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A permutation algorithm for stacking sequence optimization of composite laminates

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Abstract. Fiber-reinforced composite material structures are widely used in various industries, such as the automotive, naval, aeronautical, and construction industries. Laminated structures, made up of thin layers of materials such as carbon fiber-reinforced laminates, offer high strength and stiffness. These layers are stacked in different orientations to provide superior mechanical properties compared to conventional structures. Due to the complexity and number of variables involved, the traditional methodology of design based on trial and error is ineffective, making optimization techniques a proper alternative. In laminated structure design, it is necessary to satisfy strength, stiffness, and performance constraints. Optimization techniques find an optimal lamination scheme that satisfies these requirements. The arrangement of the layers and the orientation of the fibers have a significant impact on the final performance of the structure. The permutation problem in optimization involves determining the most effective sequence of layers and fiber orientations. This work proposes implementing a heuristic optimization technique based on layer permutation, considering a specific number of layers in each lamination. The algorithm is implemented in Octave and is demonstrated in examples of maximizing the buckling factor of laminated plates.

Keywords: Fiber-reinforced Composites, Combinatorial Optimization, Laminated Structures.

1 Introduction

In recent years, the demand for light and strong structures has driven the development of composite materials in various industries. Carbon fiber-reinforced composites stand out for their exceptional strength and stiffness properties and are widely used in critical structural components. Laminate structures, composed of thin layers of composite material with specific fiber orientations, allow the overall mechanical performance of the laminate to be optimized.
Optimizing these structures is challenging due to the large number of variables involved. Advanced optimization techniques, such as genetic algorithms (GA), are essential for finding the optimal configuration that satisfies specific strength and stiffness constraints. This work proposes an optimization technique based on GAs, implemented in Octave language, to optimize laminated structures. The effectiveness of the algorithm is demonstrated on examples of laminated plates, highlighting the improved performance of carbon fiber-reinforced structures.

2 Buckling of Laminated Plates

In this work, we address the optimization of carbon fiber reinforced laminates, made up of several thin layers of composite material in 0°, 45°, and 90° orientations, to maximize their mechanical properties. The laminate is symmetrical, balanced and has a given thickness, and is subjected to biaxial and shear loads. The formulation used to evaluate the buckling load in laminated plates is presented below.

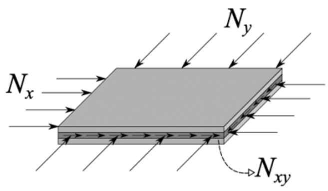


Figure 1. Laminated plate with biaxial and shear loading.

2.1 Buckling analysis

The buckling analysis is carried out to ensure that the laminated plate can maintain its stability with the applied loads. The buckling load factor is calculated considering the behavior of the plate under biaxial and shear loading. Consider a laminated plate with sides a and b, with biaxial loading N_{xx}, N_{yy} and shear loading N_{xy}. The buckling load factor due to biaxial loading is given by [1]:

$$\lambda_n{}^{(p,q)} = \frac{\pi^2(D_{11}(p/a)^4+2(D_{12}+2D_{66})(p/a)^2(q/b)^2+D_{22}(n/b)^4)}{(p/a)^2N_x+(q/b)^2\,N_y}. \tag{1}$$

Where D_{11}, D_{12}, D_{22} and D_{66} are the coefficients obtained from the ABD stress-strain matrix [1] p and q are the number of waves in each direction of the plate when it loses stability, with values ranging from 1 to 20.

In the case of laminates subjected to shear loading, the shear buckling load factor is obtained approximately from [4]:

$$\lambda_s = \begin{cases} \frac{4\beta 1(D_{11}\,D_{22}{}^3)^{1/4}}{4\beta 1(D_{11}\,D_{22}{}^3)^{1/4}}, & 1 \leq \Gamma \leq \infty \\ \frac{4\beta 1\sqrt{D_{22}(D_{12}+2D_{66})}}{b^2N_{xy}}, & 0 \leq \Gamma \leq 1 \end{cases} \tag{2}$$

Where the Gamma (Γ) and Beta1 (β1) parameters are calculated as a function of the laminate's stiffness. Gamma is calculated by:

$$\Gamma = \frac{\sqrt{D_{11}D_{22}}}{D_{12}+2\,D_{66}}\,. \tag{6}$$

Beta1 is interpolated from Gamma using Table 1.

Table 1. Coefficient β1 for the buckling load factor.

