Reliability analysis in the design of cold-formed steel built-up I section by modified Direct Strength Method

Júlia N. Mauad[1], André L. R. Brandão[2], Marcílio S. R. Freitas[1]

[1]*Civil Engineering Program, Federal University of Ouro Preto*
Ouro Preto, 35.400-000, Minas Gerais, Brazil
julia.mauad@aluno.ufop.edu.br, marcilio@ufop.edu.br
[2]*Institute of Integrated Engineering, Federal University of Itajubá*
Itabira, 37.500-903, Minas Gerais, Brazil
andreriqueira@unifei.edu.br

Abstract. Cold-formed steel (CFS) framing is an economical and efficient structural solution, as it provides high strength and low self-weight. Built-up sections, formed by combining two or more CFS members, can reduce instabilities and obtain more versatility. An approach adopted in standards from several countries for the design of CFS bars is the direct strength method (DSM), which allows calculation of axial resistance from elastic buckling loads, considering global, local and distortional buckling modes. Currently, there are proposals to modify the DSM for the design of built-up sections, aiming to better fit experimental and numerical data. This study aimed to investigate these modifications of original DSM formulae by applying them to a database of experimental compression tests on built-up "I", or back-to-back, sections. Using the database results, it was possible to calculate the professional factor (P), obtained from the ratio between experimental and theoretical results and to obtain reliability indices (β) related to theoretical methods, the professional factor was evaluated as a continuous random variable.

Keywords: *Built-up columns; DSM; FORM; Reliability; Cold-formed steel.*

1 Introduction

Cold-formed steel profiles are increasingly viable for use in the construction industry, given the speed, cost-effectiveness, and sustainability demanded by the market. This structural element can be efficiently used in warehouses, mezzanines, industrial storage systems, and Light Steel Frame (LSF) systems (Yu [1]). Cold-formed profiles (CFP) are obtained from thin flat sheets formed by a mechanical process at room temperature (ABNT [2]). Due to their high local slenderness, these elements must be carefully analyzed for structural instability phenomena such as global buckling, local buckling, and distortional buckling under compression.

Built-up sections are formed by the combinations of two or more profiles using connectors, according Andrade et al. [3] e Zhang and Young [4] combined profiles are expected to have better performance than single ones. The current design codes were verified for limited types of sections, therefore more studies are important to investigate the behaviour of build-up structures.

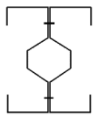
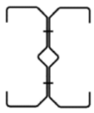
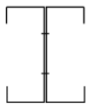
This study presents a survey of experimental compression studies on built-up profiles and proposed formulations for the design of these profiles. A comparison was made between theoretical and experimental values to assess the reliability of the evaluated formulations. For this purpose, a database of experimental results was compiled, and theoretical resistances were calculated using four methods: Direct Strength Method (Schafer and Pekoz [5], Freitas et al. [6]), and modifications of the original formulations proposed by Zhang and Young [4], Roy et al. [7] and Li and Young [8].

2 Experimental database

For the present study, it was necessary to compare theoretical and experimental values in order to assess the reliability in each evaluated formulations. Therefore, an experimental database was compiled, consisting of results of compression test on built-up section columns.

A total of 178 data points were selected for I-section profiles from the studies of Zhang and Young [9], Lu et al. [10], Roy et al. [7], Fratamico et al. [11], Aghoury et al. [12], Selvaraj and Madhavan [13], and Li and Young [8]. Four types of sections were considered: plain I-sections, stiffened I-sections, stiffened I-sections with hexagonal core folds, and stiffened I-sections with quadrilateral core folds. The following table summarizes the cross-sectional profiles and the number of data points used.

