N-M interaction curves for timber members' cross-section under ambient and high temperature

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Abstract. Widely used for structural design, axial force (*N*) - bending moment (*M*) interaction curves are suitable instruments for evaluating the structural elements' cross-section bearing capacity, relating the ultimate values of bending moment and axial force. Also known as interaction diagrams in ambient temperature and fire situation, the *N-M* curves become strongly dependent on the temperature field because of the physical and mechanical changes that occur in the material under elevated temperatures. Therefore, the present study aims to obtain the interaction curves of timber structural elements' cross-section of commonly used members in civil construction, subjected to the action of thermal conditions (ambient and high temperatures), using a generalized numerical strategy based on the strain compatibility method (SCM) and Newton-Raphson method coupling. The adopted strategy is validated through the construction of *N-M* curves for timber cross-sections subject to ambient temperature and different fire configurations and temperature distribution. The results obtained are satisfactory and within the expected range for this type of analysis. It is possible to observe how the number of faces in contact with fire influences the loss of rigidity and resistance of the cross-section of timber. In this way, it is possible to better understand the behavior of such elements under the action of high temperatures.

Keywords: timbers, N-M interaction curves, fire situation, strain compatibility methods, numerical analysis.

1 Introduction

Structural elements when subjected to high temperatures, typical of a fire situation, have their rigidity and bearing capacity affected. Timber, a typical material used in structural elements in civil construction, has its physical and mechanical characteristics deteriorate during exposure to fire. In this way, it is important to analyze temperature distribution within the element cross-section over time when exposed to fire to determine the best sizing.

The numerical analysis of structural elements in a fire situation is normally divided into two stages: thermal analysis and thermo-structural analysis, which are correlated. In thermal analysis, the first step, the temperature field along the cross-section is determined. Time is a fundamental variable, as the thermal response is obtained for each established instant of the fire through a numerical heat transfer calculation procedure. In this way, it is possible to consider the variation in the thermal and mechanical properties of the material as a function of temperature. In the second stage, thermo-structural analysis, it is possible to evaluate the influences and effects of thermal analysis on the behavior of the structure.

Therefore, this work aims to obtain the interaction curves of a typical cross-section of timber elements subjected to high temperatures, using a generalized strategy based on the strain compatibility method (SCM). To carry out this study, the computational module CS-ASA/FSA (*Computational System for Advanced Analysis/Fire Structural Analysis*; [1]), created to perform second-order inelastic analysis of structures under fire, is expanded to consider timber structural elements. It is worth noting that the solution of thermo-structural problems requires efficient numerical formulations capable of capturing nonlinear physical and geometric effects.

In the following sections, the methodology used to obtain the bearing curves will be highlighted. In sequence, the numerical results obtained are demonstrated for one timber cross-section used in other studies in the literature. A parametric study of this section is carried out to assess the influence of the surfaces exposed to fire on the bearing capacity of the structural elements. Other works in the literature share a central theme with the present research and can be cited [2-5]. Also noteworthy is the work of Pires *et al*. [6] which directly correlates with this.

2 Methodology to obtain the N-M interaction curves

The deformation at the cross-sectional level, when the structural element is subjected to internal forces, is obtained through the SCM in the present study. The evaluation of the axial and flexural stiffness of the section is based on the tangent to the moment-curvature relationship and depends on the modulus of elasticity, which is taken from the uniaxial constitutive relationship of the material. More details regarding this formulation can be found in Lemes [7].

The way to represent the behavior of a given material under the effect of tensile or compression forces is through the stress-strain diagram. With the increase in temperature, during a fire episode, materials have their stiffness and resistance altered due to the physico-chemical processes of each material. Therefore, the present study considers the constitutive relationships and thermal properties indicated for the behavior of timber under fire provided for by EN 1995-1-1:2004 [8]. Although this standard indicates an elastic-fragile constitutive relationship in tension for timber, for the model to behave in the elastic and inelastic regime, a basic requirement for advanced analysis, the section in tension of the diagram is considered linear up to the limit strain. Therefore, this research simulates timber with elastic-perfectly plastic behavior.

