Translational joint in a Finite Element Analysis program of 2D frames with straight and curved elements

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Abstract. This paper describes the development of an algorithm for the structural analysis of bidimensional frames with straight and curved elements that include a translational joint that can be set to any specified angle. The code was written in Fortran following a Finite Element Method-based structure. Three elements were considered here: a circumferential arc-shaped, a straight bar with cross-sectional height varying according to a linear polynomial function, and the traditional straight element with constant inertia. All Stiffness matrices and load vectors were derived from the inversion of their corresponding Flexibility matrices obtained using the Force Method. In a few cases, a Gaussian Quadrature was needed to calculate the integral. The resulting program was validated against solutions found in the scientific literature in addition to structures created and solved by the authors using the Virtual Work Method and is shown to give very accurate predictions for displacements and rotations as well as reactions and internal forces (axial, shear, and bending).

Keywords: translational joint, Curved Elements, Finite Element Method, Fortran.

1 Introduction

Structural analysis is described by Soriano [1] as the process in which external loads are related to internal forces, displacements, and support reactions. Numerous Finite Element-based software are available for the structural analysis of 2D frames, such as *Ftool*, developed by Dr. Luiz Fernando Martha from PUC-Rio. This work intends to develop another module for a similar program called *PAE*, which is also based on the Finite Element Method and written in Fortran by the research group of Dr. Luis Fernando Soares (IFMA-MTC). It has been in development since 2018 and aims to tackle unusual kinds of structures (variable inertias, arcs, etc.). Oliveira et al. [2] presented a GUI for one of the earlier versions of *PAE*, while Lima & Soares [3] developed a grillage analysis software with curved elements.

PAE elements include, besides the usual straight one with constant inertia, a circumferential arc-shaped and a straight bar with cross-sectional height varying according to a linear polynomial function. Figure 1 gives more details of those elements, which have two nodes with three degrees of freedom each. The module proposed here intends to add the possibility of a translational joint in the analysis of frames with unusual geometry subjected to concentrated and uniformly distributed loads. This particular type of joint is similar to a hinge, as it releases movement of one particular degree of freedom. Hinges free rotation, while translational joints liberate displacements at any direction in the *xy* plane.

2 Methodology

The stiffness matrix and load vectors for the elements with translational joints, such as the one seen in Figure 2, were derived using the Force Method individually applied for a specific element and boundary condition. The expressions for those matrices can be found by inverting the flexibility matrix, and those coefficients are calculated from their respective integrals, presented in detail at Martha [4] for the case of a traditional straight bar with a constant cross-section. All three elements shown in Figure 1 gave two different stiffness matrices and load vectors each since the translational joint can be placed at either the initial (shown in Figure 2, for example) or final node.

Neither the circumferential arc-shaped nor the straight element with cross-sectional height varying according to a linear polynomial function gave analytical solutions for its flexibility coefficients. For these particular cases, a Gauss-Legendre Quadrature was implemented in order to calculate the stiffness matrices and load vectors. Twenty points were used for both curved and variable cross-sectional elements, estimated on the basis of comparison with problems developed by the authors and solved using the Virtual Work Method.

