Space truss finite elements applied to actuated structures and origami – implementation and discussions

Rafael R. G. Almeida[1], Humberto B. Coda[1]

[1]*Departamento de Estruturas, Universidade de São Paulo*
Av. Trab. São Carlense, 400 – Parque Arnold Schimidt, São Carlos, 13566-590, São Paulo, Brazil
rafaelrga@usp.br, hbcoda@usp.br

Abstract. In the various branches of engineering, dynamic analysis of structures is always a relevant topic and the training of future professionals and researchers in this area is always important. Nowadays, with the evolution of materials and topological dispositions, structural elements and structures are becoming lighter and slender, increasing the amplitude of movements in such a way that nonlinear geometric analysis is necessary. Regarding actuated structures or origami mechanisms, there is a natural mobility between planes defined by the relative rotation in the creases that makes it possible to create foldable devices of great interest such as the production of retractable roofs, retractable antennas, wheels for light vehicles, biomechanical devices, among others. This study introduces the presence of actuators in spatial structures by directly controlling the initial length of truss elements (actuation). At the same time, to define origami or "flat" 3D structures, it is necessary to introduce the strain energy associated with the folds at the junction between flat panels (creases). To introduce this energy, vectors orthogonal to the adjacent origamic planes are defined and springs are introduced to oppose the change in the angle between the planes. Illustrative examples and discussions of future improvements are presented.

Keywords: Finite element, Nonlinear dynamics, Origami, Foldable Structures.

1 Introduction

Considering the relevance of dynamic structural analysis and its increasing evolution and improvement, the elaboration of numerical and computational models that beholds nonlinear behavior is very necessary. In this sense, the mechanisms of origamic structures, that have a natural mobility between planes defined by the rotation in their creases, allow the creation of foldable devices that have been attractive to aeronautical, mechanical and civil engineering researchers (Sorguç et al. [1], Felton et al. [2] and Nishyiama et al. [3]).

This study presents the development of a finite element computational code capable of modeling 3D truss structures, actuated truss mechanisms and origami structures using some basic concepts such as: objective strain measurements, strain energy, energy conjugate, conservative external loads, the principle of stationary mechanical energy, iterative solution and time integration (Bonnet and Wood [4], Newmark [5] and Coda and Paccola [6]).

The Finite Element Method (FEM) that uses position as parameters is chosen for the work's development; see, for instance, Greco et al. [7] and Coda et al. [8].

Specifically in this study, the origami elements are inspired in the Japanese technique of folding with creation of valleys and crests in a tridimensional mobility of paper. With that in mind, the literature references several origami patterns such as the "waterbomb" element, that has been used due to its "developable" characteristic that allows it to create many foldable structures and to adapt itself to revolution surfaces (Fonseca [9]). One of the notable uses of this pattern is the space exploration area (Dorn et al. [10] and Lee and Pelegrino

[11]). To sum up, the developed code was validated by structures with simple mechanisms and its possibilities explored in a waterbomb origami cell.

2 Finite Element Method for trusses dynamics and actuated structures

The Finite Element Method strategy of this work is called positional and is based on Greco et al. [7] and Coda [12]. Thus, from the principle of stationary mechanical energy one achieves the motion equation expressed as:

$$\delta \Pi = \delta P + \delta K + \delta U = 0. \tag{1}$$

where P is the work potential of external loads, K is the kinetic energy of the concentrated masses (truss elements), and U is the strain energy stored by all elements. From the positional approach one replaces each portion of eq. (1) by equivalent forces following direction (α) and belonging to nodes (β); (Y_α^β):

$$-\left(F_\alpha^\beta\right)^{ext} + M^{(\beta)}\ddot{Y}_\alpha^\beta + \left(F^{int}\left(Y_k^\gamma\right)\right)_\alpha^\beta = 0_\alpha^\beta. \tag{2}$$

wherein $\left(F_\alpha^\beta\right)^{ext}$ are the external loads applied at the nodes (β), $M^{(\beta)}$is the concentrated mass of each node and $\ddot{Y}_\alpha^\beta$ is the acceleration in each direction (α) of nodes (β).

It is evident that the values of the last two portions in eq. (2) are dictated by a change in the configuration of the truss element that can be represented in a general sense by Fig 1.

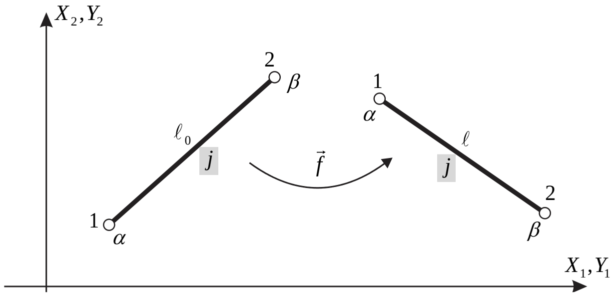


Figure 1. Change of configuration of a truss element.