Another important factor is the cross-sectional discretization, which is fundamental to describe the deformed section configuration. To build the curvature-moment relationship, using the Newton-Raphson Method, it is necessary to know the area of the fibers and their respective positions. The x and y coordinates of each fiber are referenced to the plastic centroid (PC) of the section, thus minimizing convergence problems (Lemes *et al*., [9]). The goal of discretizing the cross-section is to calculate the axial strain, ε, in the CP of each fiber and, therefore, through the constitutive relations of the materials, obtain the stresses (also in each fiber), σ_i. Thus, the axial strain in the i^{th} fiber, ε_i, is given by:

$$\varepsilon_i = \varepsilon_0 + \phi y_i \tag{1}$$

where y_i is the distance between the plastic centroids of the analyzed fiber and the cross-section; ε_0 and ϕ are respectively the axial strain in the PC of the section and corresponding curvature.

Due to the matrix notation, common to numerical procedures, the variables ε_0 and ϕ are components of the strain vector $\mathbf{X} = [\varepsilon_0 \;\; \phi]^T$. Chiorean [10] highlights that, in obtaining the axial and flexural stiffnesses, adopting the strain vector equal to null in the first iteration, convergence is quickly achieved. Lemes *et al*. [9] also states that, due to the undeformed configuration of the cross-section, convergence problems relating to the balance between the acting forces and internal forces are avoided. Therefore, section equilibrium is obtained when the following equation is satisfied:

$$\mathbf{F}(\mathbf{X}) = \mathbf{f}_{ext} - \mathbf{f}_{int} = \begin{bmatrix} N_{ext} \\ M_{ext} \end{bmatrix} - \begin{bmatrix} N_{int} \\ M_{int} \end{bmatrix} \cong 0 \tag{2}$$

in which the external force vector $\mathbf{f}_{ext}$ is given by the axial force, N_{ext}, and the bending moment, M_{ext}, and the terms N_{int} and M_{int} are the components of the internal force vector, $\mathbf{f}_{int}$.

The internal forces are obtained from the deformed configuration of the cross-section through classical integrals, given by:

$$N_{int} = \iint_{A_w} \sigma_w dA = \sum_{i=1}^{n_{fib,w}} \sigma_{wi} A_{wi} \quad \text{and} \quad M_{int} = \iint_{A_w} \sigma_w y dA = \sum_{i=1}^{n_{fib,w}} \sigma_{wi} A_{wi} y_{wi} \tag{3,4}$$

where $n_{fib,w}$ is the number of fibers in the timber cross-section; A_i is the area of the fiber and y_i is the position of the fiber in relation to the Plastic Centroid (CP).

Despite facilitating convergence, starting the process with $\mathbf{X}$ = 0 only guarantees equilibrium in the first iteration if the external forces are zero. Thus, for the next iteration, k+1, the strain vector is calculated from:

$$\mathbf{X}^{k+1} = \mathbf{X}^k + \mathbf{F}'\left(\mathbf{X}^k\right)^{-1} \mathbf{F}\left(\mathbf{X}^k\right) \tag{5}$$

in which $\mathbf{F}'$ is the Jacobian matrix of the nonlinear problem, that is:

$$\mathbf{F}' = \left(\frac{\partial \mathbf{F}}{\partial \mathbf{X}}\right) = \begin{bmatrix} \dfrac{\partial N_{int}}{\partial \varepsilon_0} & \dfrac{\partial N_{int}}{\partial \phi} \\ \dfrac{\partial M_{int}}{\partial \varepsilon_0} & \dfrac{\partial M_{int}}{\partial \phi} \end{bmatrix} \tag{6}$$

The terms of this Jacobian matrix, $\mathbf{F}'$, of the cross-section are given by the expressions:

$$\begin{aligned} f_{11} &= \frac{\partial N_{int}}{\partial \varepsilon_0} = \int_A E_t dA \cong \sum_{i=1}^{n_{fib}} E_{t,i} A_i; \quad f_{12} = \frac{\partial N_{int}}{\partial \phi} = \int_A E_t y dA \cong \sum_{i=1}^{n_{fib}} E_{t,i} A_i y_i \\ f_{21} &= \frac{\partial M_{int}}{\partial \varepsilon_0} = \int_A E_t y dA \cong \sum_{i=1}^{n_{fib}} E_{t,i} A_i y_i; \quad f_{22} = \frac{\partial M_{int}}{\partial \phi} = \int_A E_t y^2 dA \cong \sum_{i=1}^{n_{fib}} E_{t,i} A_i y_i^2 \end{aligned} \tag{7}$$

where $E_{t,i}$ is the tangent modulus of elasticity taken directly from the constitutive relationship of the component material of the i^{th} fiber. It is worth mentioning that, in a fire situation, the tangent modulus of elasticity is multiplied by the reduction factor, according to each material, in the case of the present study, timber.