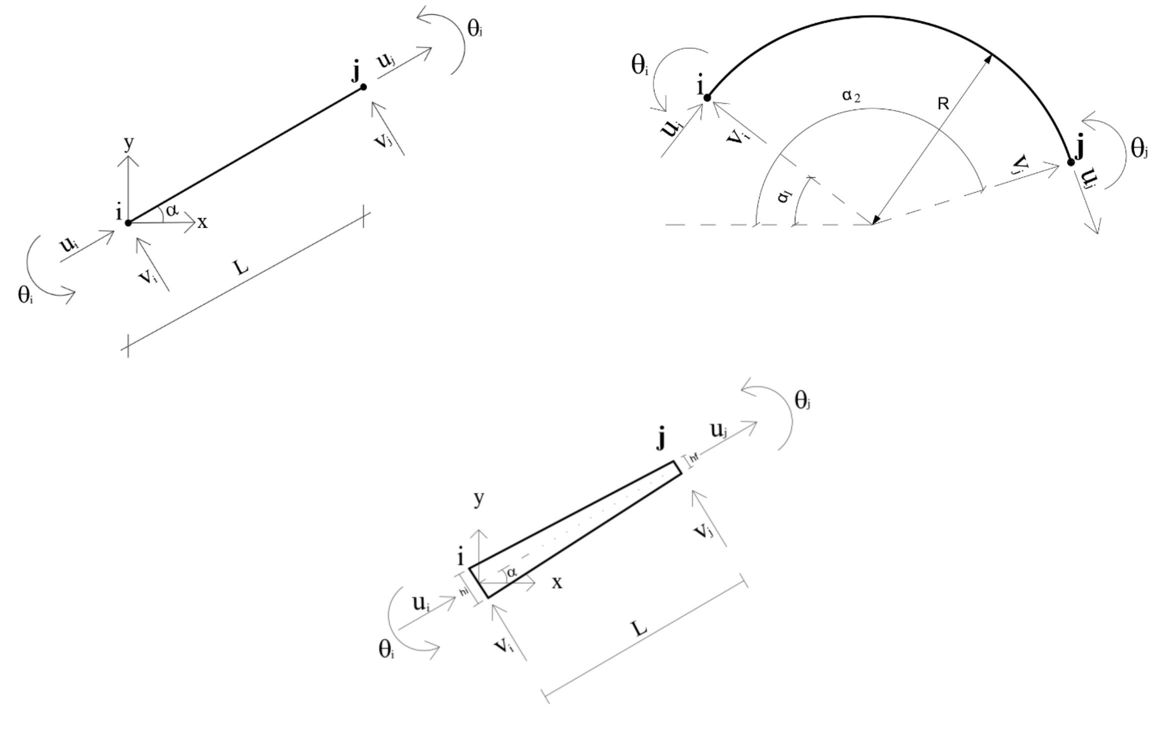

Figure 1: Some of the elements available in PAE.

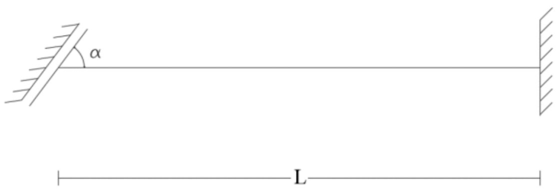

Figure 2: Translational joint at the initial node of a fixed beam element.

Once the flexibility matrix is assessed, inversion and equilibrium, along with the Virtual Work Method and Maxwell–Betti reciprocal work theorem, are enough to calculate the Stiffness Matrix and Load Vectors, which were then added to the Fortran code. Figure 3 shows the processing logic Flowchart of *PAE*.

