Advanced Computational Modeling with FVDAM for Homogenization of Reinforced Concrete Beams

Samuel R. Silva[1], Diego S. Mendonça[1], Kevin T. L. M. Souza[1], Romildo S. E. Filho[2], Francisco P. A. Almeida[1]

[1]*Technology Center, Federal University of Alagoas*
Av. Lourival Melo Mota, 57072-900, Maceió/Alagoas, Brazil
samuel.silva@ctec.ufal.br, diego.mendonca@ctec.ufal.br, kevin.tenorio98@gmail.com, patrick@ctec.ufal.br
[2]*Arapiraca Campus, Federal University of Alagoas*
Street Floriano Rosa, 57200-000, Penedo/Alagoas, Brazil
romildo.escarpini@penedo.ufal.br

Abstract. Reinforced concrete is widely used in civil engineering due to its strength, durability, and versatility. This material is heterogeneous, as it combines concrete with steel bars, giving it a tensile strength much higher than that of simple concrete. However, the computational modeling of this type of material is challenging due to the need to consider the distinct properties of the phases, generating considerable computational costs. According to Almeida (2020) [1], the simulation process of reinforced concrete beams requires a meticulous analysis of the interactions between concrete and steel, as well as the non-linear behavior of the material under different load conditions. This complexity significantly increases the time and resources needed to perform accurate computational analyses. An alternative to modeling reinforced concrete is to replace it with a homogeneous material with properties/behaviors equivalent to those of the original heterogeneous material. The Finite-Volume Direct Averaging Micromechanics (FVDAM) method emerges as a promising and efficient alternative for obtaining the effective elastic properties of composite materials with complex microstructures. In this study, the efficacy of FVDAM in the homogenization of reinforced concrete beams was evaluated. The stiffness using this technique was compared with that obtained using the calculations presented by Pinheiro (2007) [2]. In addition, two unit cell models were compared, differentiating the shape of the fiber: square and circular (Escarpini Filho and Almeida, 2023) [3]. In this work, it is expected to understand the influence of the fiber shape in the unit cell and compare the results of FVDAM with those of Pinheiro, in terms of stiffness, evaluating issues in the context of safety.

Keywords: Reinforced concrete, computational modeling, homogenization

1 Introduction

Reinforced concrete is a material widely used in civil engineering due to its strength, durability, and versatility. However, its heterogeneous nature, resulting from the combination of concrete with steel bars, makes computational modeling challenging. The need to consider the different properties of the constituent phases generates significant computational costs. According to the works of Gattu et al. (2008) [4], Escarpini Filho and Almeida (2023) [3] and Drago and Pindera (2007) [5], Finite-Volume Direct Averaging Micromechanics (FVDAM) has emerged as a promising and efficient alternative for obtaining the effective elastic properties of composite materials with complex microstructures. Traditional methods, such as the Finite Element Method (FEM), may have limitations in accurately representing the interactions between the distinct phases of composite materials, such as fibers and matrices. This often results in inaccurate predictions of global structural behavior. FVDAM overcomes this limitation by using a micromechanical approach based on finite volume discretization. This model allows the capture of the elastic and plastic behavior of the constituent phases on a microscopic scale, providing a more accurate prediction of the structural response of materials with complex and anisotropic microstructures, such as fiber-reinforced composites.

Therefore, it is interesting to analyze its effectiveness in homogenizing the materials most used in civil construction, such as reinforced concrete, seeking to reduce computational costs.

2 Methodology

The objective of this study is to compare the results obtained by Escarpini Filho and Almeida [3], who analyzed fiber-reinforced composites with circular cross-section geometry using FVDAM, with the results obtained when using fibers with square cross-section. This comparison aims to investigate and analyze the impact of fiber geometry on the characterization of the mechanical properties of composite materials. This analysis is motivated by the geometry limitations in the software developed by the authors, which only allows square discretization, exploring the differences resulting from the modeling approaches used.

Initially, some models were made to verify the differences between the Young's moduli and Poisson's ratios of the models homogenized with the FVDAM technique, using continuous circular and square fibers involved in a matrix. In addition, values obtained using the Mechanics of Structures Genome (MSG) technique were also used, Yu [6].

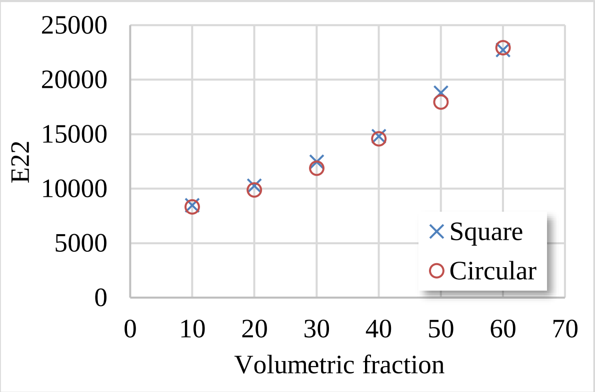
The study was then conducted on reinforced concrete models to obtain three-dimensional strength values for the homogenized model. For comparison purposes, calculations were conducted according to Pinheiro et al. [2], where the moment of inertia of a fictitious model made entirely of concrete is calculated, mixing the characteristics of steel to obtain its bending stiffness.

