PAE-Grelhas 2.0: Implementation of Gaussian quadrature in straight and curved grillages program in PYTHON.

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Abstract. This paper demonstrates the development of a Python program for linear-elastic structural analysis of grillages featuring circumferential arc-shaped and regular straight elements subjected to concentrated and distributed loads. The algorithm is based on the Finite Element Method. The force method was employed to compute the flexibility matrix, followed by the determination of the stiffness matrix and load vectors through inversion. A Gauss-Legendre Quadrature was incorporated in order to have a better control on accuracy for displacements, rotations, forces and moments. Results were validated through comparisons with literature data and similar software where curved structures are simulated using numerous straight elements. It was demonstrated that the Python-based program yields results closer to the exact solution with simplified input data, highlighting the effectiveness of both programming language and Gauss quadrature technique in structural analysis.

Keywords: grillages; Finite Element Method; Gauss-Legendre Quadrature; Python.

1 Introduction

Computer-aided design has inevitably grown in most sectors, such as civil engineering and, in particular, its structural analysis. Those programs vary from simple blueprint drawings to complex tasks such as building information modeling (BIM) or nonlinear Finite Element Analysis (NLFEA). Grillage structures are commonly used as an analogy for buildings, ships, and bridge simulations and are defined by Clarkson [1] as a structure of intersecting beams which is loaded normal to its surface. There are several available software based on the Finite Element Method (FEM) that offers grillage analysis and are very useful in calculating internal forces, support reactions, and displacements in both statically determinate and indeterminate structures. However, they hardly ever include an exclusive element for arcs, which can lead to the onerous discretization of the straight bars in order to simulate curved structures and inaccurate results.

Lima & Soares [2] presented a program for analysing grillages with circumferential arc-shaped elements and regular straight bars under various loads entitled “PAE-Grelhas 1.0”. It was Finite Element-based, written in Fortran, and presented very good predictions in comparison to analytical solutions. However, close inspection raised issues concerning the use of a significant number of arcs, causing an accumulation of minor errors that could influence the overall result. This paper then proposes the development of an updated version (2.0) of said software that rewrites it in the modern language Python to integrate with a GUI developed by Morais [3]. Also, it includes the numerical method of Gauss-Legendre Quadrature for the Stiffness Matrix and Load Vector derivations, which intends to provide better control for the calculations in cases of numerous discretization of arc elements.

2 Methodology

The transition from Fortran to Python was motivated by the aforementioned interface and various advantages provided by more present-day languages. According to the TIOBE Programming Community index [4], a known indicator of programming language's popularity, Python has been on an increasing wave since 2018, reaching and holding the first position since June of 2023. Menezes [5] comments that Python has been used in several fields of computation and programming, from games and 3D animations to biotechnology and artificial intelligence. It also offers a significant number of libraries and frameworks facilitating the development and integration of software.

PAE-Grelhas 2.0 is also based on the Finite Element Method and allows for two types of grillage elements: the traditional straight bar with constant cross-section, two nodes, and three degrees of freedom per node, and a circumferential arc-shaped element as shown in Figure 1, where α_1 and α_2 are the initial and final angles, respectively, considered from the *y* axis, which along with the radius, result in enough input data for each of the curved elements since they are assessed in polar coordinates.

Analogous to Lima & Soares [2], the Stiffness Matrix and Load Vector expressions were calculated using the Force Method by inverting the flexibility matrix. However, the numerical approximation of Gauss-Legendre was used here to find those flexibility coefficients since the integrals did not converge to a simple analytical solution. Ten points for the quadrature were deemed enough after several checks against examples developed and solved by the authors using the Virtual Work Method. A detailed description of those integrals can be found elsewhere in Sawko [6].