2.1 Actuator Element

The actuator element used in this work was presented in Coda et al. [8] and is achieved by changing the Green-Lagrange strain expression used in the truss elements to the equation below:

$$\mathbb{E} = \frac{1}{2}\left[\frac{l^2 - (l_0 + \Delta l_0)^2}{l_0^2}\right] \tag{3}$$

wherein l is the current length of the analyzed element, (l_0) is its initial length and Δl_0 is the actuated length change.

3 Origami Element

As mentioned in the introduction, the origami element consists of the union between two rigid planes constituted by truss elements and was presented in Liu and Paulino [13]. In this study, a formulation inspired by that work, is developed in the positional finite elements' version. As shown in Fig 2., the element has 4 nodes and 5 truss elements, defined here as vectors $\vec{v}^{i0}$ in the initial configuration and $\vec{v}^{i}$ in the current one; the change of configuration is also presented in Fig 2.

To incorporate this element into the equations of motion shown in Section 2, it is considered that the strain energy stored in the origami is defined by the difference in the angles formed by the two panels in its current (θ) and (θ_0) configurations. In this work, the following strain energy expression is considered:

$$U^e = \frac{K}{2}(\theta - \theta_0)^2 \tag{4}$$

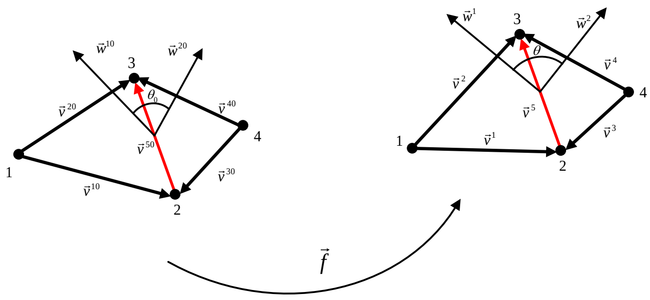


Figure 2. Change of configuration of a origami element.

With the strain energy expression defined, the conjugate internal forces are obtained as:

$$F_\alpha^{\beta int} = \frac{\partial U^e}{\partial Y_\alpha^\beta} = \frac{dU^e}{d\theta}\frac{\partial(\theta)}{\partial Y_\alpha^\beta} = K(\theta - \theta_0)\frac{\partial \theta}{\partial Y_\alpha^\beta} \tag{5}$$

considering that each vector of the origami is the difference of the nodes positions, and that $\vec{w}^1 = \vec{v}^1 \wedge \vec{v}^2$ and that $\vec{w}^2 = \vec{v}^4 \wedge \vec{v}^3$, the angles θ can be calculated as follows:

$$\theta = atan\left[\frac{(\vec{w}^1 \wedge \vec{w}^2)\cdot\vec{v}^5}{(\vec{w}^1 \cdot \vec{w}^2)\sqrt{\vec{v}^5 \cdot \vec{v}^5}}\right] \tag{5}$$

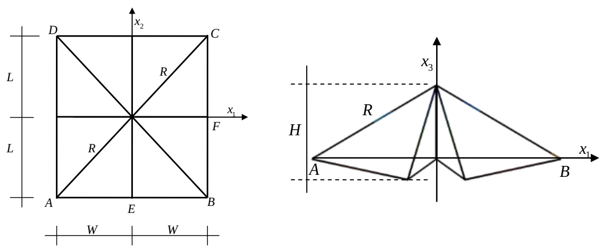
in the first step of the code, the angle obtained through the equation above is stored as θ_0, to be used in all the subsequent steps. In addition, one uses the chain rule on eq. (5) to calculate $\partial\theta/\,\partial \mathrm{Y}_\alpha^\beta$ and obtains the internal forces. Similarly, one calculates $\partial^2\theta/\left(\partial \mathrm{Y}_\gamma^\eta\,\partial \mathrm{Y}_\alpha^\beta\right)$ and obtains the origami part of the Hessian matrix as:

$$H_{\alpha\gamma}^{\beta\eta} = \frac{\partial^2 U^e}{\partial Y_\gamma^\eta \partial Y_\alpha^\beta} = \frac{\partial F_\alpha^{\beta int}}{\partial Y_\gamma^\eta} = K\frac{\partial \theta}{\partial Y_\gamma^\eta}\frac{\partial \theta}{\partial Y_\alpha^\beta} + K(\theta - \theta_0)\frac{\partial^2 \theta}{\partial Y_\gamma^\eta \partial Y_\alpha^\beta} \tag{6}$$

during the study of examples with origami cells, it was noted that the crease's stiffness is always close to instability and the choice of the Hessian matrix can affect the solution paths. In this case, it is chosen to keep only the spring material contribution to the Hessian matrix (first part of eq. (6)).