Γ	β1
0.0	11.71
0.2	11.80
0.5	12.20
1.0	13.17
2.0	10.80
3.0	9.95
5.0	9.25
10.0	8.70
20.0	8.40
40.0	8.25
∞	8.13

The critical buckling load for the case of joint biaxial and shear loading is approximated by [3]:

$$\frac{1}{\lambda_{c(p,q)}} = \frac{1}{\lambda_{n(p,q)}} + \frac{1}{\lambda_{s^2}} \tag{4}$$

Thus, the critical buckling load is given by the smallest factor considering $\lambda_{c(p,q)}$ e λ_s:

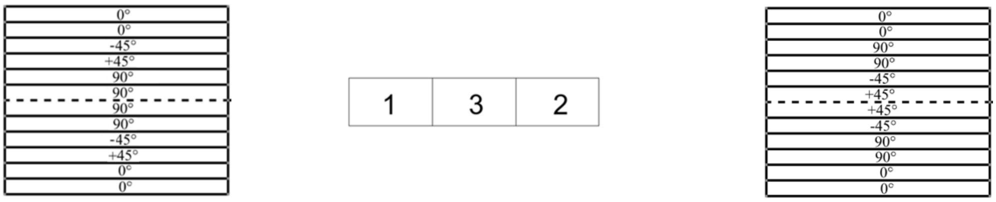
$$\lambda = \min\{|\lambda_s|, \lambda_{c(p,q)}\} \tag{5}$$

3 Genetic Algorithm for Permutation Problem

The genetic algorithm for permutation problems presented in this work, used in laminate optimization, combines sequences of individuals using gene rank crossover and introduces random modifications via mutation, which This increases the diversity of the population and avoids local solutions. Penalization ensures that solutions respect specific constraints, such as the maximum number of contiguous layers. This method is effective for finding the optimal stacking sequence that maximizes the critical buckling load, solving complex combinatorial problems with multiple constraints.

3.1 Codification

In permutation problems, the design variables represent a given permutation of a given base sequence. Thus, the individuals (laminate) consist of a sequence of integers ranging from 1 to n, where n is the number of options available. Figure 2 illustrates an example of coding an individual (genotype) and the resulting laminate obtained after performing the respective permutation (phenotype).



a) Base laminate. b) Genotype. c) Phenotype (decoded laminate).

Figure 2. Example of codification of symmetric and balanced laminate.

3.2 Gene Rank Crossover

Gene ranking crossover is an alternative crossover for permutation problems [2]. Inspired by the classification of competitors by judges, this technique combines the classifications of individuals' genes to create new configurations. The judges represent the weight assigned to each chromosome.

For example, considering three layers classified as [A, B, C] and [C, A, B], with random weights W_1 and W_2, such that que $W_1 + W_2 = 1$, the weighted classifications are combined to generate a new sequence, which is obtained after sorting the resulting rank. The procedure is discussed below.

Considering a base laminate with a specific sequence of layers, such as $[90_2/90_2/90_2/\pm45/\pm45/0_2]_s$ where its base permutation sequence is defined as [1/2/3/4/5/6]. The permutations of this sequence for two parents, described by: Permutation 1 (Parent 1): [2/5/4/3/6/1], which results in the Laminate: $[90_2/\pm45/\pm45/90_2/0_2/90_2]_s$; Permutation 2 (Parent 2): [1/2/4/5/3/6], which results in the Laminate: $[90_2/90_2/\pm45/\pm45/90_2/0_2]_s$.

O rank de cada cromossomo é dado pela posição que este ocupa no genótipo. Por exemplo, no Pai 1, o cromossomo 1 tem rank 6, pois está na sexta posição do genótipo. Assim, os ranks de cada cromossomos são dados por [6/1/4/3/2/1] e [1/2/4/5/3/6] respectivamente. Utilizando os pesos W_1=0.4634 e W_2 = 0.5366, o rank combinado é dado por:

[6/1/4/3/2/1] * 0.4635 + [1/2/4/5/3/6] * 0.5366 = [3.3170 1.5366 4.5366 3.0000 3.0732 5.5366]

The rank-ordering sequence represents the son's genotype, which is given by:

Son = [2 4 5 1 3 6], Laminate sequence: $[90_2/\pm 45/\pm 45/90_2/90_2/0_2]_s$.

It is important to note that this operator always results in a valid permutation. A second child is generated by inverting the values of W_1 and W_2.

3.3 Mutation

In mutation, variations are introduced into the children to increase the diversity of the population and avoid solutions in local minima. The code runs through all the children, generating a random number for each one. If this number is less than the defined mutation rate, a random permutation is performed on the child's genotype, changing the sequence of the layers in the laminate. This process randomly modifies the stacking configurations, allowing new solutions to be explored and increasing the genetic diversity in the population, helping to find the best solution in the optimization.

