and economically viable projects. A particular property of concrete is the bi-modularity, related to the different responses under tension and compression loadings.

In the literatura, many different models have been proposed to reproduce concrete behavior. The Continuum Damage Mechanics (CDM) enables the formulation of constitutive models based on phenomenological aspects of material deterioration when subject to loading. For instance, the CDM defines a damage variable to compute the material elastic modulus degradation, which represents the smeared cracking process that occurs in quasi-brittle material such as concrete.

The isotropic damage models [1–7] are formulated considering a scalar damage variable responsible for computing a general degradation to all material directions. A disadvantage of these models is the absence of representation of concrete anisotropic deterioration, which can be overcome with tensorial damage variables and more complex constitutive models [1, 8, 9].

To embrace simplicity and represent the asymmetric concrete behavior in compression and tension, isotropic models can be applied with a single damage variable calculating degradation by tension [10–12]. An alternative is to admit two damage variables, one related to tension and the other to compression deterioration, such as the model of Mazars [13].

Considering this context, recent damage models have been proposed to enhance the performance of classical isotropic models in reproducing concrete behavior. Ahmed et al. [14] proposed a local and non-local model based on stress decomposition into shear stress and uniaxial tension/compression stress. Caetano and Penna [15] extended the models of Mazars [13], Lemaitre and Chaboche [11], and de Vree et al. [12] to better represent the concrete responses in tension and compression. A limitation of these models is the difficulty correlating the damage evolution laws with mechanical material properties. These laws generally require constants with no physical meaning, an obstacle to material parametrization.

Based on the above, this study proposes a new isotropic bi-modular damage model. The particularity of this model is the adoption of principal strains to quantify degradation, which allows the calculus of the damage variable from stress-strain relations. The damage laws are now defined based on the material constitutive laws that have the material mechanical properties as variables, simplifying the parametrization process. Thus, this model enables numerical analyses using only data extracted directly from experimental tests.

2 Bimodular Principal Strains (BPS) model

The BPS model considers an isotropic elastic degradation, and the stress-strain relation is given by:

$$\sigma_{ij} = (1-D)E^0_{ijkl}\varepsilon_{kl}. \tag{1}$$

The damage is established based on an equivalent strain defined from the principal strains. The equivalent strain assumes the positive or negative principal strain according to the dominant state of tension or compression, respectively. The higher value is adopted when all the strain components are positive or negative.

Additionally, the dominant state of tension or compression is calculated from the first invariant (I_1) of the strain tensor:

$I_1 \geq 0 \therefore$ tension dominant state;

$I_1 < 0 \therefore$ compression dominant state.

From these definitions, the equivalent strain is obtained.

- If $I_1 \geq 0$ (tension dominant state):
  - The strain tensor is $\underset{\sim}{\varepsilon} = \langle \underset{\sim}{\varepsilon}_p \rangle_+$;
    where only positive principal strain components are considered. In case of negative components, they are replaced by zero.
  - While the equivalent strain is $\widetilde{\varepsilon}_+ = max(\widetilde{\varepsilon}_1, \widetilde{\varepsilon}_2, \widetilde{\varepsilon}_3)$.
- If $I_1 < 0$ (compression dominant state):
  - The strain tensor is $\underset{\sim}{\varepsilon} = \langle \underset{\sim}{\varepsilon}_n \rangle_-$;
    where only negative principal strain components are considered. In case of positive components, they are replaced by zero.
  - While the equivalent strain is $\widetilde{\varepsilon}_- = min(\widetilde{\varepsilon}_1, \widetilde{\varepsilon}_2, \widetilde{\varepsilon}_3)$.

