# Comparison between simplified and refined models for deflection in continuous reinforced concrete T-beams

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**Abstract.** This work performs a comparative study of different simplified methods and finite element (FE) models used to calculate RC beams' short-term deflections. Simplified methods are the one recommended by Brazilian Code ABNT NBR 6118, and the Bilinear method recommended by the fib Model Code. Two FE models are adopted, in which the material nonlinearities are considered employing moment-curvature diagrams, based on the simplified methods. The short-term deflections obtained by the different models are compared for eleven examples of continuous RC T-beams at service, by varying the beam geometry, reinforcing ratio and the compressive strength of concrete. At the end of this work, differences between the models are pointed and recommendations regarding the use of them are drawn for this specific section type.

**Keywords:** Deflections; T-beams; Reinforced concrete.

## 1 Introduction

The Brazilian Code NBR 6118:2023 [1] provides for, in addition to safety verifications (structural safety - ULS), functionality, and durability verifications (Serviceability Limit State – SLS). Among the SLS verifications for elements subject to bending, such as beams, it is necessary to analyze the maximum vertical displacement, known as deflection. The deflections must be predicted using a model that considers concrete cracking along the element (short-term deflection) and those due to the effects of concrete shrinkage and creep (long-term deflection), as these are the main factors influencing the final deflection value.

Junges [2] developed the AVSer program for the analysis of reinforced concrete (RC) beams in service conditions. Using the program, Junges and La Rovere [3] conducted a comparative study of deflection calculation methods applied to continuous RC beams with rectangular sections. The studied models included the one recommended by NBR 6118:2023, based on the method proposed by Branson [4], and the bilinear method from the CEB Manual [5], as well as two refined models using the finite element method (FEM) formulation. Continuing this work, Cruz et al. [6], [7] extended the program and conducted a comparative study for the calculation of total deflections in simply-supported and continuous beams with rectangular cross-sections. Finally, Silva and Laure [8] expanded the studies to simply-supported beams with "T" cross-sections.

A T cross-section beam present greater stiffness compared to a rectangular section of equal height and area. The concentration of more area at one end of the section raises questions about whether this type of section exhibits the same behavioral response as the rectangular section when applying the models for predicting deflection as indicated by Codes. In the case of continuous beams, there is also the presence of negative moments at intermediate supports, increasing the cracked extent of the beam. Continuing the aforementioned works, this article presents an initial comparative study between the short-term deflection calculation methods mentioned above, focusing on continuous T-beams. Thus, the aim is to observe the response of the previously studied models for this beam typology and compare their response to the findings made earlier for beams with rectangular cross-sections.

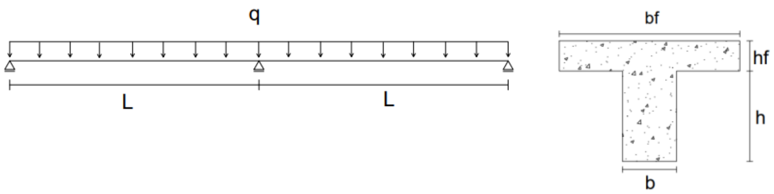
## 2 Methods to calculate short-term deflections in RC beams

The method based on Branson's model [4], indicated by NBR 6118:2023, referred to, in this work, as Branson-NBR 6118, adopts for deflection calculation an equivalent stiffness obtained through a weighting of the inertias in stages I and II, to consider the contribution of concrete between cracks in the section's stiffness. For the case of continuous beams, a weighted stiffness value is calculated according to the sections of positive and negative moments present in each span. Similarly, the simplified Bilinear method from the CEB Manual [5] consists of calculating an average value between the deflection calculated with the stiffness of stage I and that calculated with the pure stage II stiffness, using a distribution coefficient.

As described in Junges [2], the refined models consist of discretizing the beam into several small-length elements, using matrix analysis formulation to obtain nodal displacements and forces. For each element, the secant stiffness ($EI_{sec}$) is calculated from the moment-curvature diagram, using Branson's equation for the method named MEV-Branson, or from the simplified method described in the fib Model Code [9], referred to as MEV-Bilinear. The secant iterative method is used to achieve convergence in the solution of the nonlinear equilibrium equations.

## 3 Methodology

The models studied are implemented in the AVSer program (Analysis of Deformations and Efforts of Reinforced Concrete Beams in Service). For the comparative study, theoretical beams are analyzed, due to the limitation in the literature of experimentally tested continuous T-beams. Three groups were designed according to the guidelines of NBR 6118:2023, totaling 11 (eleven) two-span beams, considering class II of environmental aggressiveness. The geometry of the beams and the load typology are represented in Fig. 1.




| Dimension | Group VC-1 | Group VC-2 | Group VC-3 |
|---|---|---|---|
| L [m] | 5 | 5 | 5 |
| b [cm] | 18 | 18 | 18 |
| bf [cm] | 60 | 60 | 50 a 18 |
| h [cm] | 30 | 30 | 30 |
| hf[cm] | 10 | 10 | 10 |
| fck [MPa] | 20 | 25 a 40 | 20 |

Figure 1- Loading type, beam dimensions, and geometry.