3.4 Treatment for Constraints

In the laminate optimization problem, penalties are applied to ensure compliance with the constraints. During the evaluation of the solutions, the objective function is increased based on the violations of the limitations, multiplied by a penalty factor. This penalty is applied both to the initial population and to the children generated after the crossover and mutation steps. This ensures that solutions that violate the constraints are less favored, directing the algorithm to find permutations that maximize the critical buckling load while meeting the imposed constraints. The penalized objective function is given as:

$$\varphi = \lambda + max(N_{cont} - 4{,}0)\, r^{Penalty} \quad (7)$$

Where φ is the penalized objective function, $r^{Penalty}$ is the penalization factor, adopted as 0.1 in the specific problem. N_{cont} is the highest count of adjacent repeated layers, limited to 4 in the issue dealt with in this work. It is worth noting that this restriction is only applied to the 0° and 90° layers, while the 45° blades alternate between 45° and -45°, presenting no continuity problems.

4 Numerical Example

This example deals with maximizing the critical buckling load of laminated plates with biaxial and shear loading, with layers oriented at 0°, ±45°, and 90°, and respecting the constraint of repeated layers. The optimization problem is defined considering a specific number of layers for each orientation. The objective is to determine the most effective stacking sequence, which must be symmetrical and balanced, to maximize the critical buckling load (λ). The constraint includes a maximum of four layers with the same orientation.

The results were obtained for a 24-inch square graphite-epoxy plate, with specific properties: E_1= = 18.5 × 10^6 psi (127.59 GPa), E_2= = 1.89 × 10^6 psi (13.03 GPa), G_{12}= 0.93 × 10^6 psi (6.4 GPa), thickness of each layer t_{ply}= 0.005 in (0.0127 cm), and Poisson coefficient (v_{12})= 0.3 [2].

Table 2. Quantity of piles for charging in the three orientations.

Case	N_x (lb/in)	N_y (lb/in)	N_{xy} (lb/in)	N_0	N_{45}	N_{90}	λ
1	0	-2000	1000	4	8	4	0.775
2	0	-16000	8000	8	16	8	0.768
3	-5000	-2000	1000	6	12	6	0.870

The results for the algorithms were obtained with a population of 20 individuals, an 80% crossover rate, and a 15% mutation rate, over 200 generations. The success rate was measured considering a percentage tolerance of 1%, performing 30 optimization runs, and checking how many of these runs reached the value within the specified margin. The results demonstrated the effectiveness of the method, with a high success rate and robust solutions for laminate design.

Case (1) Case (2) Case (3)

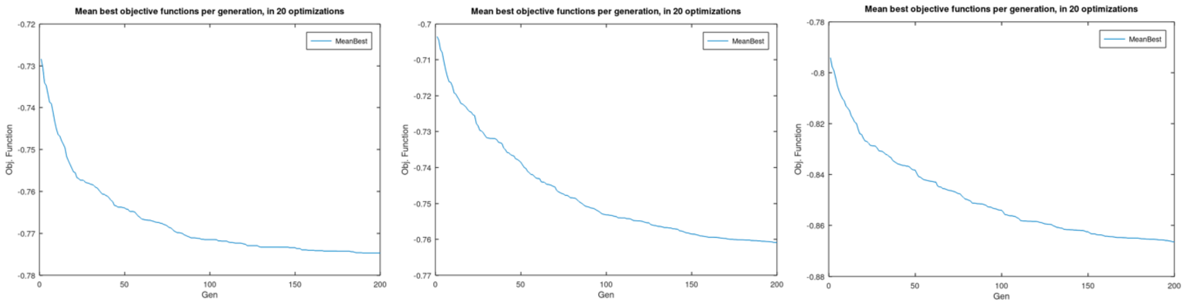


Figure 3. Average best / Gen octave output.

The graphs show the evolution of the best objective functions of 0.775 for 64 blades, 0.768 for 128 blades, and 0.870 for 96 blades over 200 generations. The success rates are 85%, 20%, and 50% respectively, indicating greater efficiency in optimization with 64 blades. All cases show a continuous reduction in the objective function, reflecting the improvement of the solutions.

Table 3. Optimal buckling factor obtained for each case.

Case	λ_c	Average of the best results λ	Stacking sequence
1	0.776	-0.775	$[(\pm45)_8/(90)_2/0/90/090/(0)_2]_s$
2	0.527	-0.762	$[(0)_8/(\pm45)_{16}/(90)_8]_s$
3	0.693	-0.870	$[0/45/90/0/45/90/0/45/90/0/45/90/0/45/90/0/45/90/0/45/90/0/45/90]_s$

5 Conclusions

This study explored the application of genetic algorithms (GAs) for optimizing composite laminates with a focus on maximizing the buckling load. A permutation genetic algorithm with gene-rank crossover was implemented and applied, which proved effective in optimizing a laminated plate, showing promising results in terms of convergence to optimal solutions.

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