Since the equivalent strain is a principal strain component, damage evolution laws directly associated with stress-strain relations can be admitted. Such relations are a function of material parameters obtained from experimental tests. Examples of stress-strain relations to concrete are the laws proposed by Carreira and Chu [16, 17] to compression and tension (Eq. 2) and by Boone and Ingraffea [18] to tension (Eq. 3).

$$\sigma_i = f_i \frac{k\left(\frac{\varepsilon}{\varepsilon_i}\right)}{k-1+\left(\frac{\varepsilon}{\varepsilon_i}\right)^k}, \quad \text{where} \quad k = \frac{1}{1-\frac{f_i}{\varepsilon_i \cdot E_0}} \quad \text{with} \quad i = t, c. \tag{2}$$

Where σ_i is the compression or tension stress, f_i is the compression or tensile strength limit, ε is the current strain, ε_i is the strain related to the elastic limit, h is the characteristic length, E_0 is the elastic modulus, and $i = c$ for compression and $i = t$ for tension.

$$\sigma = f_t e^{-k(\varepsilon - \varepsilon_t)}, \quad \text{with} \quad k = \frac{h f_t}{G_f}. \tag{3}$$

Where σ is the current tension, f_t is the tensile strength, ε is the current strain, ε_t is the strain related to the elastic limit, h is the characteristic length, and G_f is the fracture energy. This expression (Eq. 3) is restricted to the post-peak branch. Before the strength peak, the elastic relation $\sigma = E_0\varepsilon$ governs the stress-strain behavior.

Besides, the loading function of the proposed model is written as

$$f(\underset{\sim}{\varepsilon}, \varepsilon_c, \varepsilon_t) = \widetilde{\varepsilon} + \kappa(\widetilde{\varepsilon}), \tag{4}$$

where $\kappa(\widetilde{\varepsilon})$ is the historical variable, i.e., the maximum value of $\widetilde{\varepsilon}$ during the numerical analysis. This variable is initially set as the strain value related to the elastic limit.

The equivalent strain $\widetilde{\varepsilon}$ can assume positive or negative values. If $\widetilde{\varepsilon}$ is positive, κ must be obtained according to tension, while the historical parameter is obtained from compression for a negative value of $\widetilde{\varepsilon}$.

In the case of a loading inversion, when the equivalent strain is subject to a signal change, the loading conditions require adjustments to represent the regime properly.

The loading regime must attend Karush-Kuhn-Tucker conditions

$$f \leq 0 \quad ; \quad \dot{\kappa} \geq 0. \tag{5}$$

and the complementary consistency conditions, given as:

$$f\dot{\kappa} \leq 0 \quad ; \quad \dot{f}\dot{\kappa} \geq 0. \tag{6}$$

Once the loading regime is known (loading, unloading, or reloading), the damage variable is computed as a function of $\widetilde{\varepsilon}$, and the material degradation stiffness is calculated by the tangent operator. Carol et al. [19] established this operator as

$$E^t_{ijkl} = E^s_{ijkl} + \frac{1}{H} m_{ij} n_{kl}, \tag{7}$$

where:

E^s_{ijkl} are the components of the secant constitutive tensor, given by $E^s_{ijkl} = (1 - D)E^0_{ijkl}$;

E^0_{ijkl} are the components of the elastic constitutive tensor;

D is the scalar damage variable;

n_{kl} are the components of the tensor with the loading function derivation in relation to strains: $n_{kl} = \frac{\partial f}{\partial \varepsilon_{kl}}$;

m_{ij} are the components of the degradation direction tensor, obtained from de generalized degradation rule $m_{ij} = M_{ijkl}\varepsilon_{kl}$, where:

$M_{ijkl} = \frac{\partial E^s_{ijkl}}{\partial D}\mathscr{M}$, with $\frac{\partial E^s_{ijkl}}{\partial D} = -E^0_{ijkl}$ and $\mathscr{M} = 1$ for isotropic models.

Then, considering stress-strain laws written in function of material physical parameters as [16, 17] and [18] and the correlation between the secant elastic modulus and the damage variable $\left(D = 1 - \frac{E_s}{E_0}\right)$, it is possible to associate the stress-strain response with the damage evolution, as shown in Fig. 1.