Table 1. Build-up sections database

Section		Authors	Number of tests (N)
stiffened I-sections with hexagonal core fold		Zhang e Young [9]	21
		Aghoury et al. [12]	8
stiffened I-sections with quadrilateral core folds		Li e Young [8]	11
stiffened I-sections		Lu et al. [10]	18
		Fratamico et al. [11]	16
		Roy et al. [7]	59
plain I-sections		Selvaraj e Madhavan [13]	45
Total:			178

3 Design procedures

3.1 Direct strength method (DSM)

The Direct Strength Method (DSM), as described by Schafer and Pekoz [5], represents an efficient alternative for determining the strength of members in cold-formed steel (CFS) structures. To apply this method, critical axial compression forces for global, local, and distortional elastic buckling, denoted as P_{cre}, P_{crl}, and P_{crd}, respectively, must be determined. These critical forces were obtained using the finite strip method by the CUFSM program.

The equations for DSM for global, local, and distortional buckling (P_{ne}, P_{nl}, P_{nd}) are presented below.

$$P_{ne} = \begin{cases} (0.658^{\lambda_c^2})P_y & , \lambda_c \le 1.5 \\ \left(0.877/_{\lambda_c^2}\right)P_y & , \lambda_c > 1.5 \end{cases} \quad (1)$$

$$P_{nl} = \begin{cases} P_{ne} & , \lambda_l \le 0.776 \\ (1 - 0.15(1/\lambda_l)^{0.8})(1/\lambda_l)^{0.8}P_{ne} & , \lambda_l > 0.776 \end{cases} \quad (2)$$

$$P_{nd} = \begin{cases} P_y & , \lambda_d \le 0.561 \\ (1 - 0.25(1/\lambda_d)^{1.2})(1/\lambda_d)^{1.2}P_y & , \lambda_d > 0.561 \end{cases} \quad (3)$$

The reduced slenderness ratios are.: $\lambda_c = \sqrt{\frac{P_y}{P_{cre}}}$; $\lambda_l = \sqrt{\frac{P_{nl}}{P_{crl}}}$; $\lambda_d = \sqrt{\frac{P_y}{P_{crd}}}$ and $P_y = Af_y$.

3.2 Modified DSM

The Direct Strength Method (DSM) was developed as a semi-empirical approach, initially calibrated using specific individual profiles. Consequently, several authors have proposed modifications to the original formulation to the design of structures made of built-up profiles. These modifications are based on experimental results and parametric studies.

In this study, modified DSM formulations proposed by Zhang and Young [4], Roy et al. [7], and Li and Young [8] were utilized. These formulations are referred as DSM_{ZY} , DSM_{RT} and DSM_{LY} , respectively, as presented below.

Table 2. Modified DSM

Authors	Modified DSM
DSM_{LY}	$P_{nl} = \begin{cases} P_{ne} & , \lambda_l \le 0.757 \\ (1 - 0.13(1/\lambda_l)^{0.6})(1/\lambda_l)^{0.6}P_{ne} & , \lambda_l > 0.757 \end{cases}$ $P_{nd} = \begin{cases} P_y & , \lambda_d \le 0.761 \\ (1 - 0.20(1/\lambda_d)^{1.2})(1/\lambda_d)^{1.2}P_y & , \lambda_d > 0.761 \end{cases}$
DSM_{RT}	$P_{ne} = \begin{cases} (0.61^{\lambda_c^2})P_y & , \lambda_c \le 1.5 \\ \left(0.84/_{\lambda_c^2}\right)P_y & , \lambda_c > 1.5 \end{cases}$
DSM_{ZY}	$P_{nl} = \begin{cases} P_{ne} & , \lambda_l \le 0.673 \\ (1 - 0.22(1/\lambda_l)^{1.0})(1/\lambda_l)^{1.2}P_{ne} & , \lambda_l > 0.673 \end{cases}$ $P_{nd} = \begin{cases} P_y & , \lambda_d \le 0.761 \\ (1 - 0.20(1/\lambda_d)^{1.2})(1/\lambda_d)^{1.2}P_y & , \lambda_d > 0.761 \end{cases}$

4 Professional factor

The professional factor (P) is the ratio between experimental and theoretical results. The professional factor was calculated for each sample studied, considering each theoretical calculation methodology. To conduct a statistical study, the professional factor was treated as a random variable. The mean, standard deviation (SD), and coefficient of variation (COV) were calculated for each group, considering the four methods.

