Arc-shaped elements in 2D program of frames subjected to thermal loading

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Abstract. This research deals with the creation of a 2D frame program that includes circumferential and parabolical arc-shaped elements subjected to thermal loading. It was based on the Finite Element Method and written in Fortran. The elements have two nodes each and three degrees of freedom per node (both in-plane translations and out-of-plane rotation). The matrices were derived using Virtual Work and some integrals demanded a numerical approximation, where a Gaussian Quadrature approach was employed. The algorithm was validated against structures designed and solved by the authors using traditional methods and analysis software in which curves are approximated using a great number of straight elements. The results for rotations, displacements, reactions, bending, normal and shear forces are seen to be accurate at a less demanding input data in comparison to others programs.

Keywords: Thermal Load; Arc-shaped Elements; Gaussian Quadrature; Finite Element Method; Fortran.

1 Introduction

Structures are physical systems capable of receiving and transmitting forces, such as bridges, buildings, and towers. One of the main objectives of structural analysis is to relate external actions to displacements, support reactions and stresses so that any disturbances in the behavior of the material and/or the structure can be identified (Soriano and Lima [1]). According to Martha [2], temperature variation does not cause internal stresses in isostatic structures, as they have the exact number of constraints needed to ensure stability. Nevertheless, in hyperstatic structures, the temperature causes deformations and stresses, which are relevant to their design.

As stated in NBR 6118 [3], temperature variation in a structure is considered to be a uniform effect caused by atmospheric temperature and direct insolation and, thus, depends on its global location and the dimensions of its structural elements. Therefore, an alternative for investigation is to use representative mathematical models of the physical phenomenon present in a given application, which makes it possible to iterate the process using numerical methods and allows parametrical analysis, as stated by Pettres *et al.* [4].

When it comes to modeling arches, both isostatic and hyperstatic, a series of straight elements can approximate the structure's actual geometric shape. However, this approach can lead to computationally inefficient analysis in specific load and contour circumstances. In contrast, dividing the domain into curved elements (circumferential and parabolic) to determine the forces and displacements of the structure is more efficient. Studies such as those conducted by Marquis and Wang [5] and Dorfi and Busby [6] demonstrate that the use of curved elements can lead to more precise results than with straight elements.

Nowadays, performing structural analysis tasks with computer graphics and numerical models is unavoidable. In this context, this article presents the creation of another module for *PAE* (software in development since 2018 addressing unconventional types of structures with arcs, variable inertias, etc.) based on the finite element method and written in Fortran, capable of obtaining displacements, internal forces and support reactions in straight, circumferential and parabolic elements subjected to thermal loading.

2 Methodology

The development of the algorithm began with the analysis of straight elements due to the vast bibliographic reference, as can be seen in Martha [2] and Assan [7]. The internal transverse displacement related to thermal variation was assumed to be zero. It was also considered that the finite element can be heated or cooled on both upper and lower surfaces, taking into account the position of its center of gravity. Figure 1 illustrates the elements characterized by a straight axis, a circumferential arc, and a parabolic arc, respectively.

The flexibility matrix of the finite element was obtained through the Force Method, and then, using its inverse and the principles of Betti's Theorem, its stiffness matrix. The latter presents itself with unusual large components due to the complexitiy of their integrals which combine bending, axial and shear contributions and can be derived from Soriano and Lima [1], Martha [2] and Assan [7], and therefore are not included here. The Principle of Virtual Work was then employed to calculate the displacements and rotations. Finally, with the solution of this system, the vector of nodal forces corresponding to the temperature variation was obtained. To ensure the accuracy of the results, the Gauss-Legendre numerical method, known for its ability to approximate integrals, was implemented to calculate all the integrals in the code, there by minimizing errors and enhancing the reliability of the module.