2.1 Nonlocal approach

The current model is extended to a nonlocal approach to avoid strain localization phenomena. This regularization technique consists of a definition of a nonlocal variable [2]. For the proposed model, the historical value of the equivalent strains is defined as the nonlocal variable ($\tilde{\varepsilon}_{nl}$), given by:

$$\tilde{\varepsilon}_{nl} = \frac{1}{V_r(\mathbf{x})} \int_V \alpha(\mathbf{s} - \mathbf{x})\tilde{\varepsilon}(\mathbf{s})dV = \int_V \alpha'(\mathbf{x}, \mathbf{s})\tilde{\varepsilon}(\mathbf{s})dV, \tag{8}$$

where $\tilde{\varepsilon}$ is the local equivalent strain; $V_r(\mathbf{x}) = \int_V \alpha(\mathbf{s} - \mathbf{x})dV$ represents the volume of the revolution solid related to the distribution function α; $\mathbf{x}$ is the coordinates vector of the point in analysis; $\mathbf{s}$ is the coordinates vector of the points into the nonlocal domain; and $\alpha'(\mathbf{x}, \mathbf{s}) = \frac{\alpha(\mathbf{s}-\mathbf{x})}{V_r(\mathbf{x})}$.

The weight function (α) can assume different shapes. In the present work, it is adopted the Gaussian function distribution, is written as

$$\alpha(\mathbf{s}, \mathbf{x}) = e^{-(k\|\mathbf{s}-\mathbf{x}\|/r)^2}, \tag{9}$$

where r is the nonlocal radius, defining the size of the nonlocal domain; k is a constant that determines the shape of the function.

3 Numerical simulations

Numerical simulations via the finite element method are present to evaluate the proposed model characteristics. The analyses consist of a three-point bending test, resulting in the equilibrium path of the structure. The numerical results are compared with experimental data available in the literature.

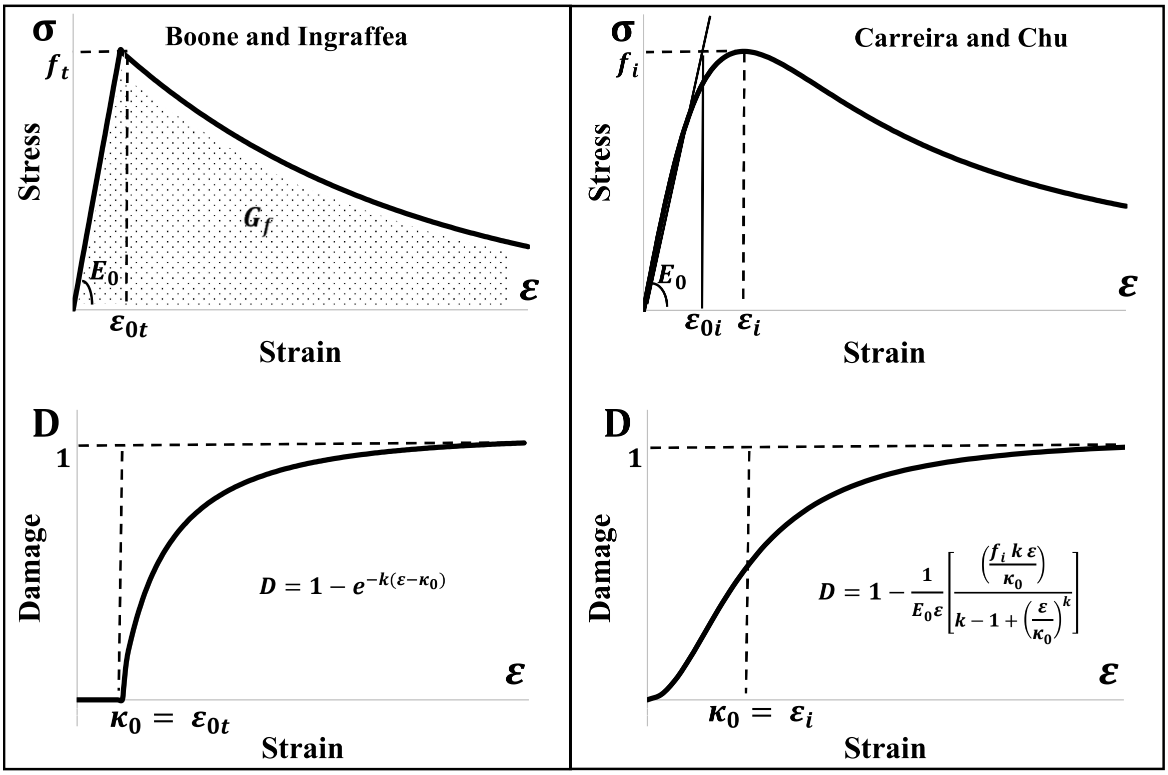

Figure 1. Stress-strain laws and damage evolution.