To identify the probabilistic distribution function (PDF) that best represents each set of data, the MINITAB program was utilized. This allowed for the generation of histograms and the results of the Anderson-Darling (AD) statistical test. Tables 3 and 4 below present the AD values obtained and the statistical data for the P variables. Figure 1 shows histograms with the fitted distributions.

Table 3 - Anderson-Darling (AD) values

PDF	DSM	DSM_{LY}	DSM_{ZY}	DSM_{RT}
Minimum Extreme Value	11.203	7.702	17.535	11.262
Maximum Extreme Value	2.164	1.419	3.581	2.207
Gamma	1.655	1.205	2.874	2.032
Weibull	4.732	3.388	7.919	5.166
Normal	2.834	2.354	4.448	3.501
Lognormal	**1.605**	**1.100**	**2.820**	**1.846**

Table 4 – Statistical data for the professional factor

	DSM	DSM_{LY}	DSM_{ZY}	DSM_{RT}
Mean	1.011	0.958	1.096	1.022
SD	0.250	0.240	0.273	0.259
COV	0.247	0.251	0.249	0.254
PDF	Lognormal	Lognormal	Lognormal	Lognormal

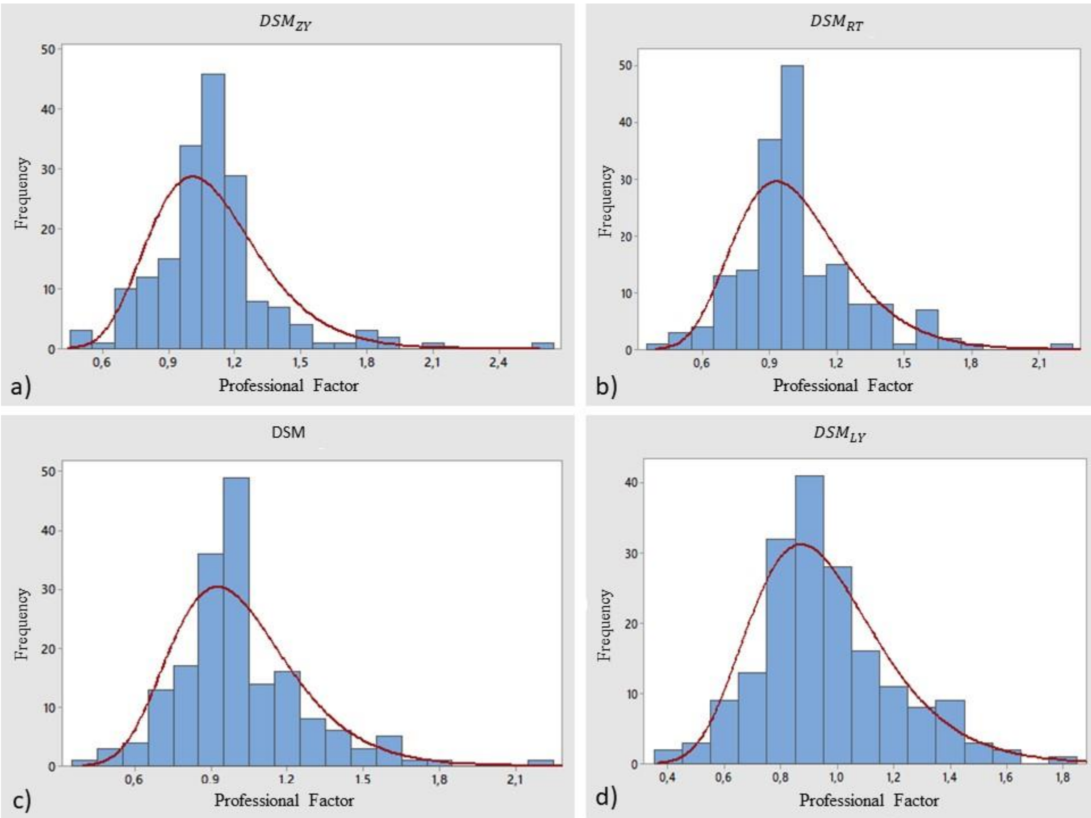


Figure 1 – Histograms: a)DSM_{ZY}; b)DSM_{RT}; c)DMS; d)DSM_{LY}

5 Reliability analysis

5.1 Reliability methods

In the case of linear limit state functions and marginal probability distributions resembling normal distributions, accurate estimates of failure probability and reliability indices are obtained using first-order analytical methods for reliability analysis.