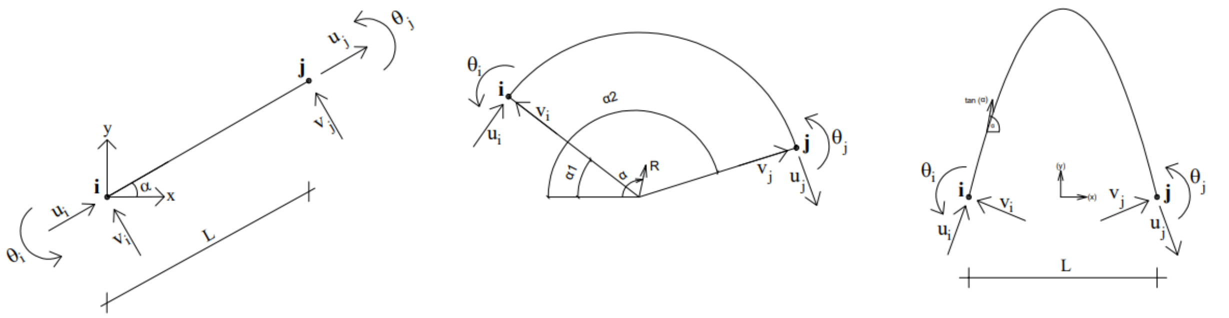

Figure 1. Generic representation of the elements: straight, circumferential and parabolic

The program requires an input file in *.txt* format for structural analysis containing information such as the nodal position, number of elements, concentrated and distributed loads, constraints, material and geometric properties, temperature, angles, and radius. Lima and Soares [8] used a similar methodology for the finite element analysis of grillages. However, to make the software accessible to the academic community, it is possible to implement it graphically, as shown in the research developed by Oliveira *et al.* [9].

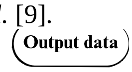

Figure 2) :

- Calculation of the element's length and orientation;
- Assembly of the local stiffness matrix;
- Assembly of the global stiffness matrix;
- Assembly of the global load vector;
- Application of the boundary conditions to the stiffness matrix and load vector;
- Calculation of the vector corresponding to the displacements;
- Determination of internal forces;
- Determination of support reactions;
- Output file (in *.txt* format).

