Comparison of Operational Modal Analysis Methods for Aerospace Structures

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Abstract.

Dynamic loads are a significant challenge to any structure projected by engineers, among these vibrations are of special concern as they can lead to damage or even failure if not controlled. Understanding the effects of these loads is crucial for ensuring structural safety. Operational Modal Analysis (OMA) is a crucial tool for this purpose, as it assesses the modal parameters (natural frequencies, damping ratios, and mode shapes) by measuring the structural responses to operating conditions. This data is then analyzed in to create the time-domain correlation functions and the frequency-domain power spectra density (PSD) functions, from which it is possible to extract modal parameters through different methods like curve-fitting techniques. OMA stands out from other methods like Experimental Modal Analysis (EMA) because it uses real loading and boundary conditions without disrupting the system's operation. This paper compares various OMA methods, specifically the numerical subspace identification (N4SID) and stochastic subspace identification (SSI) techniques: SSI-COV and SSI-DATA. Initially, the methods are validated using a steel cantilever beam (SCB) model, with results compared to theoretical and simulated data. Subsequently, the methods are applied to an aerodesign and drone structure. Overall, the paper investigates the application of these OMA methods to aerospace structures to reveal their dynamic characteristics.

Keywords: OMA, Aerospace, Aeronautics, Validation, Modal

1 Introduction

Brincker [1] defines Operational Modal Analysis (OMA) as the study of modal properties of systems under ambient or normal operating conditions. OMA gained relevance in the mid-1990s with the introduction of SSI algorithms by Van Overschee and De Moor [2] and has since been recognized as a reliable technique. It analyzes system responses to ambient vibrations, which can be due to the system's operation or external sources. This allows OMA to evaluate real loadings and boundary conditions. However, because it relies on ambient vibrations, OMA cannot directly construct frequency response functions (FRF) and instead uses correlation and PSD functions to extract modal parameters. Brincker [1] highlights OMA's importance in determining natural frequencies, verifying analytical models, assessing dynamic responses under various conditions, and monitoring structural health. In aerospace and aeronautical industries, OMA is particularly valuable as it can address phenomena difficult to replicate in a laboratory setting. This work explores different OMA techniques: SSI-DATA, SSI-COV, and N4SID, starting with a steel cantilever beam (SCB) for methodological validation and then applying them to more complex aerospace and aeronautical structures.

2 Theoretical Framework

In contrast to EMA, which measures both input and output to create frequency response functions (FRFs) and extract modal parameters, OMA is an output-only technique that relies on ambient vibrations as the input, which is unknown and assumed to be in the format of Gaussian white noise. This assumption allows OMA to model the structure and extract modal parameters without direct input measurements. Output measurements are used to create statistical tools called correlation functions in the time-domain, which are then converted into power spectral density (PSD) functions in the frequency-domain, identifying dominant frequencies indicating modal presence. However, due to the use of correlation functions OMA does not provide modal participation factors nor scaling.

Covioli [3] details the fundamental equations for both frequency-domain (eq. 1) and time-domain (eq. 2) OMA techniques. Equation 1 forms the basis for the frequency domain OMA techniques. By taking the inverse Fourier Transform (IFT) of eq. 1 it is possible to obtain the output correlation function matrix $\mathbf{R}_{yy}(\tau)$, for positive and negative time lags, in which t_s is the sampling period. Equation 2 forms the basis for the time domain OMA techniques, as it shows that the output correlation function (for $\tau > 0$) can be expressed as a sum of decaying sinusoids.