In the First Order Reliability Method (FORM), failure equation is mapped to the standard normal space, and an iterative problem is solved to find the design point, the point on the limit state function closest to the origin. The limit state function is linearized at the design point. The reliability index (β) represents the distance between the design point and the origin.

The First Order Second Moment (FOSM) method, a precursor to FORM, is based on a Taylor series approximation of the linear limit state function around the mean values of the variables. The reliability index is obtained by the ratio of the mean to the standard deviation of the limit state equation. FOSM is a simplified method that uses only second moment statistics, mean and standard deviation, without considering the probability distribution function of the random variables (Haldar and Mahadevan [12]).

Monte Carlo Simulation (MCS) has been widely used to validate the accuracy of results obtained from analytical methods, as it is a flexible and robust statistical technique capable of handling complex problems. MCS involves generating multiple sets of random values for the variables involved in the reliability problem. These sets of values are then used to assess how often the limit state function falls within the failure region, thereby calculating the probability of failure. The accuracy of this method improves with an increasing number of simulations.

By comparing the results of MCS with those obtained from FORM or FOSM, researchers can verify the accuracy of analytical approaches and understand the limitations of these methods in practical applications.

5.2 Failure mode

For reliability analysis, a function is considered that relates the resisting forces to the applied forces acting on the element. It is desirable that the resistance exceeds the applied load to prevent failure. If the applied load exceeds the resistance, failure occurs.

$$G(.) = R_n\, M\, F\, P - (D_n\, D + L_n\, L) \tag{4}$$

where M, F, P, D, and L are dimensionless independent random variables representing uncertainties related to material, geometry, model error, dead load and live loads, respectively, and R_n, D_n and L_n are the nominal strength and applied Dead and Live loads. The statistics of these variables include the bias ratio (mean to nominal value ratio) and coefficient of variation (COV). The statistical parameters used were obtained from Ellingwood et al. [13] and are summarized in Table 5. The data for variable P were obtained using a statistical study presented earlier.

Table 6 presents the resistance factors and load factors from ABNT [2] and AISI S100 [14], as well as the load ratio $\rho = L_n / D_n$ adopted in the reliability calibration of cold-formed steel structures.

Table 5: Statistical parameters

Variable	Mean	COV	PDF
M	1.10	0.10	Lognormal
F	1.00	0.05	Lognormal
D	1.05	0.10	Normal
L	1.00	0.25	Maximum Extreme Value

Table 6: Calibration data for reliability analysis

		D	L	ρ
LRFD	1.175*	1.2	1.6	5
LSD	1.25	1.25	1.5	3
NBR	1.2	1.25	1.5	3
NBR	1.2	1.25	1.5	5

* the specified value in AISI S100 (2016) is $\phi = 0.85$, with the inverse value equal to $\gamma = 1.175$.

5.3 Reliability indices

The results obtained for the reliability index for each reliability method and calculation mode are summarized in table 7 and illustrated in Figure 2. The target values for reliability index are 2.5 for LRFD and NBR, and 3.0 for LSD. It can be observed that all results are below the target, the methods proposed by Zhang and Young [5] show higher results and the method proposed by Li and Young [6] show the lowest values. This reflects in lower failure probabilities for DSM_{ZY} and higher failure probabilities for DSM_{LY}.