Table 1- Details of load and longitudinal reinforcement for the two-span beams

| Beam | q(kN/m) | Cracked span (%) | Positive Section | | | | Negative section | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | As | | As' | | As | | As' | |
| | | | Bars | cm2 | Bars | cm2 | Bars | cm2 | Bars | cm2 |
| VC-11 | 18.05 | 67.40 | 6 φ 10 | 4.70 | - | - | 6 φ 12.5 | 7.4 | - | - |
| VC-12 | 25.65 | 78.00 | 5 φ 12.5 | 6.20 | - | - | 9 φ 12.5 | 11.07 | - | - |
| VC-13 | 33.25 | 83.00 | 4 φ 16 | 8.00 | - | - | 9 φ 16 | 18.1 | 2 φ 16 | 4 |
| VC-14 | 40.85 | 86.40 | 5 φ 16 | 10.10 | - | - | 7 φ 20 | 21.99 | 2 φ 16 | 4 |
| VC-21 | 33.25 | 80.20 | 4 φ 16 | 8.00 | - | - | 8 φ 16 | 16.9 | - | - |
| VC-22 | | 77.60 | 4 φ 16 | 8.00 | - | - | 8 φ 16 | 16.9 | - | - |
| VC-23 | | 74.80 | 4 φ 16 | 8.00 | - | - | 8 φ 16 | 16.9 | - | - |
| VC-24 | | 72.40 | 4 φ 16 | 8.00 | - | - | 7 φ 16 | 14.08 | - | - |
| VC-31 | 33.25 | 83.60 | 4 φ 16 | 8.00 | - | - | 8 φ 16 | 16.9 | - | - |
| VC-32 | | 84.40 | 4 φ 16 | 8.00 | - | - | 8 φ 16 | 16.9 | 2 φ 16 | 4 |
| VC-33 | | 87.00 | 5 φ 16 | 10.10 | 2 φ 16 | 4.00 | 8 φ 16 | 14.08 | 5 φ 16 | 10.1 |

The first group consists of beams with the same geometry but varying applied loads. The same steel properties for reinforcement are adopted for all beams, with yield strength $f_{yk}$= 500 MPa and modulus of elasticity $E_s$= 21000 MPa. In the second group, $f_{ck}$varies from 25 to 40 MPa, keeping the same geometry as Group 1. The third group consists of beams with varying flange widths, ranging from 50 to 18 cm. Therefore, through the first group, the aim is to analyze the influence of the variation in load/reinforcement ratio on the short-term deflection results; through the second group, the influence of $f_{ck}$; and in the third group, the contribution of the T-beam flange. Tab. 1 presents the value of the service distributed load $q$ for each beam, as well as the amount of reinforcement in the sections of maximum positive and negative bending moments. The analyses were performed by discretizing the beams into 10 cm elements. For the analyses, an incremental process was carried out by dividing the total load into ten increments, allowing for the creation of load-deflection curves.

# 4 Results and Discussion

Table 2 presents the short-term deflection values of each beam calculated by the different methods analyzed. The rightmost columns show the percentage differences of the models in relation to the Branson-NBR method. It can be observed that the two refined models presented a similar average percentage difference (around 11%) compared to the Branson-NBR, with more conservative deflection results. The Bilinear method showed a considerable difference (22.4%), with generally lower values compared to the other methods.

Table 2- Short term deflection of all beams.

| Beam | Short-term deflection [cm] | | | | Diference (%) | | |
|---|---|---|---|---|---|---|---|
| | BransonNBR | MEVBranson | Bilinear | MEVBilinear | MEVBranson | Bilinear | MEVBilinear |
| VC-11 | 0.209 | - | 0.353 | - | - | 68.33 | - |
| VC-12 | 0.462 | 0.547 | 0.467 | 0.509 | 18.31 | 1.08 | 9.24 |
| VC-13 | 0.596 | 0.616 | 0.477 | 0.564 | 3.26 | -19.98 | 5.76 |
| VC-14 | 0.678 | 0.679 | 0.524 | 0.624 | 0.14 | -22.73 | 8.68 |
| VC-21 | 0.524 | 0.573 | 0.457 | 0.535 | 9.24 | -12.84 | 2.05 |
| VC-22 | 0.451 | 0.525 | 0.439 | 0.502 | 16.26 | -2.84 | 10.01 |
| VC-23 | 0.382 | 0.478 | 0.422 | 0.470 | 25.05 | 10.33 | 18.65 |
| VC-24 | 0.325 | 0.446 | 0.449 | 0.456 | 37.21 | 37.96 | 28.59 |
| VC-31 | 0.649 | 0.657 | 0.515 | 0.603 | 1.18 | -20.75 | 7.66 |
| VC-32 | 0.695 | 0.688 | 0.529 | 0.632 | -0.99 | -23.89 | 9.96 |
| VC-33 | 0.755 | 0.738 | 0.563 | 0.680 | -2.27 | -25.45 | 11.03 |
| | | | | Average | 11.4 | 22.4 | 11.2 |

