Reliability Analysis of Flat Slabs designed by the Simplified Punching Shear Design Method of Eurocode 2:2004

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Abstract. The punching shear phenomenon represents a limit state to be considered in the design of flat slabs. The failure is characterized by a rupture in the slab at the column connection, which, when it occurs across the entire floor, can lead to partial or total collapse of the slab. This phenomenon is empirically calculated and there are various normative methodologies for it. For this design, it is necessary to know the normal force and the bending moments at the slab-column connection in both directions of the column. However, some methods allow for an increase in the normal force and disregard the bending moment based on the position of the column in the plan, as simplified in one of the methods proposed by Eurocode 2:2004. This study aimed to discuss the reliability of slabs designed with this simplification, comparing the reliability levels of each column position in slabs with different spans. The failure prediction of ABNT NBR 6118:2023 was used, considering both normal force and bending moments in the equations. The estimation of the solicitation forces was performed using a model based on the 2D finite element method (FEM) with adaptations of the Equivalent Frame Method (EFM). The model error was calculated using the commercial and educational software TQS, which allows estimation of forces in flat slabs using a three-dimensional grid model. The reliability was simulated by the FORM (first-order reliability method). The results showed that the connections exhibited adequate reliability levels according to target reliability indices such as those of FIP MODEL CODE (2010) and ACI 318 (2019), with higher conservatism in terms of safety for corner and edge columns, which have a smaller influence area in terms of normal force. On the other hand, there was more conservatism noted in the design coefficient proposed by the simplified method of Eurocode 2:2004 for edge and corner columns.

Keywords: punching shear, reliability, FORM, FEM, EUROCODE 2:2004.

1 Introduction

The design of structural elements involves understanding the resistance and demand aspects. Although the state-of-the-art in analytical or numerical failure models for various structures shows excellent accuracy compared to experimental results, various simplifications are still permitted in design according to established norms. The challenge in design lies in finding the optimal balance between computational cost for predicting the strength and demand of a particular element without compromising reliability levels, aiming to align closely with target reliability indices specified in established standards.

This research aims to evaluate whether omitting bending moments, compensated by an increase in normal force, in the simplified design for punching shear in flat slabs as per Eurocode 2:2004 leads to conservative, unsafe, or near-target reliability levels. To achieve this, the ultimate limit state behavior of the structure in reliability analysis was based on the punching shear resistance model from ABNT NBR 6118:2023, which considers bending moments. The demand model was implemented using a 2D finite element approach in MATLAB, comparing model errors with the commercial and educational software TQS.

Reliability analysis was conducted using the FORM method through UQLab software, also in MATLAB. Results indicate that central columns exhibit reliability indices close to the target values of ACI 318 and FIB MODEL

CODE, whereas edge and corner columns show excessive conservatism. This suggests that to achieve uniform reliability indices across different types of columns in a layout, the slab thickness should be defined based on edge or corner columns, with the central column possibly having a localized thickening, known as a capital, to reduce conservatism and enhance safety margins specifically for edge and corner columns.

2 Formulation

2.1 Simplified Punching Shear Design Method of EUROCODE 2:2004

For the design of flat slabs without punching reinforcement, the following equation was used, which dispenses the analysis of bending moments at the slab-column connection by amplifying the normal force with a factor β'. This factor depends on the position of the column in the slab: edge column is 1.4, for internal column is 1.15, and for corner column is 1.5 [1,2,3]. The term τ_{Rd} represents the design resistant punching shear stress, while τ_{Sd} represents the design applied punching shear stress.
The ξ represents the implicit safety value in punching shear design according to ABNT NBR 6118:2023, d represent the effective slab depth, ρ the geometric reinforcement ratio for punching shear, f_{ck} the characteristic compressive strength of concrete, f_S the acting force at the slab-column connection and u_1 the control perimeter at $2d$ from the column.

$$g_{psd} = \tau_{Rd} - \tau_{Sd} = \xi\left(1+\sqrt{\frac{20}{d}}\right)(100\rho f_{ck})^{\frac{1}{3}} - \left(\frac{f_{Sd}}{u_1 d}\right)\beta' \tag{1}$$

The term τ_R represents the resistant punching shear stress, while τ_S represents the applied punching shear stress.
The ξ represents the implicit safety value in punching shear design according to ABNT NBR 6118:2023, d represent the effective slab depth, ρ the geometric reinforcement ratio for punching shear, f_{ck} the characteristic compressive strength of concrete, f_{Sd} the design acting force at the slab-column connection and u_1 the control perimeter at $2d$ from the column. The reduction factor for f_c is 1.4, and the increase factor for the normal force f_S is the same.

