Heterogeneity Modelling of Concrete Structures by Nonlocal Damage Models

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Abstract. The behavior of structural concrete is usually represented by considering the homogeneous material media and its macroscopic properties. The problem can be represented in more realistic models considering the material as a heterogeneous multiphase media. In representing heterogeneity, either a direct or indirect description of the phases can be considered. The direct approach considers the geometric description of the phases in the discretization of the model. In the indirect approach, phases are represented by homogenizing their physical properties or by the random distribution of points in the domain with material parameters corresponding to each phase. This work proposes a model for representing the heterogeneity of concrete, aiming at the physically nonlinear analysis of structures. Two phases will be considered to describe the heterogeneity: the cement matrix and the coarse aggregates, which will be incorporated into the discrete model using a hybrid method, which combines characteristics of direct and indirect methods. For material media degradation, a nonlocal damage model will be adopted with appropriate constitutive laws for each constituent material. Finally, numerical simulations will be presented to evaluate the model's characteristics.

Keywords: Heterogeneity; Nonlinear analysis; Nonlocal Damage Models

1 Introduction

Concrete is a composite material formed by a cement matrix (cement and water), coarse aggregates, fine aggregates, and porosity. Due to this nature, it can be analyzed at different scales of observation, such as microscale, mesoscale, macroscale, or even in a multiscale analysis. The behavior of a material at a particular scale can be explained in terms of the structure it presents at smaller observation scales, as stated by van Mier [1]. Concrete can be satisfactorily studied from the mesoscale once at the macroscale it does not rigorously establish cause-effect relationships between the physical properties of heterogeneous materials and their mechanical response, as highlighted by López et al. [2].

The treatment of heterogeneity in numerical modeling is traditionally approached by two methodologies, as described by Schlangen and van Mier [3]. The first strategy is a direct representation consisting of the geometric description of the phases in the discrete model, as performed by several authors [4, 5]. The second approach is considered an indirect method of heterogeneity representation [6–9], which distributes the physical properties of materials among the internal points of the model.

In the numerical simulation of concrete heterogeneity, different approaches are considered in the constitutive relations of materials. One possibility is modeling based on damage mechanics, where the degradation of its elastic properties phenomenologically represents the concrete fracturing process. According to Kachanov [10], damage models relate an integrity variable that ranges between 0 and 1, where 0 indicates that the material fails due to brittle rupture and 1 indicates that the material has no damage. Several studies have explored models based on damage mechanics, notably the work of Mazars [11], Lemaitre and Chaboche [12], and de Vree et al. [13], which present different deformation measures, called equivalent deformation, to obtain the evolution of the damage.

Along with damage models, regularization strategies are essential to prevent numerically induced strain localization issues. One possibility is the nonlocal approach, based on the integral formulation [14, 15] or gradients [16, 17], but both methodologies are congruent, as demonstrated by Magri et al. [18].

This work aims to simulate a concrete structure with a heterogeneous material description. It considers a hybrid methodology, which combines aspects of the direct and indirect methods, in representing the phases. For this purpose, a nonlocal damage model is adopted for the constitutive relations of the material. Finally, numerical simulations will be conducted to evaluate the performance of the model.

2 Theoretical Foundation

2.1 Standard Isotropic Damage Model

An isotropic damage model is described by the constitutive relationship given by:

$$\sigma_{ij} = (1 - D)E^0_{ijkl}\varepsilon_{kl}\,, \tag{1}$$

where σ_{ij} are the components of the stress tensor, ε_{kl} are the components of the strain tensor, and D is the damage variable, which can assume values between 0 and 1 to indicate whether the material is intact ($D = 0$) or completely degraded ($D = 1$).

The loading function in the damage model relates the evolution of stresses and deformations with resistance criteria and historical variables of the model. In general, the deformation-based loading function is given by:

$$F(\varepsilon, \kappa(D)) = \widetilde{\varepsilon} - \kappa(D)\,, \tag{2}$$

where $\widetilde{\varepsilon}$ is the equivalent strain and $\kappa(D)$ is a historical variable of equivalent strain as a function of damage given by:

$$\kappa = \begin{cases} \kappa_0, & \text{if} \quad D = 0 \\ \mathtt{max}\left[\widetilde{\varepsilon}, \kappa(D)\right], & \text{if} \quad D > 0\,, \end{cases} \tag{3}$$

where κ_0 is the initial limit value of elastic deformation. The loading function and the rate of the historical variable ($\dot{\kappa}$) must satisfy the Kuhn-Tucker conditions for loading and unloading, given by:

$$F \leq 0, \quad \dot{\kappa} \geq 0 \quad \mathsf{e} \quad F\dot{\kappa} = 0\,. \tag{4}$$