$$G_{y_i y_j}(\omega) = \sum_{n=1}^{N} \left[\tfrac{\varphi_i^{(n)} + \psi_j^{(n)}}{(j\omega - \lambda_n)} + \tfrac{\varphi_i^{(n)*} + \psi_j^{(n)*}}{(j\omega - \lambda_n^*)} + \tfrac{\psi_i^{(n)} + \varphi_j^{(n)}}{(-j\omega - \lambda_n)} + \tfrac{\psi_i^{(n)*} + \varphi_j^{(n)*}}{(-j\omega - \lambda_n^*)} \right] \tag{1}$$

$$R_{y_i y_j}(\tau) = \begin{cases} \sum_{n=1}^{N} (\varphi_i^{(n)} \psi_j^{(n)T} e^{\lambda_n \tau t_s} + \varphi_i^{(n)*} \psi_j^{(n)H} e^{\lambda_n^* \tau t_s}) for \tau \geq 0 \\ \sum_{n=1}^{N} (\psi_i^{(n)} \varphi_j^{(n)T} e^{\lambda_n |\tau| t_s} + \psi_i^{(n)*} \varphi_j^{(n)H} e^{\lambda_n^* |\tau| t_s}) for \tau < 0 \end{cases} \tag{2}$$

2.1 Reasons to use OMA

Practical/Size Limitations: Large structures are difficult to excite with traditional methods. Instead, OMA uses natural in-situ loads and accelerometers to measure structural response under the real operating conditions. ***Real-World Conditions:*** OMA accounts for actual operating conditions, providing more accurate data than laboratory tests, which may not capture real-world non-linearities. ***Structural Health Monitoring/Damage Detection:*** OMA can detect changes in modal parameters indicative of wear or damage without disrupting operations, useful for ongoing monitoring and post-event (like earthquakes) assessments.***Cost:*** OMA is more cost-effective than traditional methods (e.g., using shakers and impact hammers) as it requires less expensive equipment and avoids operational downtime.

2.2 OMA Techniques Used

Stochastic Subspace Identification - SSI: The SSI techniques are the most important time-domain techniques, as it "allows the identification of an effective state space model for a complex dynamic system subjected to stochastic excitation directly from measured data" (Van Overschee [2]). This method reduces problems of computational complexity which makes it faster when compared to other OMA techniques. SSI has two main algorithms from which it bases the identification of modal parameters: data-driven SSI (SSI-DATA) and covariance-driven SSI (SSI-COV).***Numerical Algorithm for Subspace Identification – N4SID:*** According to Li [3], the N4SID technique is a time-domain alternative to the classical system identification method based on iterative approaches. The key step of this method is the oblique projection of subspaces generated by the block Hankel matrices formed by input/output data of system. Other geometric and mathematics tools of linear algebra like singular value decomposition are used to extract the order of the system and the observability matrix which contain the parameters of the estimated model.

2.3 OMA Limitations

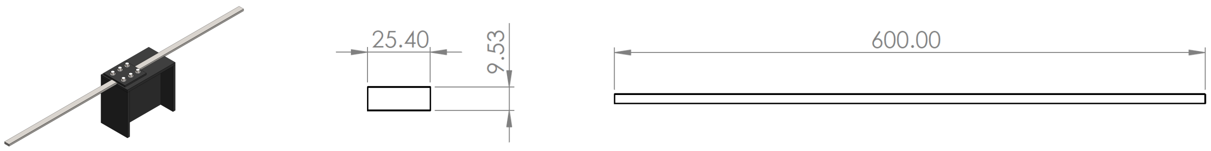
According to Bin Zahid et al. [4] and Magalhães [5], the main limitations of the OMA techniques are: ***Unscaled Mode Shapes:*** OMA provides unscaled mode shapes due to not measuring the inputs, which impact the model accuracy and sensitivity analysis to determined forces. ***Excitation Requirements:*** As OMA methods require input in Gaussian white noise format, certain types of excitations like harmonic excitations, can lead OMA methods to failure. In recent years new techniques have been developed to overcome this limitation, with the creation of new techniques such as OMAX which is according to Guillaume [6] a "unifying approach combining experimental and operational modal analysis."

3 Methodology

3.1 OMA validation

The first part of this work consisted of the OMA procedure validation. A SCB (Fig. 1) was used as the system to be studied due to the simplicity of its geometry and the consolidated theory behind its dynamic characteristics.