3.1 Three-point bending test of García-Álvarez et al. [20]

García-Álvarez et al. [20] have performed experimental bending tests of notched concrete beams of different sizes - small beam (SB), medium beam (MB), and large beam (LB). The geometry and boundary conditions of the beams are shown in Fig. 2. The dimension d assumes different values in each size (d = 80, 160, and 320 mm).

The incremental-iterative process was conducted using the direct displacement control method [21], monitoring the vertical direction of the node in the notch center. The reference load was $P = 1000$ N, with an incremental load factor of 1 N. The meshes adopted in each beam size are illustrated in Fig. 3.

The material parameters obtained from [20] are: Young's modulus $E = 33800$ MPa; Poisson ratio $\nu = 0.2$; fracture energy $G_f = 0.08$ N/mm; tensile strength $f_t = 3.5$ MPa. The compression strength was estimated according to the relation $f_{ctm} = 0.3 f_{ck}^{2/3}$, with $f_{ctm} = f_t$. The strain limits were admitted as $\varepsilon_c = 0.002$ for compression and $\varepsilon_t = 2.0 \times 10^{-4}$ for tension. Since the characteristic length of concrete is between 2.7 to 3.0 times the maximum aggregate size [22], which in the experimental tests was 12 mm, the characteristic length (h) adopted is 36 mm.

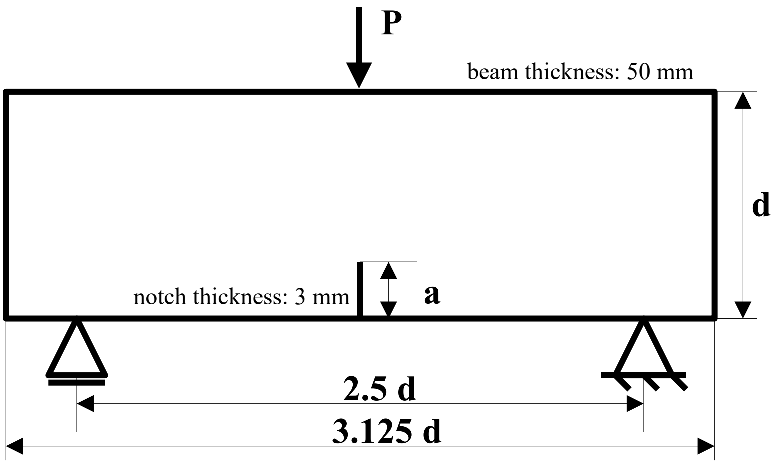

Figure 2. Three-point bending specimen.

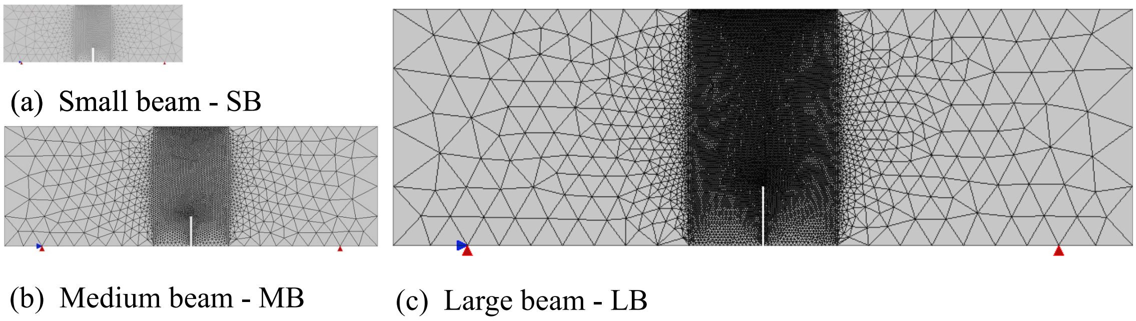

Figure 3. Finite element meshes adopted for each beam.