Table 7: Reliability indices

Proposed Formulae	Reliability Method	Calibration LRFD		LSD		NBR (ρ=3)		NBR (ρ=5)	
		β	***P_f*** ***(%)***	***β***	***P_f*** ***(%)***	***β***	***P_f*** ***(%)***	***β***	***P_f*** ***(%)***
DSM	**FOSM**	2.015	2.193	2.064	1.950	1.940	2.617	1.927	2.702
	FORM	1.993	2.316	2.028	2.129	1.902	2.860	1.901	2.863
	SMC (200,000)	2.004	2.251	2.028	2.129	1.915	2.772	1.914	2.778
DSM_{LY}	**FOSM**	1.842	3.274	1.884	2.981	1.761	3.914	1.754	3.973
	FORM	1.813	3.494	1.842	3.273	1.717	4.297	1.722	4.249
	SMC (200,000)	1.818	3.450	1.858	3.161	1.717	4.296	1.733	4.155
DSM_{ZY}	**FOSM**	2.242	1.247	2.299	1.076	2.176	1.480	2.154	1.562
	FORM	2.225	1.305	2.266	1.172	2.140	1.616	2.134	1.644
	SMC (200,000)	2.240	1.255	2.285	1.115	2.167	1.511	2.128	1.668
DSM_{RT}	**FOSM**	2.017	2.185	2.064	1.951	1.942	2.607	1.929	2.684
	FORM	1.991	2.326	2.025	2.146	1.900	2.869	1.901	2.866
	SMC (200,000)	1.995	2.300	2.035	2.095	1.921	2.738	1.907	2.827

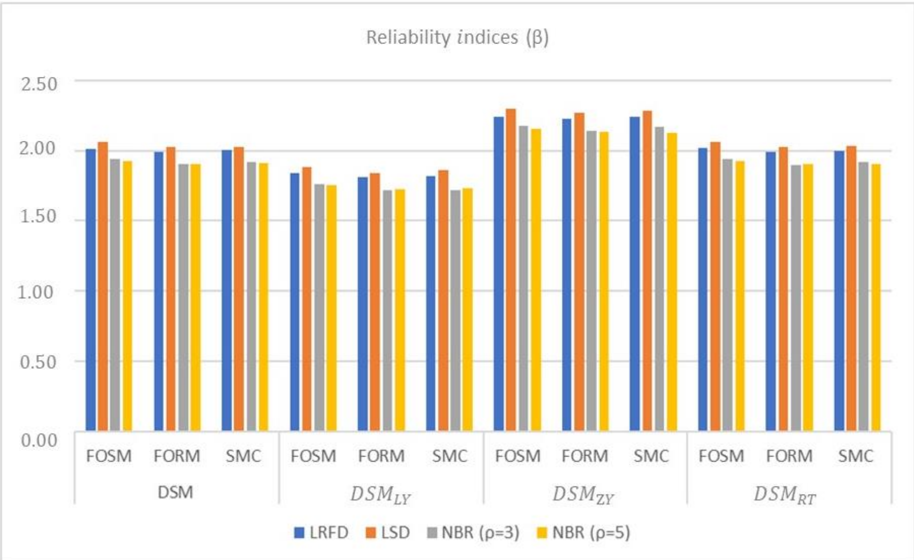


Figure 2 – Reliability indices.

All four adopted methods showed significantly high dispersion values, ranging between 0.24 and 0.273, with only the LY method showing a mean value below one. It is interesting to note that the proposed methods were formulated based on experimental and numerical data, but in this work, only experimental values were considered in the database. Also, the evaluated experiments show variations in several random variables such as length, connector spacing, and cross-sectional area, that can result in higher dispersion.

In the analysis, three reliability methods were adopted, FORM and FOSM being analytical methods. FORM is expected to be the most accurate since it considers the PFD of the random variables. However, FOSM is widely used and serves as the calibration basis for the AISI standard. The Monte Carlo method is used to verify the other methods. It is observed that the values among the three methodologies do not differ much, but

FOSM results in slightly higher values. Regarding the design philosophies considered, it is noticeable that values calculated by LSD were the highest and those by the Brazilian standard were the lowest.