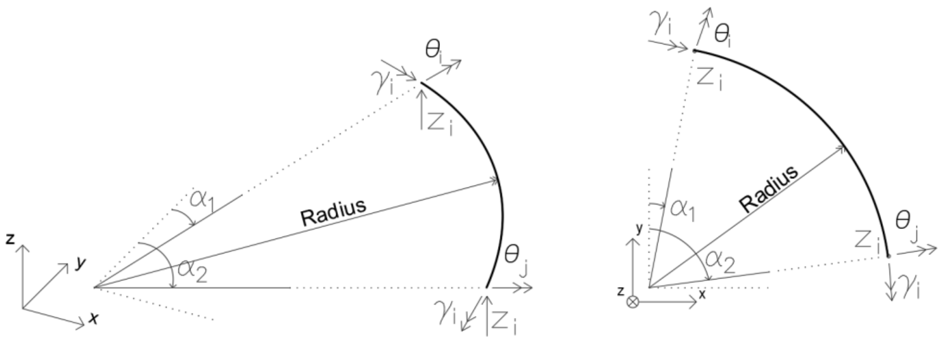

Figure 1: Generic circumferential arc-shaped element in both isometric and plane view.

At the current stage, **PAE-Grelhas 2.0** uses a *.txt* generated by the GUI (Morais [3]) as input since it gathers the required data for its linear-elastic analysis. It includes node coordinates, concentrated loads (moments along *x* and *y*, and force in *z*) and uniformly distributed loads, boundary conditions, material and geometric properties like Young's and shear modulus, and both moments of inertia (bending and torsion), as well as angles and radius for the curved elements. Figure 2 shows the program's flowchart logic.

After the processing, **PAE-Grelhas 2.0** displays two different *.txt* files. The first is a report that summarizes every input data as well as all stiffness matrices and load vectors. The idea behind this report is to be used in classes or for any particular reason that would demand those processing values to be known. The second report presents only the resulting support reactions, internal forces (shear, bending, and torsion), displacements, and rotations.

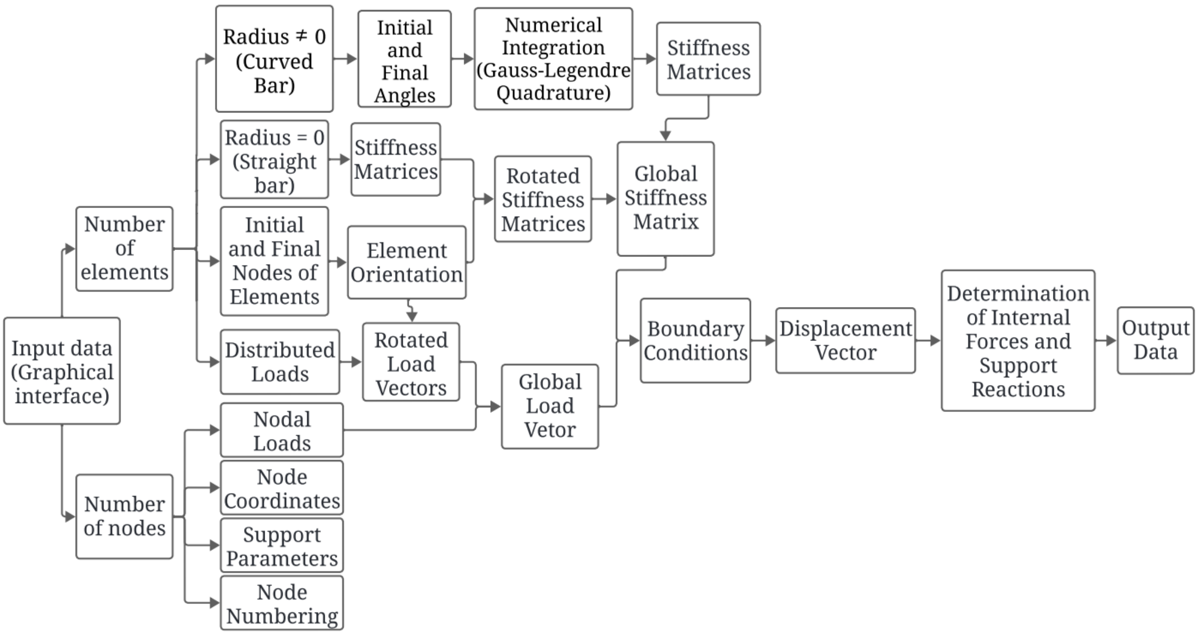

Figure 2: Flowchart of the program's processing logic.