The validation was divided into four parts:



(a) SCB model (b) Cross section (mm) (c) Beam length (mm)

Figure 1. SCB model used for the OMA validation, the material properties correspond to the AISI 1020 steel.

1. Theoretical analysis: the first four natural frequencies and mode shapes of the SCB were obtained through the Euler-Bernoulli beam theory equations, which would be used as the basis of comparison for all the other methods.

2. Simulated modal analysis: A beam element model (BEM) of the SCB was created to perform a simulated modal analysis to obtain its first four natural frequencies and mode shapes in the software ANSYS® Mechanical. The beam of 600 mm was divided into 6 parts and 7 nodes equally distributed at 100 mm to create the mesh.

3. Simulated OMA: Using the BEM, two transient simulations were performed using a white noise (10N) and impulse (100 N) excitations on the tip of the SCB, setting a damping ratio of 1% as a common value for structural steel. The nodes created in the BEM were used as the measurement points, from which the output acceleration data was extracted at a sampling frequency of 8192 Hz and used for an OMA using the N4SID, SSI-COV and SSI-DATA methods in the MATLAB® Output-Only Modal Analysis (OoMA) Toolbox and the Siemens® Simcenter Testlab 2306® software from which all the modal parameters were extracted.

4. OMA on the SCB: OMA tests were performed using an impact force on the tip of the beam. The outputs were recorded with a PCB® 35A21 accelerometer using the BEM nodes positions from which the modal parameters were extracted with the software previously mentioned. However, due to limitations in the acquisition equipment, a reference accelerometer was not available which had a significant impact on the mode shapes. A manual synchronization was necessary, which was possible due to the nature of the impact force, which allowed to synchronize the measurements at the moment of the impact. However, other types of forces like white noise are impossible to manually synchronize. Once the four steps were concluded, a comparison of the modal parameters and MAC analysis was made between all the results to observe if the OMA techniques applied were able to extract the modal parameters from the SCB with accuracy.

3.2 OMA in other structures

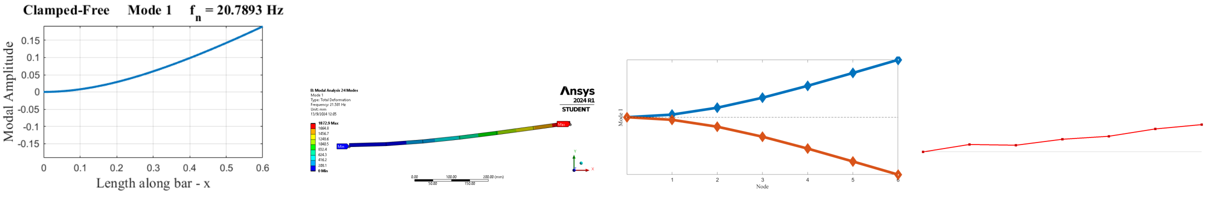
1. Mamutes' Barbie aerodesign: The Barbie aerodesign from the Mamutes competition team was tested using the excitation from the motor (without its propeller), which was suspended in the air to recreate a free-free condition as it was not possible to fly the plane due to limitations of the measurement equipment. The acceleration recording was done at a sampling frequency of 4196 Hz for a total time-length of 15 seconds along all the measurement points defined for the structure.

2. EDRA's Hyarra Drone: The Hyarra drone of the EDRA competition team was tested using the four motors as the excitation for the structure, however due to technical problems only one propeller was available during the test. The drone's frame was fixed in place to prevent it from flying and the recording was done at a sampling frequency of 4196 Hz for a total time-length of 08 seconds across all the measurement points defined for this structure. After the acquisition of all the acceleration measurements, the data was processed only in the Simcenter Testlab 2306® software from which the natural frequencies and damping ratios were extracted and then compared to the natural frequencies and mode shapes from FEM simulations provided by the teams, however due to privacy reasons no other information regarding the FEM was shared. As previously mentioned a reference accelerometer was not available for the tests, as such the mode shapes could not be obtained accurately as manual synchronization was not possible. Finally, it is important to note that the OoMA toolbox was not used during these tests. As such, the Operational PolyMAX method was used as well to determine the modal parameters for this second test. Future works to adapt the OoMA Toolbox software into more complex structures are needed.