The simulations were performed using Carreira and Chu [16, 17] to compression and tension (CC) or Carreira and Chu [16, 17] to compression and Boone and Ingraffea [18] to tension (CB).

The nonlocal approach was established from a Gaussian weight function with a nonlocal radius of $r = 36$ mm and the constant $k = 6.0$. This constant regulates the spread of the distribution function.

The results are illustrated in Fig. 4. Comparing the experimental spectrum with the numerical curves, a good representation of the experiments is verified for all beams. The load peak was achieved with more precision by the CB laws, while the CC law better approximated the softening branch. In bending tests, the tension state is the main responsible for the degradation process, so the pronounced softening presented by the CB laws is associated with the exponential behavior of Boone and Ingraffea [18] law, as indicated in Eq. 3. On the other hand, the polynomial law by Carreira and Chu [16, 17] showed a better performance in reproducing the smooth softening under tension.

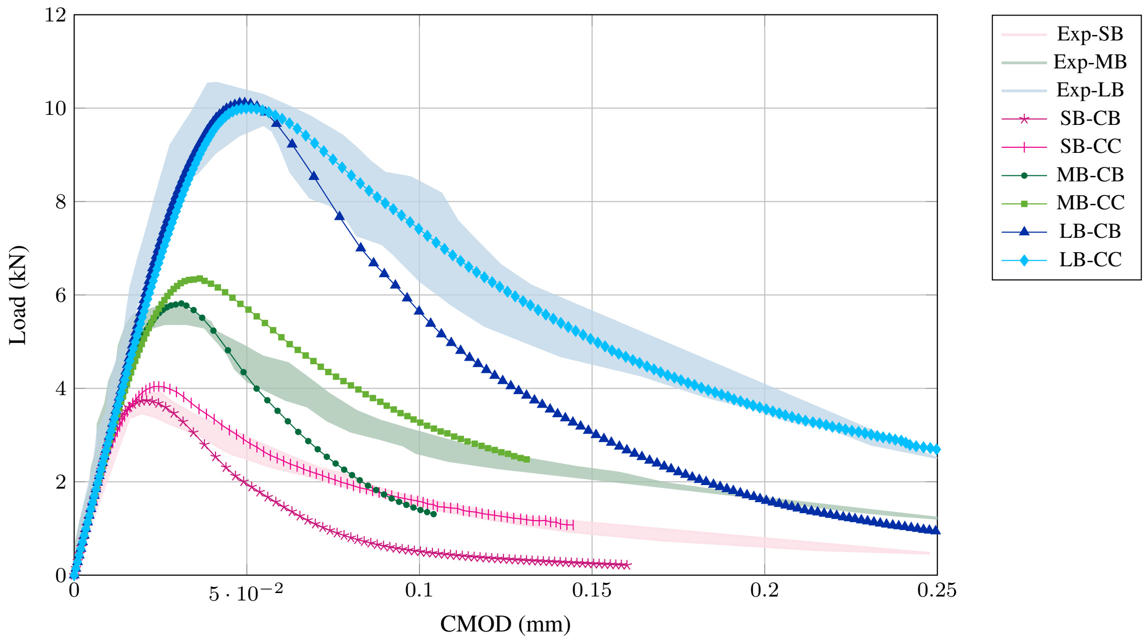

Figure 4. Load *versus* Crack Mouth Opening Displacement - CMOD.

4 Final remarks

Finally, the main conclusion of the present study can be summarized:

i The proposed BPS model was capable of describing concrete behavior considering material physical parameters to describe degradation;

ii The BPS model was validated from comparison with experimental data of the three-point bending test of García-Álvarez et al. [20];

iii The nonlocal approach was efficient in avoiding localization phenomena;

iv Numerical simulations of different structural models must be performed with the BPS model to consolidate model characteristics and limitations.

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