The convergence criterion of this process is based on the ratio of the Euclidean norms of the vector of unbalanced forces, $\mathbf{F}$, and the vector of external forces, $\mathbf{f}_{ext}$, that is:

$$\frac{\|\mathbf{F}\|}{\mathbf{f}_{ext}} \leq Tol \tag{8}$$

with Tol being the assumed tolerance equal to 10^{-5}.

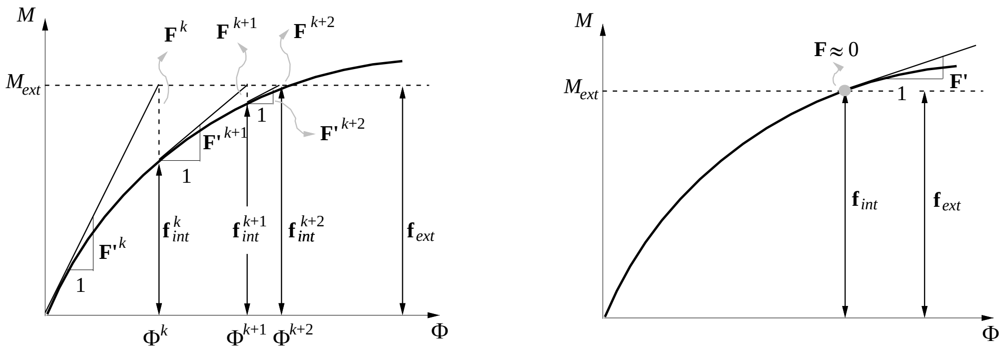
Once the convergence criterion of the iterative cycle is met (Fig. 1a), an equilibrium point is reached. At this point, the generalized stiffness parameters, EA_t and EI_t, are calculated using the strains in the fibers in relation to the deformed configuration. The fibers strains are used to calculate the Jacobian matrix at the equilibrium point, as shown in Fig. 1b. The terms EA_t and EI_t are given by [9]:

$$EA_t = (f_{11} f_{22} - f_{12}^2) / f_{22} \quad \text{and} \quad EI_t = (f_{11} f_{22} - f_{12}^2) / f_{11} \tag{9,10}$$

When, for a given axial force, the maximum moment of the moment-curvature relationship is reached, total plastification of the section occurs. This pair of forces is considered a point on the normal force-bending moment (N-M) interaction curve. The iterative process described previously is repeated for each bending moment increment until the Jacobian matrix becomes singular. The bending moment increment strategy is based on the proposal of Zubydan [11], that is:

$$M_{n+1} = M_n + d\phi EI_t \tag{11}$$

in which $d\phi$ is a curvature increment, EI_t is the tangent bending stiffness given by Eq. (10), and the sub-indices n and n+1 indicate the current increment and the next increment. The algorithm for obtaining interaction curves is shown in Table 1.



a) Iterative process b) Equilibrium point

Figure 1. Moment-curvature relationship [9]

Table 1. Numerical solution adopted to construct the interaction curve

1. Read the section and material data	**16. if** $\lVert\mathbf{F}\rVert \div \lVert\mathbf{f}_{ext}\rVert \leq Tol$ **then**
2. Discretization of the section into fibers	**17.** Stop the iterative process and go to line 26
3. Obtaining the plastic centroid (PC)	**18. end if**
4. Translation from the reference system to the PC	**19.** Assembles the tangent stiffness matrix of the section **F'**
5. Determines the maximum axial forces (tension - $N_{t,max}$ e compression - $N_{c,max}$)	**20.** Checks the uniqueness of **F'**
6. Calculates the normal effort increment $\Delta N = (N_{t,max} - N_{c,max})/100$	**21. if F'** is singular **then**
7. First normal effort value $N = N_{c,max}$	**22.** The critical bending moment found
8. for each increment of axial force, N, **do**	**23.** Stores M and N as a point on the interaction curve
9. Initialize $\mathbf{X} = \mathbf{0}$	**24.** Stop the process and go to line 29
10. for each increment of bending moment, M, **do**	**25. end if**
11. Set $\mathbf{f}_{ext}$	**26.** Corrects the strain vector **X**
12. for k = 1, *nmax* **do**	**27. end for**
13. Determines ε	**28. end for**
14. Set $\mathbf{f}_{int}$	**29.** $N = N + \Delta N$
15. Calculate **F(X)**	**30. end for**