The code was validated against several examples found in the literature, such as Süssekind [7], and developed by the authors. One of these examples is shown in the next topic. It also presents results from a few other software that deal with grillage analysis for comparison: VGPlan [8] and LESM 3.0.0 [9]. Neither of those includes a specific element for curved bars.

3 Results

An example of a grillage with both straight and curved bars was developed and answered using the code proposed here, analytically with the Virtual Work Method, and with the programs previously mentioned ([8] and [9]). Figure 3 presents the details of the structure in isometric and plan view, while Table 1 summarizes the geometrical properties of the cross-section and both bending and torsion modulus. Table 2 gives information about the circumferential arc-shaped elements, presenting their radius and initial and final angles as shown and explained in Figure 1.

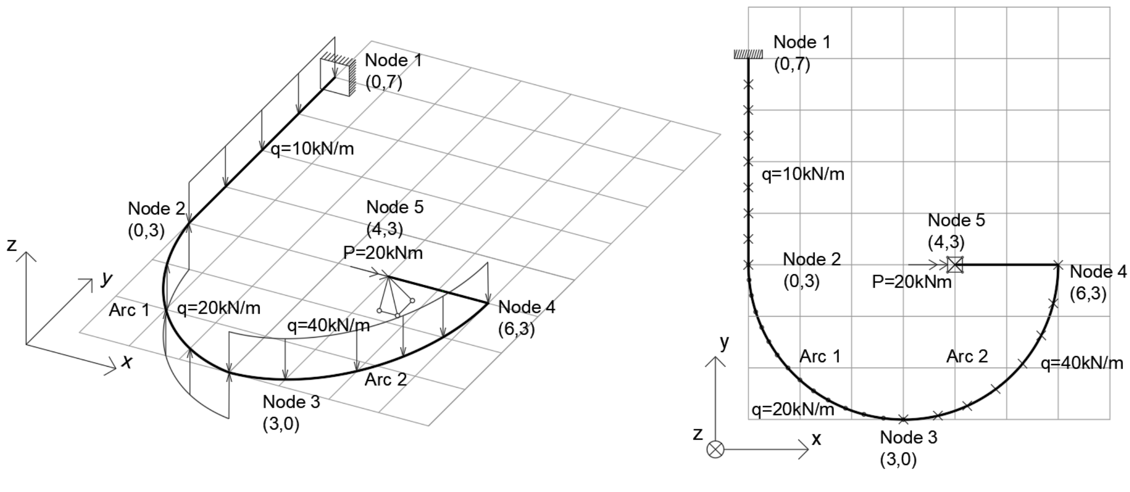

Figure 3: Grillage structure used as an example.

Table 1. Cross-sectional geometrical and material properties.

Parameters	Notation	Value
Young's modulus (Bending)	E	$2{,}0 \cdot 10^7$ kN/m^2
Shear modulus (Torsion)	G	$1{,}0 \cdot 10^7$ kN/m^2
Moment of Inertia (Bending)	I_b	$7{,}2 \cdot 10^{-3}$ m^4
Moment of Inertia (Torsion)	I_t	$7{,}526 \cdot 10^{-3}$ m^4

Table 2. Input data for the circumferential arc-shaped elements.

Name	Notation	Value
Arc 1	Radius	3 m
	α_1	270°
	α_2	180°
Arc 2	Radius	3 m
	α_1	180°
	α_2	90°

Table 3 gives the Input data extracted and adapted from the GUI ([3]) for the example of Figure 3. It starts by summarizing the total number of nodes and elements, followed by their coordinates and nodal loads. The boundary conditions (Displacement in Z, Rotations in X and Y) are defined as *0* when free and *1* when fixed. Sign convention for the UDL and concentrated loads follow the established axes. No units are required, though the parameters need to be in equivalent magnitude. For the example, *kN* and *m* were used, and angles were read in degrees.