2.2 Mechanical Model for Failure Estimation in Flat Slabs

The requested portion for evaluating failure in slab-column connections in flat slabs was based on the finite element method (FEM), wich consists breaking down a continuous medium into several elements. These elements are described by differential equations and, depending on the magnitude of the problem, must be solved using computational resources. The Equivalent Frame Method [4,5] is a method for estimating forces in flat slabs that allows analyses under the assumption of linear elastic material, simulating the slab as a bar element. The finite element formulation for this research is limited to the frame element.
In addition to assuming the material is linear elastic, the FEM formulation for linear structural analysis also assumes small displacements and constant boundary conditions, meaning they do not vary during load application [6]. Based on these conditions, the vector of nodal displacements of the structure $\boldsymbol{u}$ can be described, which is linearly related to the vector of external forces $\mathbf{F}_{ext}$, according to:

$$\mathbf{Ku} = \mathbf{F}_{ext} \tag{2}$$

In this equation, $\mathbf{K}$ represents the global stiffness matrix of the structure. If a coefficient ψ multiplies the force vector, such as $\psi\mathbf{u}$, the displacements will also increase by $\psi\mathbf{u}$. This reflects Hooke's Law, which states that stress is proportional to the strain multiplied by the material's elastic modulus.
The Saint-Venant principle is also considered, which indicates that the stress distribution can be considered equivalent in a section sufficiently far from the point of load application.
Returning to the frame element, it is assumed that each frame element has two nodes and three degrees of freedom for each respective node. Figure 1 depicts these elements and the nodal forces at each node, both for the global system and the local system.

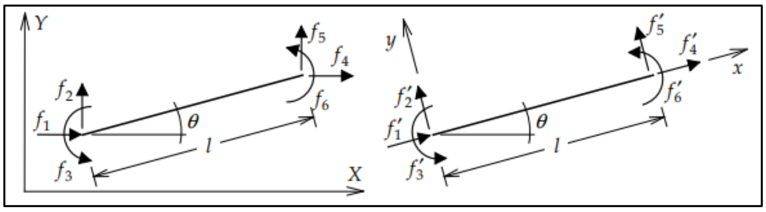

Figure 1. Representation of generalized forces for a bar in global and local systems

It is also possible to identify the forces from the vector $\mathbf{F}_{ext}$ in the global coordinates:

$$\mathbf{F}_{ext} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix} \tag{3}$$

In the global coordinate system, the nodal displacement vector $\mathbf{u}$ for planar frame elements consists of:

$$\mathbf{u} = \begin{bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{bmatrix} \tag{4}$$

In Equation 5, u_i, v_i and θ_i represent the horizontal displacement, vertical displacement, and rotation, respectively, where $i = 1, 2$ denotes the initial node 1 and the final node 2 of the element.
The relationship between displacements in the global and local coordinate systems is given by:

$$\begin{bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{bmatrix} = \mathbf{R} \begin{bmatrix} u_{L1} \\ v_{L1} \\ \theta_{L1} \\ u_{L2} \\ v_{L2} \\ \theta_{L2} \end{bmatrix} \tag{5}$$

The $\mathbf{R}$ represents the rotation matrix, described by:

$$\mathbf{R} = \begin{bmatrix} cos\alpha & sin\alpha & 0 & 0 & 0 & 0 \\ -sin\alpha & cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & cos\alpha & sin\alpha & 0 \\ 0 & 0 & 0 & -sin\alpha & cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

This formulation developed is valid for a single frame element. However, for the case simulated in this research, there is an integration of multiple frame elements. Thus, the local stiffness matrices and local force vectors for each element must be appropriately rotated and superimposed in order to generate a global equilibrium equation for the structure.
For the assessment of resistance, the equations used in the reliability analysis considered the bending moments acting on the slab-column connection, based on the resistance models of ABNT NBR 6118:2023. For the case of a central column, the following was used:

$$g_{psc}(\mathbf{X}) = \tau_R - \tau_S = \xi\left(1+\sqrt{\frac{20}{d}}\right)(100\rho f_c)^{\frac{1}{3}} - \left(\frac{f_S}{u_1 d}\right) \quad (7)$$