The model can be developed according to the formulation presented by Carol et al. [19]. Thus, the equation eq. (1) is taken and differentiated in time, resulting in:

$$\dot{\sigma}_{ij} = E^t_{ijkl}\dot{\varepsilon}_{kl}\,, \tag{5}$$

where the upper point indicates a variation in pseudo-time and E^t_{ijkl} is given by:

$$E^t_{ijkl} = E_{ijkl} - \frac{\partial D}{\partial \tilde{\varepsilon}}\frac{\partial \tilde{\varepsilon}}{\partial \varepsilon_{ij}}E^0_{klpq}\varepsilon_{pq}\,. \tag{6}$$

For the evolution of the damage, the exponential law will be adopted, presented by de Borst and Gutiérrez [16], which is given by:

$$D(\tilde{\varepsilon}) = 1 - \frac{\kappa_0}{\tilde{\varepsilon}}\left[1 - \alpha + \alpha e^{-\beta(\tilde{\varepsilon} - \kappa_0)}\right]\,, \tag{7}$$

where $\tilde{\varepsilon}$ is the equivalent deformation measure, κ_0 is the equivalent strain value from which the damage process begins, α is the maximum allowable damage value for the material, β is the intensity of the damage evolution.

In this work, the equivalent deformation proposed by de Vree et al. [13] was adopted, which is defined by:

$$\tilde{\varepsilon} = \frac{k_r - 1}{2k_r\left(1-\nu\right)} I_1^{\varepsilon} + \frac{1}{2k_r}\sqrt{\frac{\left(k_r-1\right)^2}{\left(1-2\nu\right)^2}\left(I_1^{\varepsilon}\right)^2 + \frac{12k_r}{\left(1+\nu\right)^2} J_2^{\varepsilon}}\,, \tag{8}$$

where $k_r = f_c/f_t$, where f_c is the compressive strength of the material and f_t is the tensile strength of the material, I_1^{ε} is the first invariant of the strain tensor, and J_2^{ε} is the second invariant of the deviator strain tensor.

2.2 Nonlocal Model

Numerically induced localization is a recurring problem in damage models based on elastic degradation. In this sense, the abovementioned problem can be addressed using a nonlocal strategy. Among many other authors, Jirásek [15] presents the basis of the integral formulation in which, in general, the approach consists of replacing a given variable at a point with its nonlocal value obtained by the weighted average at the points in its neighborhood, as presented in:

$$\bar{f}(\boldsymbol{r}) = \frac{1}{V_r}\int_V w(||\boldsymbol{r}-\boldsymbol{s}||)f(\boldsymbol{s})dV\,, \tag{9}$$

where $w(||\boldsymbol{r}-\boldsymbol{s}||)$ is the weight function that depends on the Euclidean distance $||\boldsymbol{r}-\boldsymbol{s}||$ between the point $\boldsymbol{r}$, where the local variable will be replaced by its nonlocal value, and the point $\boldsymbol{s}$ being analyzed in the weighting. The local magnitude associated with a point $\boldsymbol{s}$ is given by $f(\boldsymbol{s})$, while $\bar{f}(\boldsymbol{r})$ is the nonlocal quantity associated with the point $\boldsymbol{r}$. The domain of influence of the point $\boldsymbol{r}$ is V, given by:

$$V_r = \int_V w(||\boldsymbol{r}-\boldsymbol{s}||)dV\,, \tag{10}$$

that is, an integral of the weight function in the domain of influence of the point $\boldsymbol{r}$. In this work, the Gaussian weight function was adopted given by:

$$w(||\boldsymbol{r}-\boldsymbol{s}||) = \exp\left[-\left(\frac{k||\boldsymbol{r}-\boldsymbol{s}||}{\ell}\right)^2\right]\,, \tag{11}$$

where k is a parameter that defines the shape of the curve and ℓ is the nonlocal radius that defines the size of the domain V_r.

Considering the heterogeneity, nonlocal models with phase division must be used, as presented by Fish et al. [20], Magri et al. [18] and Lenz and Mahnken [21]. Therefore, in this work, only points with the same phase were considered for calculating the nonlocal variable. Therefore, the weight function was redefined to:

$$w = \begin{cases} w, & \text{if} \quad \mathbf{s} \in V_{r\eta} \\ 0 & \text{otherwise}\,, \end{cases} \tag{12}$$

where $V_{r\eta}$ are the within the domain V, with phase η, which is the same phase as the analysis point $\boldsymbol{r}$.

