# Creep Functions Based on the Hydration Degree of Oil Well Cement Pastes

Alfredo Quiroga Flores[1], Ana Beatriz de Carvalho Gonzaga e Silva[2], Vivian Karla Castelo Branco Louback Machado Balthar[3], Eduardo de Moraes Rego Fairbairn[4]

*[1] Civil Engineering program, Instituto Alberto Luiz Coimbra de Pós-Graduação e Pesquisa em Engenharia, Universidade Federal do Rio de Janeiro,*
*Centro de Tecnologia, Rio de Janeiro, Brazil*
*alquirogaf@coc.ufrj.br*
*[2] Polytechnic Engineering school, Universidade Federal do Rio de Janeiro,*
*Centro de Tecnologia, Rio de Janeiro, Brazil*
*anabeatrizgonzaga@coc.ufrj.br*
*[3] Faculdade de Arquitetura e Urbanismo, Universidade Federal do Rio de Janeiro,*
*Rio de Janeiro, Brazil*
*vivian@numats.coc.ufrj.br*
*[4] Civil Engineering program, Instituto Alberto Luiz Coimbra de Pós-Graduação e Pesquisa em Engenharia, Universidade Federal do Rio de Janeiro,*
*Centro de Tecnologia, Rio de Janeiro, Brazil*
*eduardo@coc.ufrj.br*

**Abstract.** Creep is the phenomenon of increasing deformation under sustained loads occurring in cementitious materials. Cement pastes, used for cementing oil wells, are subjected to this phenomenon through the application of mechanical, temperature, and pressure loads. For this study cement paste specimens were cast and tested under basic creep conditions at NUMATS/LABEST laboratory of the COPPE research institute of Universidade Federal do Rio de Janeiro. A constant compressive load started to be applied on the specimens at three different ages, 24 hours, 3 and 7 days lasting 83 days approximately. The suitability of the equation of the Bureau of Reclamation was assessed successfully to represent the experimental creep strain results. This equation also allowed to generate additional creep curves corresponding to 2-day and 28-day age load initiation. These five curves were employed to define the Generalized Kelvin units, as Dirichlet series, to represent the creep behavior of this material. The coefficients of every Generalized Kelvin spring stiffness equation, dependent on the hydration grade, were determined. This was carried out by using the Speckel software created by Professor Eduardo Fairbairn and later improved by Vitor Colimodio in his thesis carried out at COPPE. The creep functions and the experimental curves were compared.

**Keywords:** creep, oil well cement pastes, creep functions

## 1 Introduction

The creep of cementitious materials occurs when sustained stress is applied on concrete, then the calcium silicate hydrate (CSH) loses adsorbed water. Other contributing factors are the removal of water held by hydrostatic tension in small capillaries of the hydrated cement paste [1]. The nonlinearity of the stress-strain behavior of cementitious materials also affects mainly at stress levels higher than 30-40% of the ultimate stress. This happens due to the microcracking of the interfacial transition zone. Additionally, if there is a drying condition, more microcracking in the interfacial transition zone occurs. Also, the delayed elastic strain in aggregates, components of the cementitious matrix, increase the total creep. Therefore, the total creep of a cementitious material is composed of basic creep and drying creep. In the former the strain occurs only due to the action of the stresses. In the latter deformation occurs due to exchange of moisture with the environment. There are various theories to explain the mechanisms during creep: microcracking and seepage of water into cracks [2], dissolution of calcium silicate hydrate (C-S-H) [3], polymerization of C-S-H [3], forms of C-S-H sliding/dislocation [4], solidification [5], , relaxation of microprestress [5], clinker dissolution [6].

To simulate the basic creep behavior of a cementitious material, several models can be used. However, to represent creep in a finite element method (FEM) computer code, the most part of the models have a mathematical representation that corresponds to the integral formulation that demands an enormous quantity of memory. Such use of memory makes modeling unfeasible. Two models allow the development of algorithms that overcome this problem because they require storage of only a few state variables. These are the generalized Maxwell and Kelvin models. In this paper we use the generalized Kelvin model that consists of a series of units composed of a spring and a damper arranged in parallel position (see Figure 1) [7]. The generalized Kelvin model corresponds to the creep strain represented by a Dirichlet series that requires definition of parameters such as the spring stiffnesses and the characteristic times of the dashpots. During the hardening process of a cementitious material, these parameters change according to the evolution of the hydration reaction.