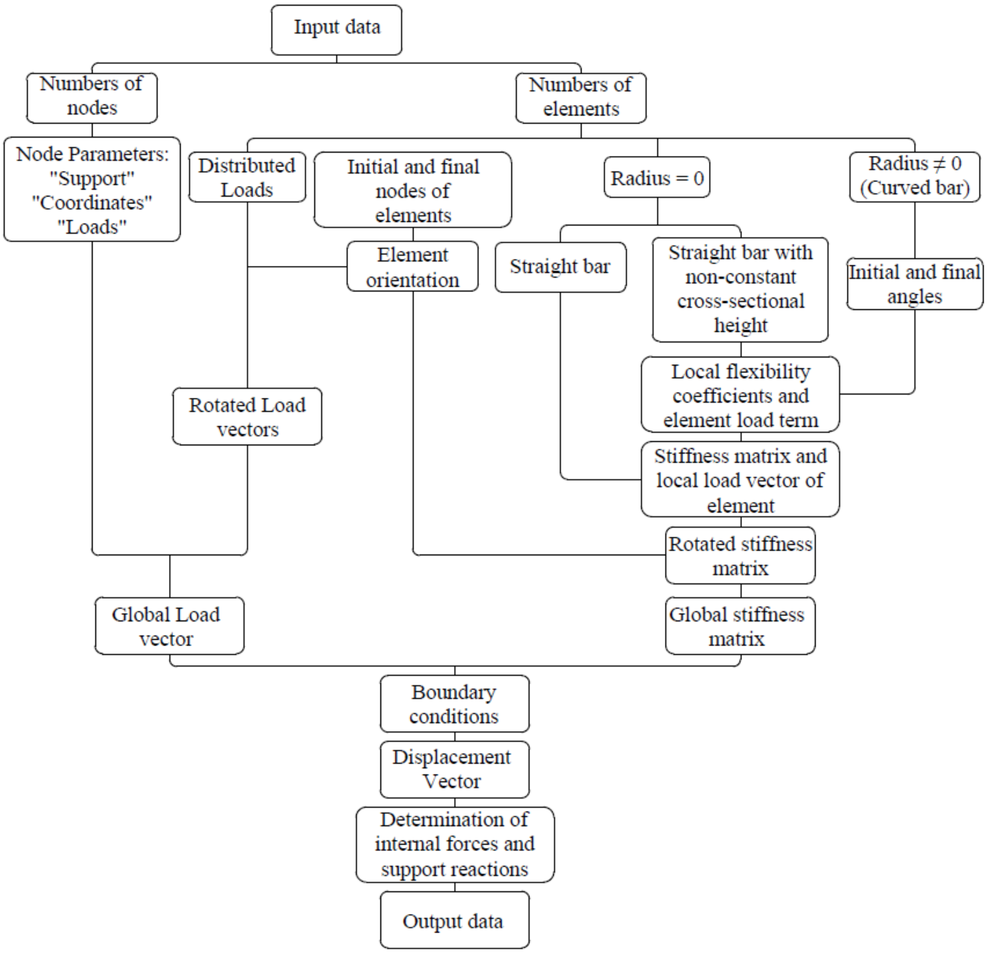

Figure 3: Processing logic Flowchart of the program.

3 Results

Figure 4 shows an example of a statically indeterminate frame that presents translational joints at nodes *2* and *4*, a circumferential arc-shaped and a straight bar with cross-sectional height varying according to a linear polynomial function as elements *4* and *2*, respectively. A column and an inclined beam are also included in the structure. The loads are three uniformly and one linearly distributed. More details are displayed in It can be seen that despite both programs presenting reasonable predictions, *PAE* gives results closer to the analytical solutions (calculated with the Virtual Work Method) than *SAP2000*. The greatest errors for the example in Figure 4 were 4% for *SAP2000*, whereas *PAE* gave precise results, most likely due to the angles of 0º, 45º and 90º for the translational joint in nodes *4* and *2*, respectively.

Table 1, which illustrates the Input Data for the code. That input states the number of nodes, their coordinates, the elements, and their initial and final nodal points. Support conditions are read as "fixed" and "free". The parameter *type* identifies the chosen element: *0* for straight with constant cross-section, *1* for circumferential arcs, *2* for parabolical arcs (not included in this particular example), and *3* for straight ones with variable cross-sectional height. *Connection* implies the type of joint at the end of the element, which varies from a rigid one (transfer all types of internal forces – *0*); hinges at the initial (*2*), final (*1*), or at both nodes (truss element – *3*); and translational joint at either the beginning (*5*) or the end (*4*) of the element. For elements with variable cross-sections, both moment of inertia and area is calculated later in the code.

Table 2 shows a comparison between *PAE* and the commercial software *SAP2000*, a general-purpose civil engineering tool for the analysis and design of structural systems. The popular *Ftool* was not included in this analysis as it does not allow translational joints.