As expected, in the beams of Group 1, the greater the applied load, the greater the deflection. An important observation is that the differences between the models did not remain constant across all the beams in the group. For example, Fig. 2 shows that for beam VC-12, despite curves with different trajectories, the two simplified methods reached very close final values, which were still the lowest estimated values among the models. However, for VC-14, the Branson-NBR method yielded a value very close to MEV-Branson, with these being the highest among the methods. The MEV-Branson method presented the most conservative deflection values for this group of beams. The simplified bilinear method generally presented the lowest values, despite a stiffer slope in the post-cracking curve of the concrete; however, since the onset of cracking occurs at a lower load value in this method, the final deflection value is not significantly reduced compared to the other methods. Beam VC-11 did not converge for the refined methods, and new analyses will need to be conducted to identify the cause.

In comparison to the analyses of rectangular cross-section beams conducted by Junges and La Rovere [3], some similarities and differences can be noted. Unlike the observations in the present study, the MEV-Branson method generally presented the lowest values. The simplified Bilinear method exhibited the same behavior, with a stiffer curve, sometimes resulting in lower values and other times higher. In the cited study, the simplified NBR 6118 model generally presented higher and more dispersed results compared to the MEV-Branson, contrary to what was observed in the group of T-section beams shown here, where the two methods tend to converge, especially for the two most heavily loaded beams in the group.

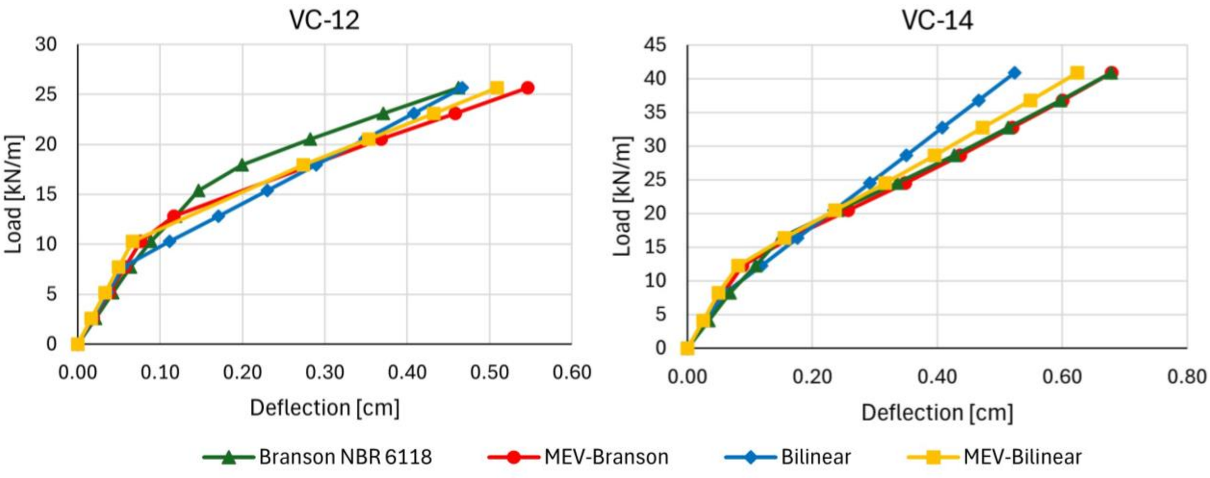



Figure 2 - Load-deflection graphs for beams VC-12 and VC-14

Figure 3 shows the deflection results obtained for the Branson-NBR model for all beams in Group 2 (left graph), where $f_{ck}$ varies, and Group 3 (right graph), where the flange width varies. It can be seen that for both groups, the stiffness (slope of the curves) of the beams after concrete cracking is very similar within each group; however, what differs most is the onset of cracking, determined in the models by the cracking moment $M_r$. This depends on the tensile strength of the concrete and the moment of inertia of the cross-section. Thus, the lower the $f_{ck}$ , the lower the tensile strength, and the narrower the flange, the lower the moment of inertia of the section, causing the concrete to crack at lower load values. It is also observed that the relative difference between the final deflection values of the beams in Group 2 is slightly greater than that of Group 3. In Group 2, the refined MEV-Branson and MEV-Bilinear methods resulted in very similar values, as shown in Table 2, with a percentage difference between the two averaging 3.68%.