4 Results

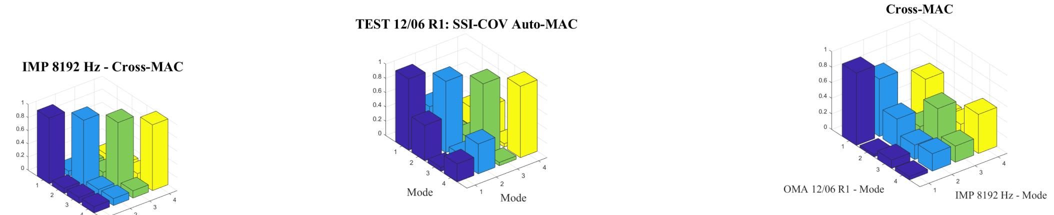
4.1 OMA validation

Table 1 shows that the results for the natural frequencies among all four procedures yielded good results when compared to the analytical values expected. The simulated OMA (3) and the OMA tests (4) were done using all



(a) Analytical (b) BEM (c) Sim. Impulse - SSI-DATA (d) T. 12/06 R1 - SSI-DATA

Figure 2. First mode shape of the SCB obtained with all the procedures.



(a) Simulations Cross-MAC (b) T.12/06 R1 SSI-COV Auto-MAC (c) Cross-MAC between the impulse simulation (SSI-DATA) and TEST 12/06 (SSI-DATA)

Figure 3. MAC analysis. The Cross-MAC between the simulations shows no signs of modal coupling nor incoherences between all modes, however the 1st OMA test performed on June 12th (TEST 12/06 R1) shows that the lack of a reference accelerometer is detrimental to the results, confirmed even further by comparing the SSI-DATA impulse simulation modes to the TEST 12/06 R1 SSI-DATA results, where a high level of incoherences is shown in the MAC.

the OMA techniques mentioned in section 2.2. However, due to space limitations only two techniques are shown in this work. It is also notable that the f_n values obtained with the real SCB (columns 6 and 7) show slightly lower natural frequencies, which is expected as the exact material of the beam is unknown as no material certification was available. ζ in the simulations showed good results, being close to the value of 1% specified.

Table 1. Natural frequencies in Hz obtained during the OMA validation. The 1st column shows the mode, the 2nd column shows the results obtained with the Euler-Bernoulli equations, the 3rd shows the modal simulation done with the BEM. Columns 4 and 5 show the results from the impulse and white noise simulations. Columns 5 and 6 show the 1st OMA test performed on the SCB on June 12th, 2024.

M	**1. An.**	**2. FEM**	**3. IMP: N4SID**		**3. WN. SSI-COV**		**4. T. 12/06: SSI-DATA**		**4. T. 12/06 Simc.**	
	f_n **(Hz)**	f_n **(Hz)**	f_n **(Hz)**	ζ	f_n **(Hz)**	ζ	f_n **(Hz)**	ζ	f_n **(Hz)**	ζ
1	20.79	21.58	20.79	1.00%	21.58	1.00%	19.18	0.61%	19.17	0.93%
2	130.28	135.19	130.28	1.02%	135.06	1.02%	134.77	2.13%	134.18	2.09%
3	364.80	379.67	364.80	1.06%	377.01	1.06%	351.48	0.08%	351.08	0.15%
4	714.86	753.08	714.86	1.08%	733.11	1.09%	683.41	0.08%	682.30	0.07%

The mode shapes obtained showed good results for most procedures, as seen in figure 1, where the 1st mode shape of the SCB is shown. However, in the OMA Test 12/06 R1 the lack of a reference accelerometer had a negative impact on the results, as manual synchronization is not the ideal case. This is further observed in figure 3, where Fig. 3a shows the Cross-MAC between the impulse and white noise simulations, where the MAC shows very good results for all modes analyzed. However, in Fig. 3b where the impulse simulation is compared to the OMA 12/06 R1 test, the Cross-MAC reveals incoherences between the two sets, which is expected due to the lack of a reference accelerometer.