Considering the nature of the functions used to represent the creep phenomenon by a generalized Kelvin chain, at least 5 creep curves with 5 different ages at loading are necessary. The problem is that in most cases it is very difficult to have such many creep tests. Therefore, in this work we present a procedure that allows, from a reduced number of experimentally obtained functions, to interpolate functions that satisfactorily represent the creep phenomenon.

For this aim, this study presents the experimental results of cement paste specimens casted and tested under compressive load under basic creep conditions at NUMATS/LABEST laboratory of the COPPE research institute of Universidade Federal do Rio de Janeiro during 83 days after the load initiation for three ages at loading. The equation of the Bureau of Reclamation was used to represent creep and served to interpolate two curves belonging to two additional ages at loading. These curves allowed us to obtain the parameters that define the generalized Kelvin model for this oilwell cement paste. It then becomes possible to provide input of creep functions to computational codes that use the generalized Kelvin chain as a model and algorithm.

## 2 Basic creep functions of cementitious materials

### 2.1 The creep function of the US Bureau of Reclamation

The creep function of the US Bureau of Reclamation [[8] is presented in Equation (1) where $t$ is the time variable, $t'$ is the age at loading, $E(t')$ is the Young´s modulus, $F_k(t')$ is a creep coefficient and $J$ is the creep function.

$$J(t,t') = \frac{1}{E(t')} + F_k(t') \log(1 + t - t') \tag{1}$$

### 2.2 Hydration model

The hydration model proposed by Ulm e Coussy was employed in this work. The complete derivation of this model can be found elsewhere [9,10] and subsequent applications [11]. The hardening of a cementitious material is a complex process which comprehends chemical reactions and heat release. The hydration degree $\xi$ was proposed as a parameter to measure at what extent the reaction process is as defined in Equation (2). It cannot be measured directly. The equation of thermal fields in a determined volume in terms of the hydration degree can be expressed as in Equation (3). Furthermore, the Arrhenius law can also be expressed in terms of this variable, Equation (4). On the other hand, the hydration heat can be measured directly and is defined in Equation (5). A relation between the hydration heat and the hydration degree was proposed in Equation (6). The adiabatic test allows to establish an adiabatic curved defined by Equation (7) where $\Delta T_j^{ad}$ is the increase of temperature of the cementitious sample at a step $j$, $\Delta T_{max}^{ad}$ is the total temperature increase during the test, $k$ and $n$ are parameters and $t$ is the time.

$$\xi = \frac{Quantity\ of\ hydrated\ cement}{Original\ quantity\ of\ cement} \tag{2}$$

$$C_e\dot{T} = Q + L\dot{\xi} + k\nabla^2 T \tag{3}$$

$$\frac{dm}{dt} = \frac{1}{\eta(m)}A(m)e^{\frac{-E_a}{RT}} \rightarrow \frac{d\xi}{dt} = \widetilde{A(\xi)}e^{\frac{-E_a}{RT}} \tag{4}$$

$$\xi_T = \frac{Quantity\ of\ heat\ gerated\ during\ time\ t}{Quantity\ of\ heat\ gerated\ during\ t \rightarrow \infty} \tag{5}$$