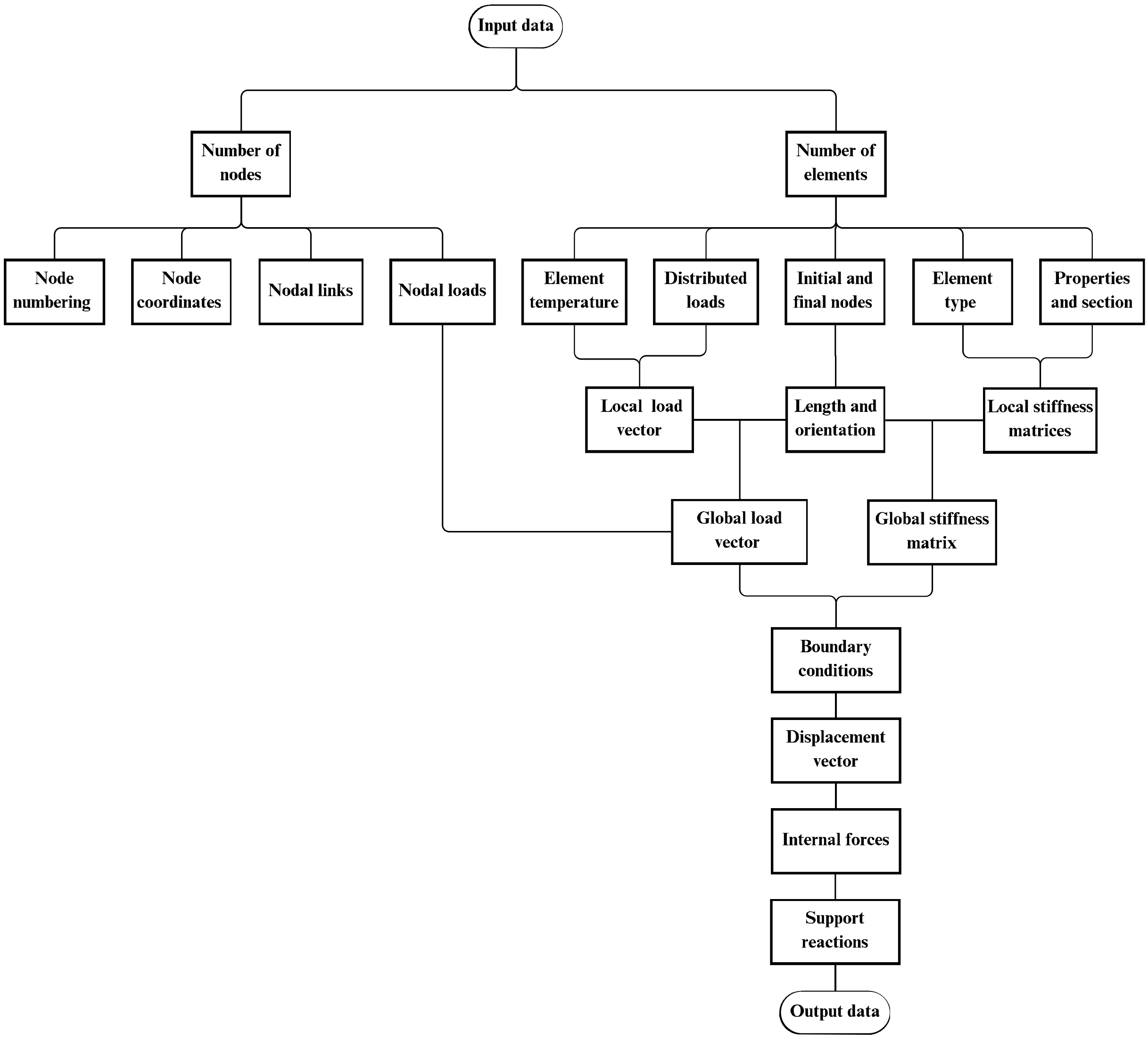

Figure 2. Programming flowchart

3 Results

Figure 3 represents a structure inspired by the Juscelino Kubitschek Bridge, located in Brasilia, the capital of Brazil. This unique structural masterpiece has four supports and three spans supported by asymmetrical arches located in different planes. The example presented here includes two circular arch elements, one parabolic arch element, and a straight beam subjected to temperature variation, uniformly distributed loads, and nodal loads. It should be noted that the example is not an idealization of the structural model but rather a verification of the efficiency of the code developed.

The input data is shown in Fig. 4. For circumferences, the initial and final angles and radius are required. For parabolic arcs, the coefficients of the quadratic function are needed. Table 1 shows the results for the proposed structure with *PAE* and *FTOOL* solution.

For the purpose of comparison, the structure was discretized in *FTOOL*, a process that was both time-consuming and laborious due to the presence of only straight elements. For this task, 185 points were used. Despite the similarity of the results, the *PAE* module significantly streamlines the structural analysis process, making it more efficient, simple, and straightforward.

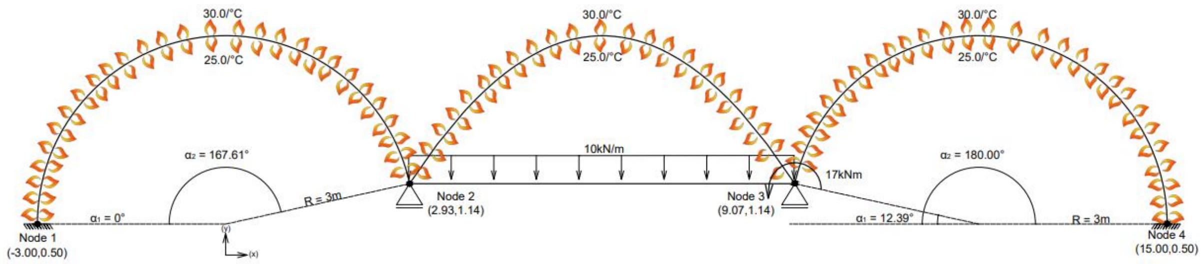

Figure 3. Structure used for validation

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|Nº NÓS| & |Nº ELEMENTOS| DA ESTRUTURA EM ANÁLISE