4 Example 1: 8 creased Waterbomb cell

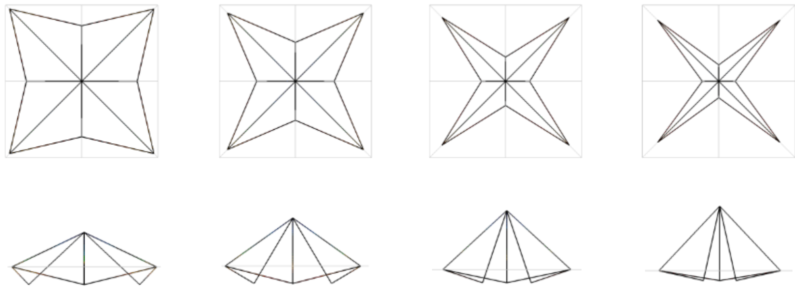
In this example, the Waterbomb cell will be explored, specifically in how its dimensions vary with the application of forces. Fig. 3a represents a vertical view of the cell and shows the dimensions (L and W) that will change along the analysis, whereas the dimension $R = \sqrt{L_0^2 + W_0^2}$ is constant. Fig. 3b shows a side view of the deformed cell and where the height H is defined. The input data are: $L_0 = W_0 = 1m$; for the trusses' bars: $E = 3MPa$, $A = 10^{-2}m^2$ and $\rho = 70kg/s^2$ and for the hinges: $K = 1N.m$. Points A, B, C and D are constrained in the vertical direction, point E is constrained in direction x_1, point F in the direction x_2 and finally the central point in the directions x_1 and x_2.



(a) Vertical view (b) Lateral View

Figure 3. Geometry of the Waterbomb cell.

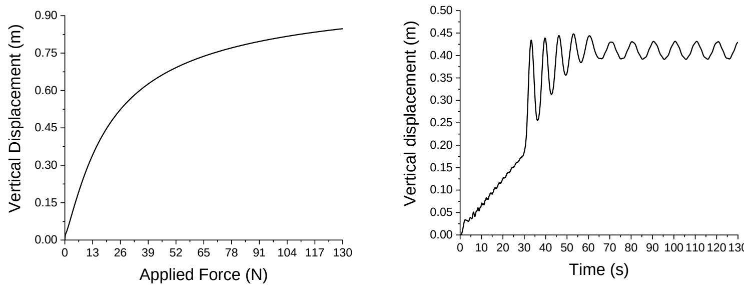
In the static analysis, a vertical force is applied in the central point of $130N$ along with negative vertical forces of $32.5N$ in points E, F and their symmetrical partners. These loads are divided in 650 equally spaced steps. Figure 4 shows the general geometrical behavior of the cell in function of the central applied load.



$F = 20N$ $F = 40N$ $F = 80N$ $F = 130N$

Figure 4. Configurations of the static origami analysis.

The static analysis is also graphically shown in Fig. 5a with the central vertical displacement. Through the analysis the theoretical prediction of the planar Poisson's ratio (Fonseca [9]) was exactly reproduced with the value -1 constant. Fig. 5b shows the dynamic behavior of the central point considering a linear applied load until time 65s with the final value being $130N$, after that the force is kept constant. The damping considered was $C = 0.1s^{-1}M$ and the adopted time step was $\Delta t = 0.1s$.

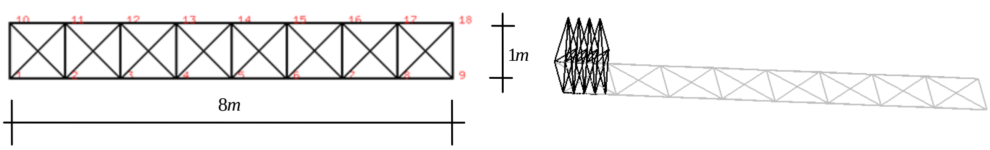


(a) Static responce (b) Dynamic response

Figure 5. Vertical displacement of the central point.

5 Example 2: Foldable structure

The foldable structure and its discretization are depicted in Fig. 6a. It is constituted of 49 truss elements (including actuators) with same properties of the previous example and origami elements with transverse junctions with K = 3.32Nm. We adopted K = 1000Nm for diagonal origami elements. Nodes 1 and 10 are restricted for all translations, nodes 2, 4, 6, 8 11, 13, 15 and 17 are free to move and have an initial vertical different from 0 (1mm), while other nodes are restricted in vertical direction. Actuation is imposed by a total contraction of 7.2m spread in actuators of 2m (connecting nodes 1,3 etc) divided in 1800 steps, i.e., a $\Delta\ell = -0.5x10^{-3}m$ for each actuator for each step. In Fig. 6b one can see the folded structure.



(a) Discretization (b) Folded structure

Figure 6. Initial and final positions of a foldable structure

As one can see in this example, the use of origami, truss and actuators elements makes possible to simulate 3D foldable structures with a very good response, as the considerable total contraction exemplifies.

6 Conclusions

In this work, a successful formulation for origami and actuated structures was implemented and, even with simple examples, it is shown the great potential of this approach, specifically in the packaging of foldable structures and the analysis of transient nonlinear dynamics. The analysis of the single waterbomb cell reproduced the expected behavior and leaves confidences for bigger waterbomb structures, such as cylindrical ones obtained through waterbomb tesselation. The retractable beam example showcases the possible applications of foldable structures, like retractable roofs and deployable solar pannels. In the continuity of this study, it is aimed to improve the algorithm, including actuated origami elements and nonlinear spring strain energy. Moreover, it is intended to generate more complex and larger structural devices to be studied.

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7 References

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