$$\frac{\xi}{\xi_T} = 1{,}0618\xi^2 - 1{,}058\xi + 0{,}9994 \tag{6}$$

$$\Delta T_j^{ad} = \Delta T_{max}^{ad}\frac{t_j^n}{k^n + t_j^n} \tag{7}$$

### 2.3 Kelvin model

The complete derivation of the Generalized Kelvin (GK) model should be reviewed elsewhere [7]. The basic creep behavior of a cementitious material can be simulated by GK units as presented in Figure 1. A GK unit $i$ consists of a damper with a damping coefficient $\eta_{kv}^i$ and a spring with elastic stiffness $k_{kv}^i$. The stress in a GK unit $i$ is the sum of the stress in the spring and in the damping (Equation (8)). Two and three GK units were used. The derivation of the basic creep strain based on a Dirichlet series, Equation (9), where $J_b(t,t')$ is the basic creep strain at time $t$, the loading started at time $t', n$ is the number of GK units, and $\tau_i$ is the delay time or characteristic time defined in Equation (10). The spring stiffness of the GK unit varies is dependent on time as presented in Equation (11), where $c_1^i, c_2^i, c_3^i$ and $c_4^i$ are coefficients obtained from experimental results. Given that the hydration degree was derived from time and the $k_{kv}^i$ also depends on time, this stiffness can also be obtained from the hydration degree as presented in Equation (12) [12]. In Equation (12), $\bar{\xi}$ is the normalized hydration degree defined in Equation (13), where $\xi_0$ is hydration degree at yield point and $.\xi$ is the hydration degree at a certain time during the hydration process of cement.

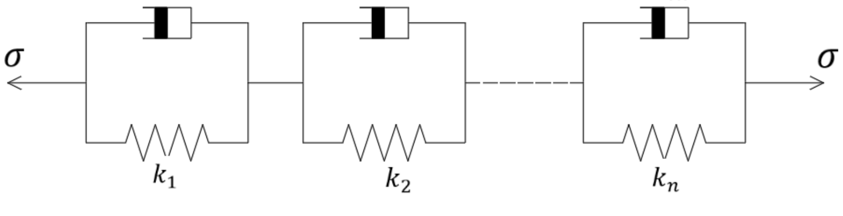

Figure 1 The Kelvin-Voigt model

$$\sigma^i(t) = k_{kv}^i \varepsilon_{kv}^i(t) + \eta_{kv}^i \dot{\varepsilon}_{kv}^i(\mathrm{t}) \tag{8}$$

$$J_b(t,t') = \sum_{i=1}^{n} \frac{1}{k_{kv}^i(t')} \left(1 - e^{\frac{t-t'}{\tau^i}}\right) \tag{9}$$

$$\tau^i = \frac{\eta_{kv}^i}{k_{kv}^i} \tag{10}$$

$$k_{kv}^i(t) = c_1^i + c_2^i \log\left(1 + \frac{t}{1\ dia}\right) + c_3^i\, log^2\left(1 + \frac{t}{1\ dia}\right) + c_4^i\, log^3\left(1 + \frac{t}{1\ dia}\right) \tag{11}$$

$$k_{kv}^i(\xi) = c_1^i + c_2^i 10^{\bar{\xi}} + c_3^i 10^{\overline{2\xi}} + c_4^i 10^{\overline{3\xi}} \tag{12}$$

$$\bar{\xi} = \frac{\xi - \xi_0}{1 - \xi_0} \tag{13}$$

# 3 Results and Discussion

The three experimental results of the basic creep tests are presented in Figure 2. They are not enough to adjust the $c_{\mathrm{j}}^i$ parameters of Equation (12) thus two additional curves are approximated by the equation of the Bureau of Reclamation, Equation (1) at $t'$= 2 days and 28 days, (Table 1). Figure 3 presents that the adjustment is suitable to the experimental curves and the additional ones are consistent with the laboratory results.