   4           4

 |NÓ|    |COORDENADA X|    |COORDENADA Y|    |Fx|    |Fy|    |MOMENTO Z|    |VINCULAÇÃO X|    |VINCULAÇÃO Y|    |GIRO Z|

  1        -3.00             0.5          0.00    0.00       0.00             1               1             1
  2         2.93             1.14         0.00    0.00       0.00             0               1             0
  3         9.07             1.14         0.00    0.00      17.00             0               1             0
  4        15.00             0.5          0.00    0.00       0.00             1               1             1

|ELEMENTO| |TIPO| |NÓ INICIAL (i)| |NÓ FINAL(j)| |PSI|     |G|          |E|       |Iz|    |S|    |Qxi|    |Qxj|   |Qyi|   |Qyj| |TEMP INF| |TEMP SUP| |ALTURA| |COEF DILATAÇÃO| | RÓTULA|

  1         1          1              2         1.2   10416666.67  25000000.0  0.01   0.12   0.00    0.00    0.00    0.0   25.00     30.00     1.00      0.00001        0
  2         2          2              3         1.2   10416666.67  25000000.0  0.01   0.12   0.00    0.00    0.00    0.0   25.00     30.00     1.00      0.00001        0
  3         1          3              4         1.2   10416666.67  25000000.0  0.01   0.12   0.00    0.00    0.00    0.0   25.00     30.00     1.00      0.00001        0
  4         0          2              3         1.2   10416666.67  25000000.0  0.01   0.12   0.00    0.00  -10.00  -10.0    0.00      0.00     1.00      0.00001        0

------------------------------------------------ ARCO DE CIRCUNFERÊNCIA ------------------------------------------------
|ELEMENTO|                 |RAIO|                 |ÂNGULO INCIAL|                 |ÂNGULO FINAL|
------------------------------------------------------------------------------------------------------------------------
  1                         3.00                       0.00                            167.61
  2                         0.00                       0.00                              0.00
  3                         3.00                      12.39                            180.00
  4                         0.00                       0.00                              0.00
------------------------------------------------------------------------------------------------------------------------

-------------------------------------------------- ARCO DE PARÁBOLA ----------------------------------------------------
|ELEMENTO|                  |A|                     |B|                          |C|
------------------------------------------------------------------------------------------------------------------------
  1                         0.00                     0.00                          0.00
  2                        -0.25                     3.00                         -5.50
  3                         0.00                     0.00                          0.00
  4                         0.00                     0.00                          0.00
------------------------------------------------------------------------------------------------------------------------
```

Figure 4. Input Data

Table 1. Results

Program	Node	Support Reaction Fx (kN)	Support Reaction Fy (kN)	Support Reaction Mz (kNm)	Displacement Dx (m)	Displacement Dy (m)	Rotation Rz (rad)
PAE	1	53.75	5.01	-106.34	0.00	0.00	0.00
FTOOL		53.79	5.01	-106.39	0.00	0.00	0.00
PAE	2	0.00	27.93	0.00	-8.860E-07	0.00	-4.211E-05
FTOOL		0.00	27.93	0.00	-8.783E-07	0.00	-4.220E-05
PAE	3	0.00	24.18	-0.00	5.846E-05	0.00	7.681E-05
FTOOL		0.00	24.18	0.00	5.841E-05	0.00	7.689E-05
PAE	4	-53.75	4.28	107.36	0.00	0.00	0.00
FTOOL		-53.79	4.28	107.42	0.00	0.00	0.00

4 Conclusions

The effects of temperature are significant and must be considered, especially in hyperstatic models. The advantages of having a dedicated element for curved bars are noted specially during discretization, where curved structures demand numerous straight elements whilst a circumferential arc-shaped element, for example, would only require two angles and one radius, enabling a significantly reduction in processing time.

Thermal variation causes deformations and stresses that can generate additional stresses and, consequently, when not considered in design, can cause structural pathologies. Therefore, the algorithm developed in this paper allows the calculation of displacements, internal forces, and support reactions in straight and curved axis structures subject to temperature variation. The main objective was to include this command in *PAE* in order to promote its use by students and professionals in a simple, efficient, and freely available way.

Acknowledgements. The authors would like to acknowledge FAPEMA (Maranhão Research Foundation) and IFMA (Federal Institute of Maranhão) for financing this research.

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