Therefore, two examples are presented: the study of inclusion geometry and the determination of the bending stiffness of a reinforced concrete beam, respectively.

3 Results

3.1 Geometry of fibers involved in matrix

The study compares the longitudinal and transverse moduli of elasticity, as well as the Poisson's ratio, of six diverse types of fibers in relation to the total volume. The analysis focuses on parameters v23, G23, E22 and E33 due to the evaluation of two-dimensional models, highlighting the differences between the square geometries made by the authors and the circular geometries used by Escarpini Filho and Almeida [3]. The aim is to show the impact of geometric variations on these elastic constants. Figures 1 to 4 show the numerical results obtained.



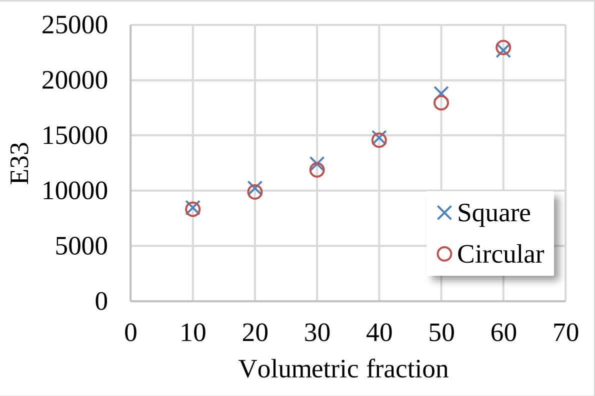
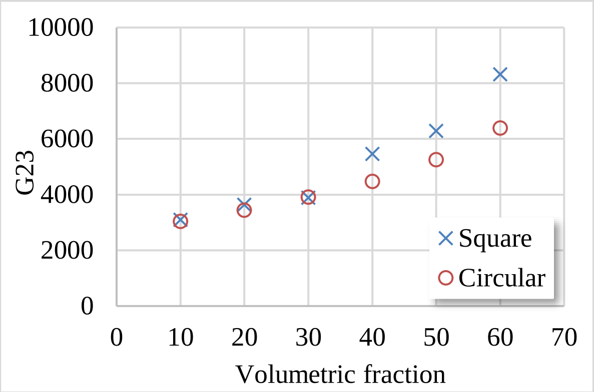


Figure 1 and 2: Variation of Young's moduli (E22 and E33) in relation to fiber volume fraction for different fiber cross-section geometries.



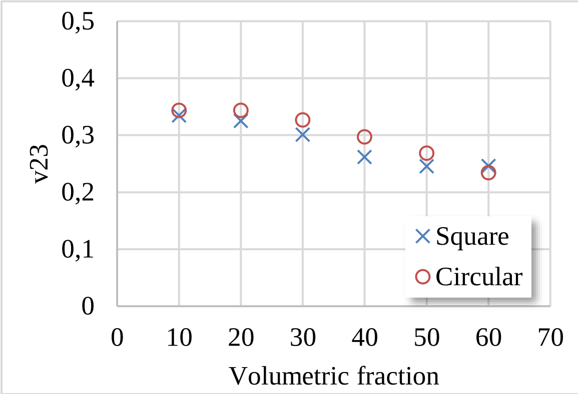


Figure 3 and 4: Variation of transverse shear modulus (G23) and Poisson's ratio (v23) in relation to fiber volume fraction for different fiber cross-section geometries.

The largest and smallest percentage differences of each parameter between the two geometries are presented in Table 1.

Table 1: Percentage Difference of Parameters

Parameters	Smallest percentual difference	Largest percentual difference
E22	1.01%	4.57%
E33	1.04%	4.49%
G23	0.57%	23.21%
v23	2.60%	11.88%

Based on the results obtained, it was found that the differences between the circular and square fiber models are minimal, with the constants G23 and v23 being the most sensitive to changes in geometry. This finding makes it possible to move towards practical application, exemplified in the modeling of a reinforced concrete beam subjected to bending using the software developed by the authors, which adopts square geometry for the cross-section of the steel bars.

3.2 Stiffness analysis of reinforced concrete beams

To investigate the effectiveness and accuracy of the FVDAM technique in this problem, two beam stiffness analyses were conducted. One approach was performed manually, using an approximate theoretical model, while the other was performed computational, using software developed by the authors. For better observation of the results, an example of a beam under bending similar to the one used by Pinheiro et al. [2] was selected, which consists of verifying the moment of inertia of stage I for the simply supported beam indicated in Fig. 5, considering a section of 22 cm x 40 cm, l = 410 cm, concrete C25, steel CA-50, longitudinal reinforcement 3φ20 (9.42 cm^2), d = 35.9 cm, class II of Environmental Aggressiveness, $E_s = 210000\ MPa$, $E_c = 0.85 * 5600 * \sqrt{25} = 23800\ MPa$.