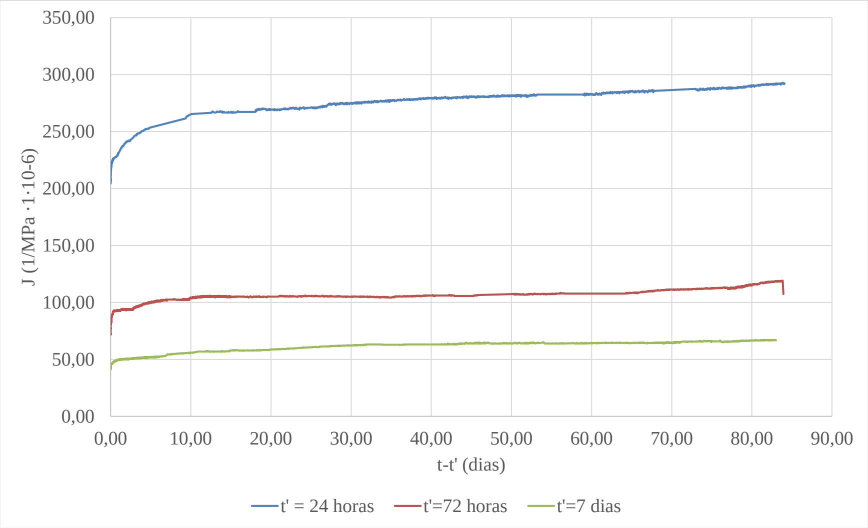

Figure 2. Experimental results

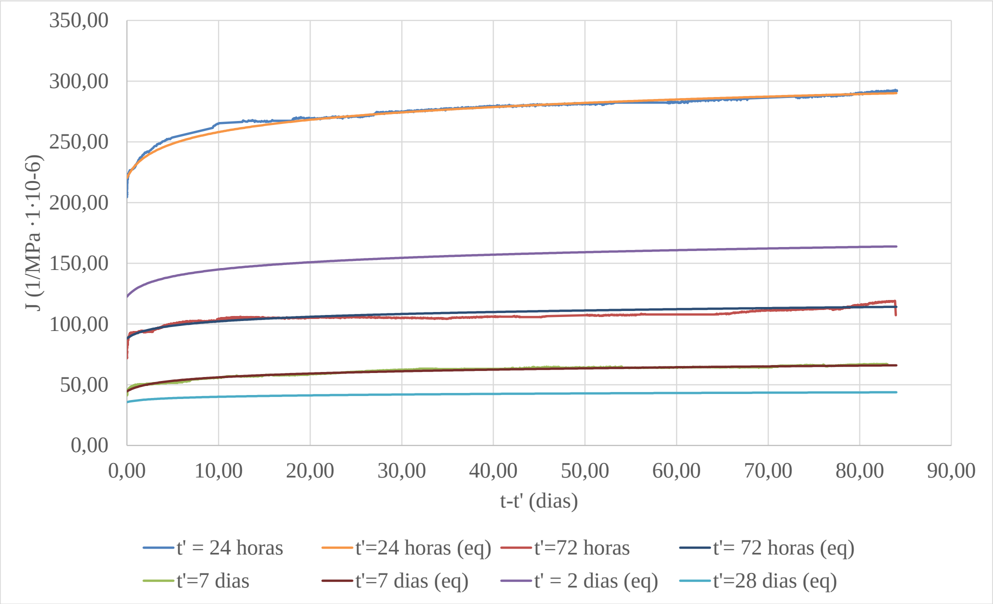

Figure 3. The suitability of the equation of the Bureau of Reclamation

Table 1. Factors of the Bureau of Reclamation equation

| t' (dias) | E(t') (MPa) | Fk(t') |
|---:|---:|---:|
| 1 | 4535,17 | 3,610E-05 |
| 2 | 8158,55 | 2,143E-05 |
| 3 | 11345,29 | 1,349E-05 |
| 7 | 22457,17 | 1,112E-05 |
| 28 | 28000,0 | 4,179E-06 |

Subsequently the parameters of Equation (12) were found. To do so, the characteristic times $\tau^i$ of Equation (9) were assumed by varying $\tau^{i+1}/\tau^i$ ratios: 5, 10 and 25. The results were compared by the average regression coefficient $r$ which had the highest value when $\tau^{i+1}/\tau^i$=10 and $\tau^1$ =0,5 days, $\tau^2$ =5 days and $\tau^3 = 50$ days.