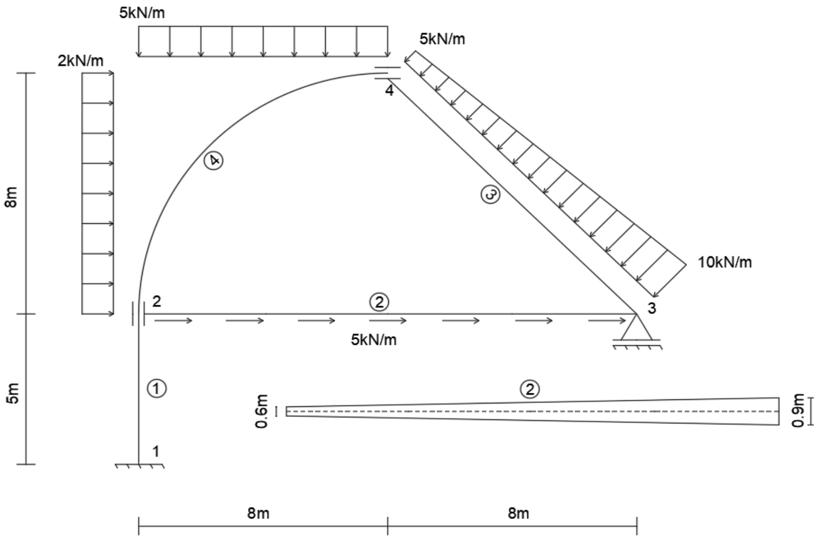

Figure 4: Frame used as an example.

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Table 1: Input Data.

Nº. of Nodes	Nº. of Elements
4	4

Node	Coordinates X	Coordinates Y	Nodal Loads Fx	Nodal Loads Fy	Nodal Loads Mz	Nodal Restraints Displ. X	Nodal Restraints Displ. Y	Nodal Restraints Rotat. Z
1	0.0	0.0	0.0	0.0	0.0	Fixed	Fixed	Fixed
2	0.0	5.0	0.0	0.0	0.0	Free	Free	Free
3	16.0	5.0	0.0	0.0	0.0	Free	Fixed	Free
4	8.0	13.0	0.0	0.0	0.0	Free	Free	Free

Element	Node i	Node j	Type	Connection	Angle	E	Inertia	Area	Distributed Load Qxi	Qxj	Qyi	Qyj
1	1	2	0	0	-	$2.0 \cdot 10^7$	0.0036	0.12	0	0	0	0
2	2	3	3	5	90	$1.5 \cdot 10^7$	-	-	5	5	0	0
3	3	4	0	4	45	$2.5 \cdot 10^7$	0.01215	0.18	0	0	10	5
4	2	4	1	4	0	$2.0 \cdot 10^7$	0.0036	0.12	2	2	-5	-5

Circumferential Arc-shaped element			
Element	Radius	Initial Angle	Final Angle
4	8.0	0.0	90.0

Straight Element with variable Cross-Sectional Height			
Element	Base	Initial Height	Final Height
2	0.2	0.6	0.9

Table 2: Result comparison for the example.

Node		**Analytical Result**	**PAE**	**SAP2000 v25 3.0**
Support Reactions NODE 1	Fx [kN]	-36.0	-36.0	-36.0
	Fy [kN]	53.69	53.69	53.44
	Mz [kNm]	196.29	196.29	192.31
Support Reaction NODE 3	Fy [kN]	46.31	46.31	46.56
NODE 2	Horiz. Displac. [m]	0.02366	0.02366	0.02315
	Vert. Displac. [m]	-0.00011	-0.00011	-0.00011
	Rotation [rad]	-0.00738	-0.00738	-0.00710
NODE 3	Horiz. Displac. [m]	0.02354	0.02354	0.02303
	Rotation [rad]	0.00818	0.00818	0.00801
NODE 4	Vert. Displac. [m]	-0.05159	-0.05159	-0.05096
	Rotation [rad]	0.00282	0.00282	0.00279

4 Conclusions

This paper developed a code module to include a translational joint in *PAE*, a structural linear-elastic analysis program of *2D* frames with straight and curved elements subjected to various loads. It allows for the calculation of support reactions, nodal displacements, rotations, and internal forces (Bending, Shear, and Axial). It is worth mentioning that the implemented joint can be added at any angle, not just 0º, 45º and 90º, as it was used in the example of Figure 4. That allows for some interesting analysis that can go beyond structural engineering.

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