Therefore, the interaction curve is constructed through a series of ordered *N-M* pairs. Once the analysis has started, it is necessary to know the value of the reduced plasticized moment, M_{pr}, for a given value of normal force, N. Therefore, any point on the interaction curve can be obtained by interpolation linear from:

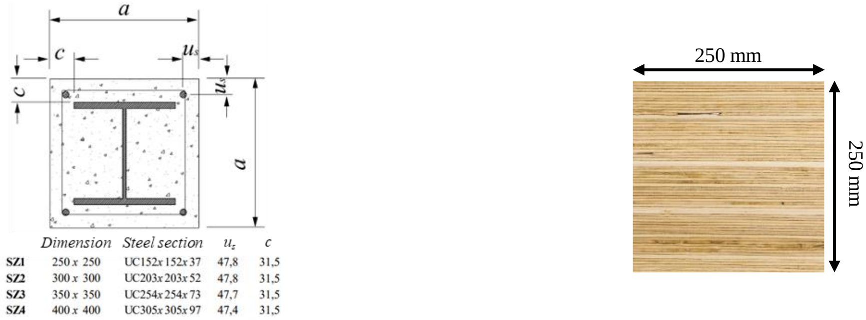
$$M_{pr} = \left(\frac{M_{j+1} - M_j}{N_{j+1} - N_j} \right) \left(N - N_j \right) + M_j \tag{12}$$

To account for the modification of material mechanical properties, it is essential to construct *N-M* interaction curves for each step of fire duration under elevated temperatures.

3 Numerical example

The N-M interaction curves in a fire situation for one timber element cross-section commonly used in civil construction are constructed and a parameter study is made. The thermal, physical, and mechanical properties of timber used at high temperatures are those indicated by EN 1995-1-1:2004 [8]. Quadrilateral isoparametric finite elements (4 nodes) were used in the cross-sectional discretization.

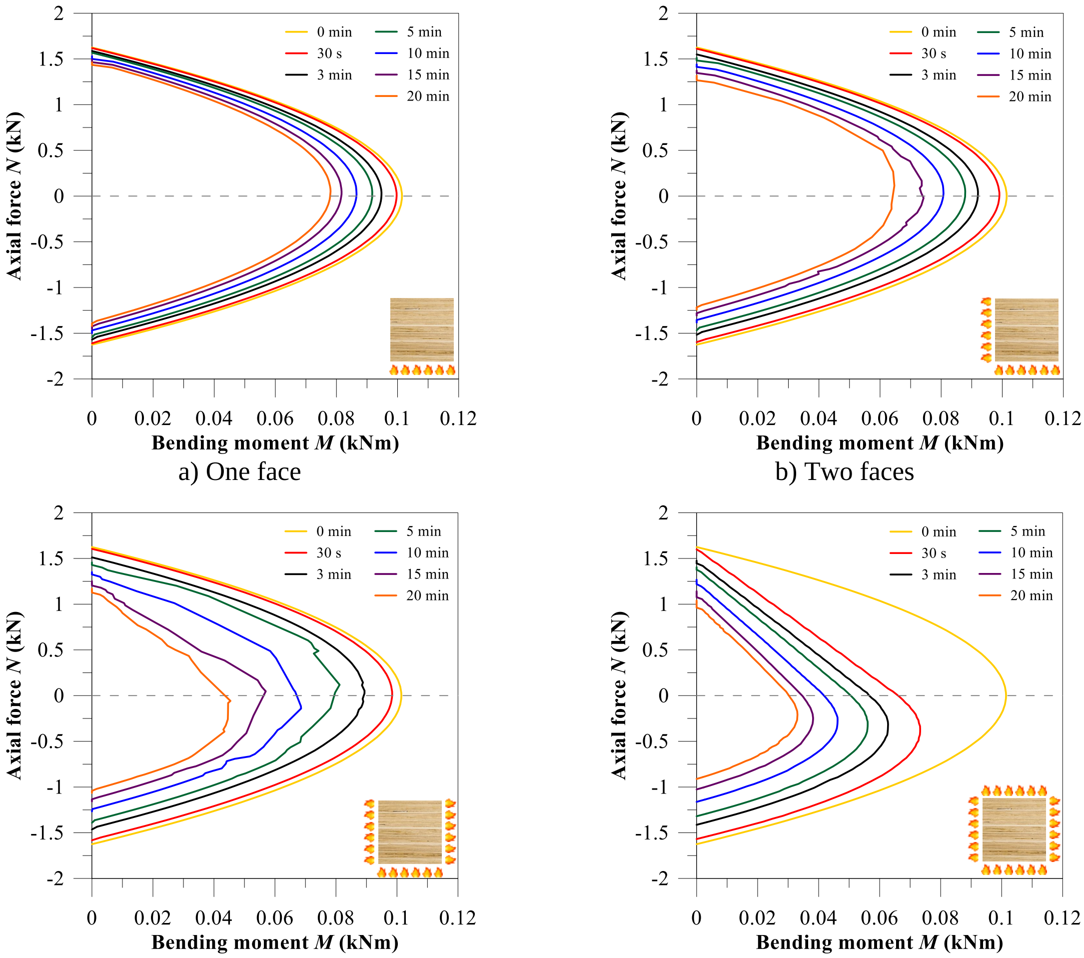
The timber's cross-section evaluated is an adaptation of the composite section used by Huang *et al*. [12] in the series of numerical analyses on the bearing capacity of composite columns under fire action. These composite sections were made of I profiles surrounded by concrete, as shown in Fig. 2a (dimensions in mm). The authors used the FEMFAN-3D software to perform the study. Subsequently, Barros [13] used such sections to validate the implementations carried out in CS-ASA/FSA [1]. This work then adapted one of these sections for softwood, as shown in Fig. 2b.