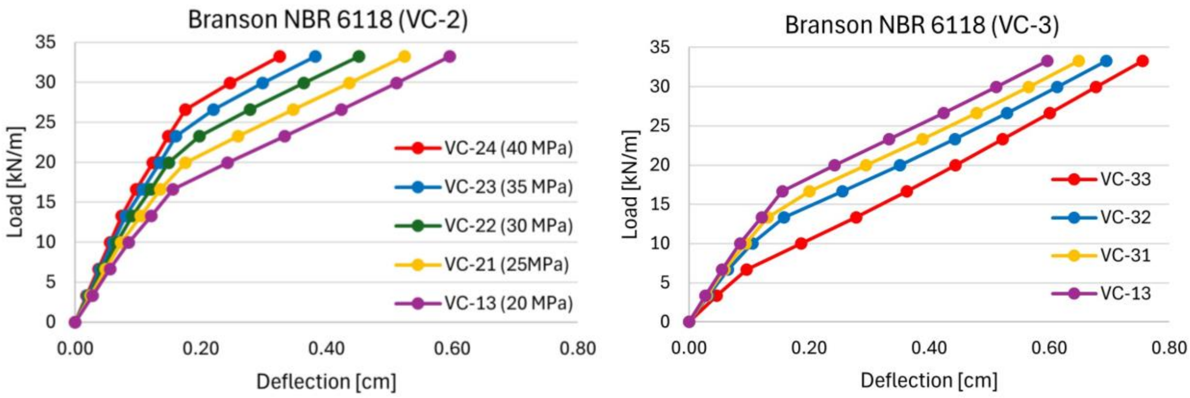



Figure 3- Load-deflection graph for Group 2 and Group 3 beams according to the Branson NBR 6118 method

Through the load-deflection graphs of each beam, as shown in Fig. 4 for beams VC-32 and VC-33, it is observed that in Group 3, the variation of the flange size did not alter the relative results between one model and another; the Bilinear method presented the lowest values, and the two variants of the Branson equation showed higher and similar results. This group included one more beam; however, since it presented some issues in the results that have not yet been clarified, it was omitted from this work.

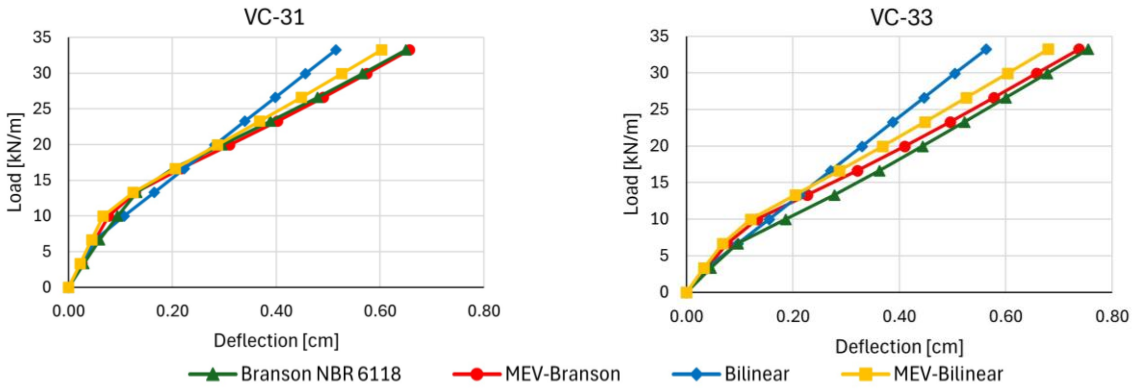



Figure 4- Load-deflection graphs for beams VC-31 and VC-33.

# 5 Conclusions

The initial study presented demonstrates the influence of the variation in reinforcement ratio, concrete compressive strength, and the presence of a flange in the cross-section on the short-term deflection value of the continuous beams studied. From Group 1, it was observed that the reinforcement ratio strongly influences the beam's deflection and affects the model's response, with a change in relative behavior among the studied models as the load and reinforcement increase. Unlike what was observed for continuous rectangular section beams, the simplified method recommended by NBR 6118 showed results closer to the refined MEV-Branson model. This is believed to be due to the high level of cracking in the beams studied here. On the other hand, the Bilinear method, similar to the case for rectangular section beams, presented a stiffer response, with generally lower values than the other methods. For more conclusive results, it will be necessary to expand the studies by analyzing more beams and adopting a more refined model to serve as a reference for comparison among models.

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