6 Conclusions

The results show a low reliability of the DSM method and its adaptations in this study, due to the high data dispersion among groups. In all cases, the reliability indices were below 2.5, especially in the adaptation proposed by Li and Young [8], where, besides the high data dispersion, the mean of the Professional Factor was slightly below 1. Zhang and Young's adaptation [4] was the closest to the target of 2.5 (LRFD), but still well below the target of 3.0 (LSD). The results suggest the need to reduce the resistance factors (ϕ) to meet the target reliability indices. However, for a more consistent reliability analysis of built-up members, expanding the database is recommended for a stronger statistical study of the Professional Factor.

Acknowledgements. This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001. The authors are grateful to the Federal University of Ouro Preto (UFOP).

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] C. Yu, Recent Trends in Cold-Formed Steel Construction. Woodhead Publishing. Woodhead Publishing. Series in Civil and Structural Engineering: Number 65, 2016.

[2] ABNT. NBR 14762: Dimensionamento de estruturas de aço constituídas por perfis formados a frio. Rio de Janeiro: Associação Brasileira de Normas Técnicas, 2010.

[3] C. P. Andrade, M. S. R. Freitas and A. L. R. Brandão. “Reliability of built-up cold-formed steel columns designed by the direct strength method”. In: XLIII IberianLatin American Congress on Computational Methods in Engineering, pp. 1-7, 2022.

[4] J. Zhang, B. Young, Finite element analysis and design of cold-formed steel built-up closed section columns with web stiffeners, Thin-Walled Structures, 131, 223–237, 2018.

[5] B.W. Schafer, T. Peköz, Direct strength prediction of cold-formed steel members using numerical elastic buckling solutions. In: Yu, W. W., Laboube, R. (Eds), Proceedings of 14th International Conference on Cold-formed Structures, pp. 69-76, 1998.

[6] M. S. R. Freitas, A. L. R. Brandão, A. M. S. Freitas, Resistance factor calibration for cold-formed steel compression members. REM. Revista Escola de Minas, v. 66, pp. 233-238, 2013.

[7] K. Roy, T.C.H. Ting, H. H. Lau, J.B.P. Lim, Effect of thickness on the behaviour of axially loaded back-to-back cold-formed steel built-up channel sections - experimental and numerical investigation, Structure 16, 327–346, 2018.

[8] Q.Y. Li, B. Young, Experimental and numerical investigation on cold-formed steel built-up section pin-ended columns. Thin-Walled Structures, Volume 170, 2022.

[9] J. H. Zhang, B. Young, Compression tests of cold-formed steel I-shaped open sections with edge and web stiffeners. Thin-Walled Structures, Volume 52, pp. 1-11, 2012.

[10] Y. Lu, T. Zhou, W. Li and H. Wu, Experimental investigation and a novel direct strength method for cold-formed built-up I-section columns. Thin-Walled Structures, Vol. 112, pp. 125-139, 2017.

[11] D. C. Fratamico, S. Torabian, X. Zhao, K. J. R. Rasmussen, B. W. Schafer, Experiments on the global buckling and collapse of built-up cold-formed steel columns. Journal of Constructional Steel Research, Vol. 144, pp. 65-80, 2018.

[12] M. E. Aghoury, M. Tawfic and E. Amoush, Compressive Strength of Axially Loaded Built-up Sigma Cold Formed Sections Columns. Future Engineering Journal: Vol. 1: Issue. 1, Article 6, 2020.

[13] S. Selvaraj, M. Madhavan, Design of cold-formed steel built-up columns subjected to local-global interactive buckling using direct strength method. Thin-Walled Structures, Vol. 159, 2021.

[14] A. Haldar, S. Mahadevan, Probability, Reliability and Statistical Methods in Engineering Design. New York: John Wiley & Sons, 2000.

[15] B. Ellingwood, J. G. Macgregor, T. V. Galambos, C.A. Cornell, Development of a Probability-Based Load Criterion for American National Standard A58 – NBS Special Publication. National Bureau of Standards, United States Department of Commerce, Washington: D.C., 1980.

[16] American Iron and Steel Institute. AISI-S100-16: North American specification for the design of cold-formed steel structural members. Washington, D.C.: ANSI/AISI, 2016.