a) Cross-sections adopted by Huang *et al*. [12] b) Softwood cross-section adopted in this study

Figure 2. Cross-section adapted

Initially, it is performed a parametric analysis considering the number of faces under fire. In this way, it is possible to understand how the configuration of burning surfaces influences the bearing capacity of the timber's cross-section. To perform the analyses, the following data were used: 3600 quadrilateral linear finite elements (Q4) in the mesh; a density of 338 kg/m³; and a humidity of 11.6%. The *N-M* curves for the intervals of 30s, 3, 5, 10, 15 and 20 minutes are shown in Fig. 3 for all fire scenarios. The time instant t = 0 represents the ambient temperature of 20 °C.



a) One face b) Two faces

c) Three faces d) All faces

Figure 3. N-M interaction curves for scenarios of cross-section face fire exposition

The degradation of resistance limits as the time of exposure to fire and, consequently, the temperature increases are observed. The stiffness and resistance reduction factors considered in the analysis lead to such degradation. The timber, a combustible material, shows marked degradation above 300 °C, completely losing its structural capacity. Therefore, for higher temperature values, the resistance limits are low. The asymmetry presented by the curves is caused by the difference between the tension and compression reduction factors provided by EN 1995-1-1:2004 [8], and the results obtained fit within what was expected.

It is possible to observe that the more surfaces exposed to fire, the faster the temperature propagates through the section. This result is within the expected and highlights the importance of protecting the faces to avoid the fast spread of fire and ensure more time for evacuation of the compartments. Figure 4 demonstrates more clearly this importance by comparing the *N-M* interaction curves for fire exposure times of 5, 10, 15, and 20 min for all fire scenarios.

Figure 4 shows how the number of faces exposed to fire substantially influences the bearing capacity of the timber cross-sections. The asymmetry of the exposed faces leads to more complex numerical results since the heat transmission occurs less "orderly" than in other scenarios. These observations enhance the understanding of analyzing timber structures in fire situations, leading to more accurate and reliable results.