2.3 Heterogeneity Representation

To represent heterogeneity a hybrid methodology was adopted. This strategy distributes the material properties at the integration points similar to the indirect method, however the integration points will spatially represent the aggregates like the direct method. This way, the aggregate is represented without it being directly discretized in the mesh of elements. This hybrid approach is similar to the unaligned mesh proposal presented by Zohdi and Wriggers [22] and Zohdi [23], in which the finite elements of the mesh may internally present a material discontinuity.

A cumulative distribution function for particle size must be adopted in numerical models of concrete heterogeneity. The most traditional in the literature, and used in this work, is proposed by Fuller and Thompson [24], given by:

$$F^{\texttt{Fuller}}(d) = \left(\frac{d}{d_{Max}}\right)^{n^{\texttt{Fuller}}} . \tag{13}$$

where $n^{\texttt{Fuller}}$ is the Fuller number that varies from 0.45 and 0.7 according to Wriggers and Moftah [25] and modifies the accumulated value in the particle size distribution curve, d_{Max} is the diameter maximum of the aggregate and d is the value of the diameter of the aggregate considered in the function.

3 Numerical Simulation

A numerical test on a L-shaped panel of Winkler et al. [26] was conducted to evaluate the numerical response of the model with a heterogeneous medium using the hybrid method. An isotropic damage model was used with an exponential law given by eq. (7) and with an equivalent deformation measure proposed by de Vree et al. [13].

The data from the experiment performed by Winkler et al. [26] are for macroscopic homogeneous concrete, and there is no specification of the properties of the aggregates. Therefore, the properties presented by Winkler et al. [26] are the modulus of elasticity $E_0 = 24469$ N/mm^2, Poisson's coefficient $\nu = 0.18$, tensile strength of the material $f_t = 2.7$ MPa, fracture energy $G_f = 0.065$ N/mm.

There are no further details about the mesoscopic parameters, and as reported by Sun [27], the characteristics of the mesostructure are difficult to obtain, and several tests present in the literature [28–30], focus only on the parameters of concrete. Therefore, the homogeneous material was parameterized and subsequently, to describe the heterogeneity, the phase parameters in the heterogeneous modeling were adjusted based on the properties of the homogeneous material.

The parameterization was carried out considering constitutive laws defined with the parameters of the experimental test and comparing them with the parameters used in the damage model. Thus, for the homogeneous material we have the following damage law properties: $E_0 = 24469$ N/mm^2, $\nu = 0.18$, $k_r = 10$, $\alpha = 0.999$, $\beta = 950$, $\kappa_0 = 1.35 \times 10^{-4}$.

For the data used in the heterogeneous model, eq. (14), used by Eckardt and Könke [31], was used to find the values of the elastic modulus of each material.

$$\frac{1}{E^C} = \frac{\phi^M}{E^M} + \frac{\phi^A}{E^A} , \tag{14}$$

where E^C is the modulus of elasticity of the homogeneous concrete, E^M is the modulus of elasticity of the cement matrix, E^A is the modulus of elasticity of the aggregate, ϕ^M is the percentage of mortar in the concrete and ϕ^A is the percentage of aggregate in the concrete.

Considering that the value of the elastic modulus of the mortar is lower, as highlighted by Eckardt and Könke [31], for a proportion of 30% aggregate and 70% cement matrix, the values of $E^M = 23000$ N/mm^2, and $E^A = 28750$ N/mm^2.

The parameters for the cement matrix are adjusted, taking the macroscopic properties of the concrete as a reference, similar to what was done for the elastic modulus. Thus, the data for the cement matrix adopted were: $\alpha^M = 0.999$, $\beta^M = 760$ and $\kappa_0^M = 1.28 \times 10^{-4}$. Additionally, the aggregate parameters are adjusted with the aim of having a linear elastic response, causing the damage to propagate preferentially in the mortar. This methodology is commonly applied by several authors and is reported in the literature review developed by Thilakarathna et al. [32]. Therefore, the properties for the aggregates adopted were: $\alpha^A = 0.999$, $\beta^A = 20000$ and $\kappa_0^A = 3.25 \times 10^{-4}$.

The geometric data from the numerical test of the L-shaped panel, the adopted mesh, and the heterogeneity representation are presented in Fig. 1. The mesh is made up of CST (Constant Strain Triangle) elements, the integration points in gray represent the mortar, while those in red represent the aggregates. The aggregates size ranges from 4 mm to 8 mm, according to the accumulated distribution function exposed in eq. (13) with $n_{fuller} = 0.5$.