4.2 OMA in other structures

The natural frequencies and damping ratios obtained for the Barbie aerodesign and Hyarra drone structures can be seen below in table 2. It is possible to see that both the SSI-DATA and Op. PolyMAX methods obtained similar results between them with a significant difference with regards to the FEM values expected. This can be explained as there is no information available as to how the FEM simulations were made by the competition teams and as differences between the models and real structures are also expected.

Table 2. Natural frequencies and damping ratios obtained in the OMA tests were performed on complex structures. All OMA results were obtained with the Simcenter Testlab 2306

	Barbie Aerodesign			Hyarra Drone		
M.	**FEM f_n**	**SSI-DATA f_n**	**SSI-DATA ζ**	**FEM f_n**	**PolyMAX f_n**	**PolyMAX ζ**
1	19.14	10.62	3.61%	64.83	75.8	0.08%
2	98.06	91.93	0.06%	65.64	76.88	0.12%
3	146.56	179.92	0.03%	314.12	302.23	0.16%
4	347.17	360.22	0.21%	315.55	302.27	0.13%

5 Conclusions

The validation process for the OMA techniques implemented showed satisfactory results at estimating the natural frequencies and damping ratios of the SCB when compared to the data obtained from the analytical and simulated results for all OMA methods assessed. However, the effect of having a single accelerometer for the tests proved to be detrimental for the mode shapes obtained in the OMA tests performed on the real SCB model, as a fixed accelerometer is fundamental for the synchronization between all the points measured. Nevertheless, as the excitation used in the tests was an impulse force, it was possible to possible to manually synchronize all the measurements with some negative effects on the mode shapes obtained. However, for the Barbie and Hyarra structures, the synchronization problem proved to be impossible to overcome manually, thus leading to imprecise mode shapes. In addition to that, the harmonic forces encountered during the Barbie and Hyarra tests created by the motors proved challenging to the interpretation of the results, provoking the necessity applying a narrow-band approach to correctly identify the modes of interest. OMA proved to be capable of asses most of the modal parameters of all the structures evaluated, even in cases of very weak excitations as with the of the white noise simulations and the Barbie aerodesign tests. Future works with the addition of a second fixed accelerometer are required to observe if this is enough to overcome the problems encountered in the mode shapes due to the multiple-run approach used for the tests.

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References

[1] R. Brincker and C. Ventura. *Introduction to Operational Modal Analysis*. Wiley, 2015.

[2] P. Van Overschee and B. De Moor. *Subspace identification for linear systems. Theory, implementation, applications. Incl. 1 disk*, volume xiv, pp. xiv + 254, 1996.

[3] H. Li, Q. Wei, J. Wang, and Z.-G. Wu. Subspace identification algorithm based on non-negative matrix factorization for hammerstein model. *International Journal of Automation and Computing*, vol. 11, n. 1, pp. 45–52, 2014.

[4] F. Bin Zahid, O. Chao, and S. Khoo. A review of operational modal analysis techniques for in-service modal identification. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, vol. 42, 2020.

[5] F. Magalhães. *Operational Modal Analysis for Testing and Monitoring of Bridges and Special Structures*. PhD thesis, Universidade de Porto, 2010.

[6] P. Guillaume, T. De Troyer, C. Devriendt, and G. De Sitter. Omax - a combined experimental-operational modal analysis approach. vol. 5, pp. 2985–2996, 2006.