Table 4 presents a comparison between the results obtained from the Virtual Work Method, **PAE-Grelhas 2.0**, VGPlan [8], and LESM 3.0.0 [9] for the reactions over the support in Node 1 and Displacement in Z for Node 2.

It can be seen that all programs gave good predictions for the evaluated parameters in comparison to the analytical result. However, since neither VGPlan [8] nor LESM 3.0.0 [9] has a dedicated arc element, straight bars were used to simulate the curved structure. Around fifteen elements were employed for each arc in the example of Figure 3. Since the results can be very discretization-dependable, different analyses aiming for more accurate values tend to demand even more effort. However, **PAE-Grelhas 2.0** only requires two angles and a radius, presenting closer answers to the exact solution.

Table 3. Input data in **PAE-Grelhas 2.0**.

|N. Nodes| & |N. Elements|										
5	4									
|Node|	|X Coord.|		|Y Coord.|	F_z	M_x	M_y	Displac. Z	Rot. X	Rot. Y	
1	0.0		7.0	0.0	0.0	0.0	1	1	1	
2	0.0		3.0	0.0	0.0	0.0	0	0	0	
3	3.0		0.0	0.0	0.0	0.0	0	0	0	
4	6.0		3.0	0.0	0.0	0.0	0	0	0	
5	4.0		3.0	0.0	20.0	0.0	1	0	0	
|Element|	| i |	| j |	|Radius|	$\|\alpha_1\|$	$\|\alpha_2\|$	|E|	$\|I_b\|$	|G|	$\|I_t\|$	$\|Q_z\|$
1	1	2	0.0	0.0	0.0	$2.0{\cdot}10^7$	$7.2{\cdot}10^{-3}$	$1.0{\cdot}10^7$	$7.526{\cdot}10^{-3}$	-10.0
2	3	2	3.0	180.0	270.0	$2.0{\cdot}10^7$	$7.2{\cdot}10^{-3}$	$1.0{\cdot}10^7$	$7.526{\cdot}10^{-3}$	20.0
3	4	3	3.0	90.0	180.0	$2.0{\cdot}10^7$	$7.2{\cdot}10^{-3}$	$1.0{\cdot}10^7$	$7.526{\cdot}10^{-3}$	-40.0
4	4	5	0.0	0.0	0.0	$2.0{\cdot}10^7$	$7.2{\cdot}10^{-3}$	$1.0{\cdot}10^7$	$7.526{\cdot}10^{-3}$	0.0

Table 4. Comparison of results

	Analytical	VGPlan 0.2	Error (%)	LESM 3.0	Error (%)	**PAE-Grelhas 2.0**	Error (%)
Support Reaction Node 1 Displac. z (kN)	**-19,96168**	-19,62251	0,01699	-19,28337	0,03398	**-19,96168**	0,0
Support Reaction Node 1 Rotation x (kN·m)	**40,15328**	41,03824	0,02204	42,42020	0,05646	**40,15328**	0,0
Support Reaction Node 1 Rotation y (kN·m)	**-205,90550**	-202,40309	0,01701	-203,88031	0,00984	**-205,90550**	0,0
Vertical Desloc Node 2 (m)	**-0,00445**	-0,00447	0,00541	-0,00430	0,03373	**-0,00445**	0,0

4 Conclusions

The updated version of PAE-Grelhas 1.0 [2] presented in this paper kept the interesting features of its first version, including a dedicated circumferential arc-shaped element that only requires two angles and a radius as input, while improving some aspects related to the precision of results due to the Gauss-Legendre Quadrature applied in the calculation of the Stiffness Matrix and Load Vectors, as well as the conversion to Python language and posterior association with its GUI [3].

A newer version of this software is already being developed, focusing on shape functions for both straight and curved elements to calculate and display the internal forces and deflections along the element.

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