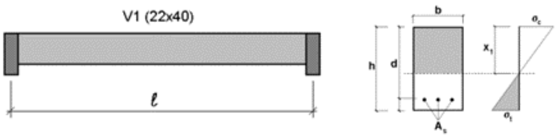


Figure 5. Reinforced concrete beam

Applying the formulation presented by Carvalho and Figueiredo Filho [7] for the theoretical calculation of beams, the following results are obtained:

$$\alpha_e = \frac{E_s}{E_c} = \frac{210000}{23800} = 8.82$$

$$X_1 = \frac{\frac{b*h^2}{2} + (\alpha_e - 1) \cdot A_s \cdot d}{b*h + (\alpha_e - 1) * A_s} = 21.23\ cm$$

$$I_1 = \frac{b*h^3}{12} + b \cdot h \cdot \left(X_1 - \frac{h}{2}\right)^2 + (\alpha_e - 1) \cdot A_s \cdot (d - X_1)^2 = 134517.92\ cm^4$$

$EI_1 = 28000 \cdot 134517.92 = 3766501760\ MPa.cm^4$(considering initial modulus of elasticity)

$EI_1 = 23800 \cdot 134517.92 = 3201526496\ MPa.cm^4$(considering secant modulus of elasticity)

Using the FVDAM technique, the model was discretized into 315 elements, Fig. 6, and the results obtained are:

E11 = 25788.79 MPa;
E22 = 24191.36 MPa;
E33 = 24176.22 MPa.

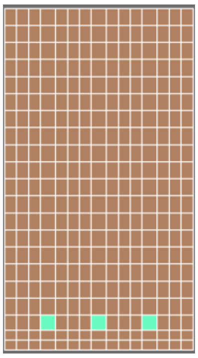


Figure 6: Discretization of the model analyzed using the FVDAM technique.

Thus, using the moment of inertia considering the theoretical model, we arrive at the following homogenized stiffness results:

$EI11 = 25788.79 \cdot 117333.33 = 3025884607.37\ MPa \cdot cm^4$;
$EI22 = 24191.36 \cdot 11733.33 = 2838452826.03 MPa \cdot cm^4$;
$EI33 = 24176.22 \cdot 11733.33 = 2836676399.41\ MPa \cdot cm^4$.

The percentage differences between the stiffness obtained from each Young's modulus are presented in Tab. 2.

Table 2: Stiffness Difference

Young's moduli	EI11	EI22	EI33
Initial	24%	33%	33%
Secant	6%	13%	13%

Thus, considering the results obtained with the secant modulus of elasticity of the theoretical model, the average difference with respect to the FVDAM was 10.67%, using the model with square cross-section steel bars.

In addition, a study was also conducted with a more discretized model containing 14904 elements. However, the results were less than 1% different from the coarse mesh model.

4 Conclusions

The comparative analysis between the use of continuous fibers with circular and square cross-sections in models applied with the Finite-Volume Direct Averaging Method (FVDAM) technique revealed that the variations are minimal. When investigating the performance of square fibers in the representation of steel bars in reinforced concrete models, it was observed that the FVDAM presented consistent results that were close to the values referenced in the literature. The divergence was acceptable, with a difference of only 6% when comparing the theoretical values of concrete stiffness using the secant modulus of elasticity (Es) with the result of the constant E11 obtained by the software developed by the authors.

These results demonstrate the robustness of the FVDAM as an effective tool for analyzing complex problems in structural mechanics. Specifically, the method demonstrates its ability to homogenize composite materials in an agile and cost-effective manner, providing a reliable platform for detailed investigations. The practical application of these findings can bring significant benefits to the advancement and optimization of structures, offering valuable insights for future designs.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] ALMEIDA, M. Análise de Elementos Finitos para Vigas de Concreto Armado. Revista de Engenharia Civil, São Paulo, v. 10, n. 2, p. 45-58, jul. 2020.

[2] PINHEIRO, Libânio M.; MUZARDO, Cassiane D.; SANTOS, Sandro P. Estados Limites de Serviço. In: PINHEIRO, Libânio M.. Fundamentos do Concreto e Projeto de Edifícios. São Paulo: Universidade de São Paulo, 2007. Cap. 14.

[3] ESCARPINI FILHO, Romildo S.; ALMEIDA, Francisco PA. Reinforced masonry homogenization by the finite-volume direct averaging micromechanics—FVDAM. Composite Structures, v. 320, p. 117185, 2023.

[4] GATTU, Mahendra et al. Parametric finite-volume micromechanics of uniaxial continuously-reinforced periodic materials with elastic phases. 2008.

[5] DRAGO, Anthony; PINDERA, Marek-Jerzy. Micro-macromechanical analysis of heterogeneous materials: Macroscopically homogeneous vs periodic microstructures. **Composites science and technology**, v. 67, n. 6, p. 1243-1263, 2007.

[6] YU, Wenbin. A unified theory for constitutive modeling of composites. Journal of Mechanics of Materials and Structures, v. 11, n. 4, p. 379-411, 2016.

[7] CARVALHO, Roberto Chust; FIGUEIREDO FILHO, Jasson Rodrigues de. Cálculo e detalhamento de estruturas usuais de concreto armado: segundo a NBR 6118: 2014. São Carlos: EdUFSCar, 2014.