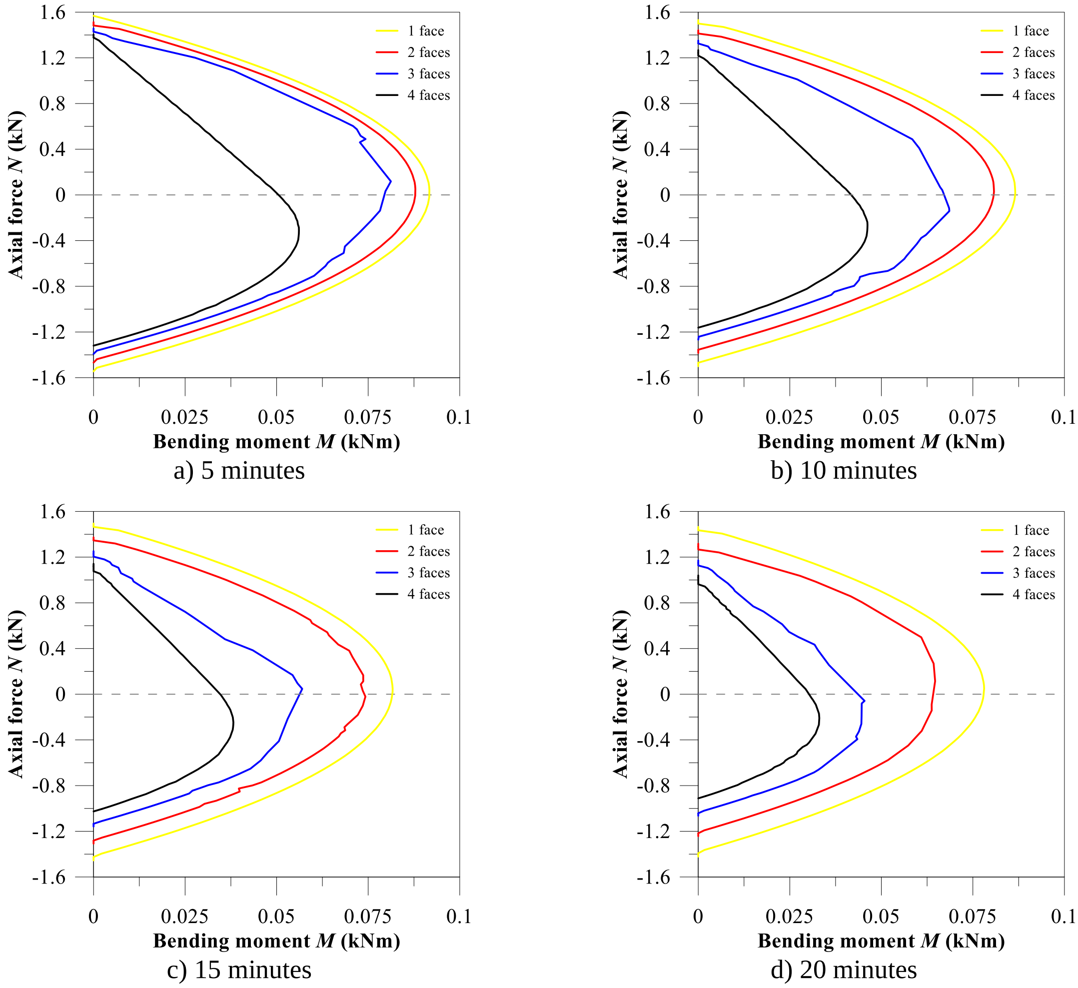


Figure 4. *N-M* interaction curves for different times of fire exposure

4 Conclusions

The present study aimed to obtain the *N-M* interaction curves of a typical timber element cross-section commonly used in civil construction, under fire conditions. A SCM-based strategy was adopted. The implemented numerical changes expanded the capabilities of the CS-ASA/FSA computational module to analyze timber *N-M* bearing curves.

The example presented in the previous section demonstrates that the program can assess and construct *N-M* interaction curves, thereby proving the successful implementation and validity of the proposed methodology.

The impact of fire and material degradation on the ultimate strength of the timber cross-section can now be assessed. Also noteworthy is the asymmetry observed in the interaction diagram, resulting from the different values for the reduction factors for wood traction and compression. Such factors greatly influence the resistance capacity of wooden elements, as observed in the interaction curves obtained in the analyses. Furthermore, it is important to mention that this research is a preliminary phase for investigations that will assess the structural strength of timber components in high-temperature conditions. Hence, additional research will be conducted, including parametric studies on fire-exposed surfaces, analyzing the effects of timber density and moisture on structural fire resistance, and more.

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