Table 2. $\tau^i$ parameters

| | | | | | | | |
|---|---:|---:|---:|---:|---:|---:|---:|
| $\tau^{i+1}/\tau^{i}$ | | 5 | | | | | |
| Modelo | A | B | C | D | E | F | G |
| $\tau^1$ | 0,01 | 0,05 | 0,1 | 0,5 | 1 | 5 | 10 |
| $\tau^2$ | 0,05 | 0,25 | 0,5 | 2,5 | 5 | 25 | 50 |
| $\tau^3$ | 0,25 | 1,25 | 2,5 | 12,5 | 25 | 125 | 250 |
| *r* | 0,3733 | 0,6094 | 0,7416 | 0,97238 | 0,99697 | 0,9977 | 0,9899 |
| $\tau^{i+1}/\tau^{i}$ | | 10 | | | | | |
| Modelo | A | B | C | D | E | F | G |
| $\tau^1$ | 0,01 | 0,05 | 0,1 | 0,5 | 1 | 5 | 10 |
| $\tau^2$ | 0,1 | 0,5 | 1 | 5 | 10 | 50 | 100 |
| $\tau^3$ | 1 | 5 | 10 | 50 | 100 | 500 | 1000 |
| *r* | 0,5553 | 0,8535 | 0,9497 | **0,9995** | 0,6005 | 0,9978 | 0,9892 |
| $\tau^{i+1}/\tau^{i}$ | | 25 | | | | | |
| Modelo | A | B | C | D | E | F | G |
| $\tau^1$ | 0,01 | 0,05 | 0,1 | 0,5 | 1 | 5 | 10 |
| $\tau^2$ | 0,25 | 1,25 | 2,5 | 12,5 | 25 | 125 | 250 |
| $\tau^3$ | 6,25 | 31,25 | 62,5 | 312,5 | 625 | 3125 | 6250 |
| *r* | 0,8856 | 0,9971 | 0,9952 | 0,9983 | 0,9967 | 0,9979 | 0,989 |

Consequently, the creep functions defined by Equation (9) were drawn along with the experimental curves in Figure 4 where the creep functions defined as a Dirichlet series describe the behavior satisfactorily.

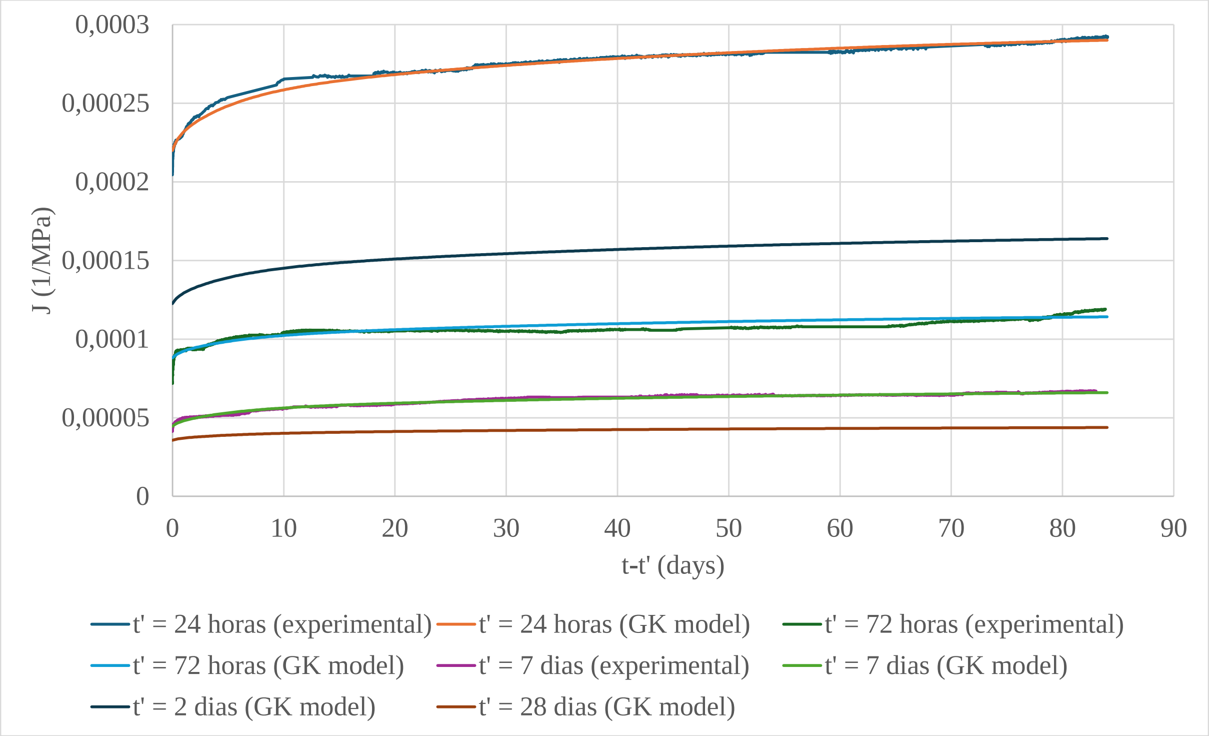

Figure 4. The comparison of the creep functions with the experimental results

## 4 Conclusions

It is concluded that:

- From the experimental results it is known the creep strain contribution corresponds to 43%, 64% and 62% of the elastic strain of the tested specimens at $t'$ = 24 hours, 72 hours and 7 days, correspondingly. It is very rare to carry out creep tests at various ages at loading.
- The equation of the Bureau of Reclamation formulated for mass concrete of dams adjusted well to the experimental creep strain curves of oilwell cement pastes.
- The GK model successfully represented the experimental basic creep behavior. Nowadays the unique way that makes feasible the numerical modeling of creep in a finite element method software is the use of the Kelvin and Maxwell chains whose mathematical representations are the Dirichlet series. These models allow the use of a reasonable amount of computational memory.
- The algorithmic representation of the models of Dirichlet series require the availability of five ages at loading.
- This paper presents then a procedure that allows the interpolation of creep functions and makes possible the utilization of these functions in a finite element method computer code.

**Acknowledgements:** This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001.

**Authorship statement:** The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

## References

[1] P. Mehta, P. Monteiro, Concrete: Microstructure, Properties, and Materials, McGraw-Hill Professional, 2005.

[2] P. Rossi, J.-L. Tailhan, F. Le Maou, L. Gaillet, E. Martin, Basic creep behavior of concretes investigation of the physical mechanisms by using acoustic emission, Cem Concr Res 42 (2012) 61–73. https://doi.org/10.1016/j.cemconres.2011.07.011.

[3] J.J. Thomas, H.M. Jennings, A colloidal interpretation of chemical aging of the C-S-H gel and its effects on the properties of cement paste, Cem Concr Res 36 (2006) 30–38. https://doi.org/10.1016/j.cemconres.2004.10.022.

[4] M. Shahidi, B. Pichler, Ch. Hellmich, Viscous interfaces as source for material creep: A continuum micromechanics approach, European Journal of Mechanics - A/Solids 45 (2014) 41–58. https://doi.org/10.1016/j.euromechsol.2013.11.001.

[5] Z.P. Bažant, A.B. Hauggaard, S. Baweja, F.-J. Ulm, Microprestress-Solidification Theory for Concrete Creep. I: Aging and Drying Effects, J Eng Mech 123 (1997) 1188–1194. https://doi.org/10.1061/(ASCE)0733-9399(1997)123:11(1188).

[6] X. Li, Z.C. Grasley, E.J. Garboczi, J.W. Bullard, Modeling the apparent and intrinsic viscoelastic relaxation of hydrating cement paste, Cem Concr Compos 55 (2015) 322–330. https://doi.org/10.1016/j.cemconcomp.2014.09.012.

[7] G.J. Creus, Viscoelasticity — Basic Theory and Applications to Concrete Structures, Springer Berlin Heidelberg, Berlin, Heidelberg, 1986. https://doi.org/10.1007/978-3-642-82686-3.

[8] US Bureau of Reclamation, Concrete Manual, 8th edition, Denver, USA, 1975.

[9] F.-J. Ulm, O. Coussy, Modeling of Thermochemomechanical Couplings of Concrete at Early Ages, J Eng Mech 121 (1995) 785–794. https://doi.org/10.1061/(ASCE)0733-9399(1995)121:7(785).

[10] F.-J. Ulm, O. Coussy, Strength Growth as Chemo-Plastic Hardening in Early Age Concrete, J Eng Mech 122 (1996) 1123–1132. https://doi.org/10.1061/(ASCE)0733-9399(1996)122:12(1123).

[11] E.M.R. Fairbairn, I.A. Ferreira, G.C. Cordeiro, M.M. Silvoso, R.D. Toledo Filho, F.L.B. Ribeiro, Numerical simulation of dam construction using low-CO2-emission concrete, Mater Struct 43 (2010) 1061–1074. https://doi.org/10.1617/s11527-009-9566-z.

[12] V.S. Colimodio, Modelos numéricos para fluência básica do concreto: Revisitando a cadeia de Kelvin, Instituto Alberto Luiz Coimbra de Pós-graduação e Pesquisa de Engenharia (COPPE) da Universidade Federal do Rio de Janeiro, 2016.