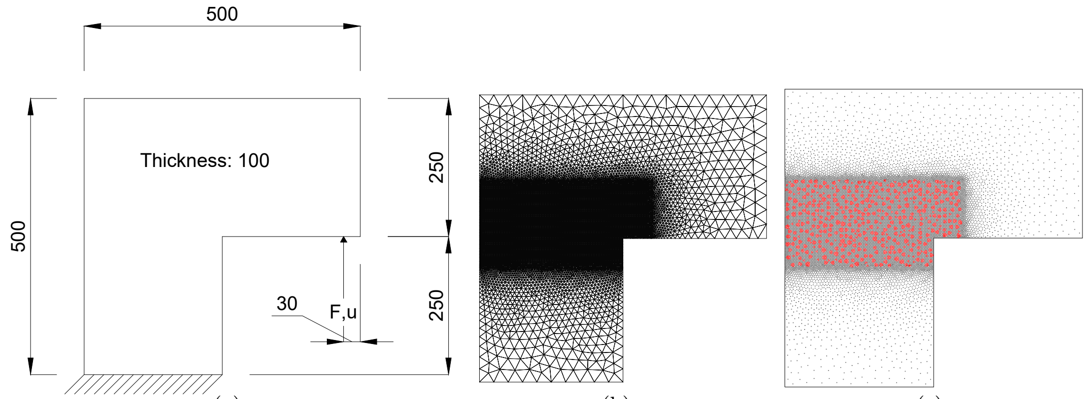

Figure 1. L-shaped panel. (a) Geometry of the model with measurements in millimeter; (b) T3 mesh ($h = 25$ mm in the coarse region and $h = 2.5$ mm in the fine region); (c) Representation of heterogeneity in the model

In the solution process, the direct displacement control method was adopted with an increment 4×10^{-3} at the point of force application. Two Load x Displacement curves for the force application point are presented in Fig. 2a. In both curves, the nonlocal model with phase division was used, with the Gaussian weight function, considering a nonlocal radius of 28 mm and each curve has a value of k. The size of the non-local radius is based on a value close to 3 times the maximum diameter, as highlighted by Bažant and Pijaudier-Cabot [33]. The experimental result obtained by Winkler et al. [26] is also illustrated together with the responses from the numerical simulations. In Fig. 2b the damage is shown considering $k = 1.5$, while in Fig. 2c the damage scale is shown.

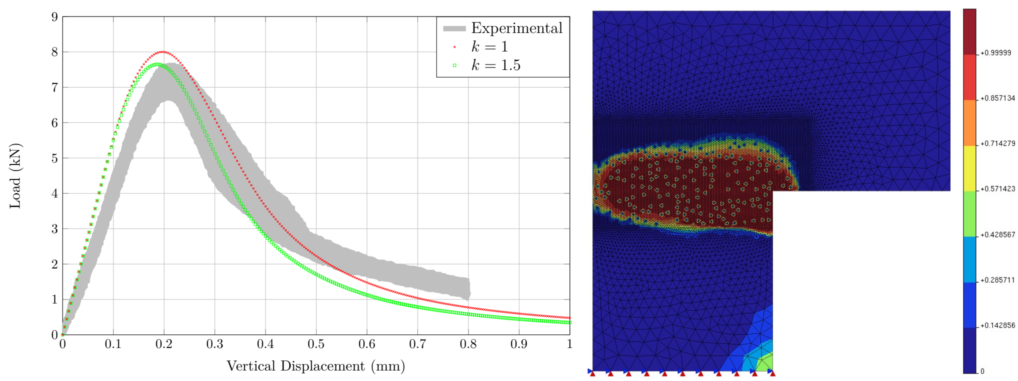

Figure 2. (a) Load-displacement curves for heterogeneous simulations.(b) Contour plots for damage in L-Shaped Panel. (c) Damage scale.

Both results are partially within the experimental curve, present softening behavior similar to expected, and have a peak load relatively close to the experimental one. The phase parameterization must be developed and improved so that the heterogeneous modeling corresponds with the experiment. However, the results obtained already show a very reliable representation of the Winkler et al. [26] test. It is observed that a higher value of k results in a curve with a lower peak load. The contour plots for damage show consistency with the crack path observed in the literature but present a spread degradation zone due to the nonlocal character of the modeling.

4 Final Remarks

In this work, numerical simulations with a heterogeneous description of concrete were conducted. A hybrid methodology was adopted in the model conception, using integration points for spatial representation of heterogeneity, and a nonlocal damage model with phase division was employed due to the heterogeneity. During the study, it was observed that the literature contains data on homogeneous concrete. However, more studies are needed on the physical properties of the phases that compose the concrete. Therefore, a parameterization must be developed to obtain the input data when performing numerical simulations with heterogeneity and comparing them with experimental results. The parameterization in this study obtained results that were close to the experimental one, but further studies should be conducted to improve it.

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