For the case of an edge column, the following was used:

$$g_{pse}(\mathbf{X}) = \tau_R - \tau_S = \xi\left(1+\sqrt{\frac{20}{d}}\right)(100\rho f_c)^{\frac{1}{3}} - \left(\frac{f_S}{u_1 d}\right) - \left(\frac{K_1 M_{S1}}{W_{p1} d}\right) - \left(\frac{K_2 M_{S2}}{W_{p2} d}\right) \quad (8)$$

The K_1 term represents the coefficient used that provides the portion of the moment transmitted to the column by shear, depending on the relevant parameters C_1 and C_2, lengths of the column's. The M_{S1} or M_{kx} represents the bending moment in the x direction of the column acting on the slab-column connection, according to Figure 2. While M_{S2} or M_{ky} represents the bending moment in the y direction. The terms W_{p1} and W_{p2} represents the plastic resistance modulus perpendicular to the free edge, calculated for the perimeter u.

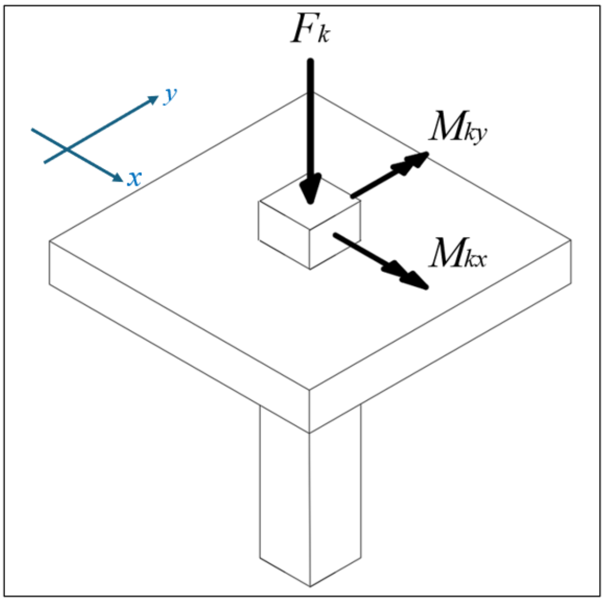

Figure 2. Forces acting on the slab-column connection in the coordinate system

The same provisions apply to the corner column when no moment acts in the plane parallel to the free edge. However, since the corner column has two free edges, the verification must be carried out separately for each one, considering the bending moment whose plane is perpendicular to the adopted free edge.
The model error for the resistance portion consisted of the variable E_{Rp}, wich is gaussian and has the mean value as 0.187 and the standard deviation as 0.02 [7]. This value aims to correct the resistant portion of the resistance model from ABNT NBR 6118:2023 with experimental results used as a basis to assess the safety factor adopted in the standard.
The model error for the requested portion is calculated by comparing the FEM implementation with a spatial bar software using a discretized grid slab, TQS, which allows for the assessment of forces in flat slabs. The ratio between the effort from TQS and the effort from the FEM implementation was used to multiply the load factor in the limit state equation for reliability analysis.

2.3 Reliability Analysis

The method used was FORM (First Order Reliability Method). It is commonly used in structural engineering, reliability engineering, and risk assessment to predict the likelihood of structural failure, evaluate safety margins, and design systems with adequate reliability [8]. It helps engineers make informed decisions by quantifying uncertainties and assessing the likelihood of adverse events. It is based on the transformation by Hasofer and Lind [9]. As a transformation method, it involves solving an optimization problem to find the design point.

In this method, it is possible to incorporate into the analysis, in addition to the mean and covariance of the random variables, information regarding probability distributions as well as the correlation between the random variables of the problem.
The method essentially consists of mapping the joint probability function, $f_{\mathbf{X}}$, from the standard design space $\mathbb{X}$ to the isoprobabilistic space $\mathbb{Y}$. To do this, the normal approximation principle [10] and the Nataf transformation [11] must be applied.
The normal approximation principle involves determining, for a point x_i^*, an equivalent normal distribution that preserves the probability content of the original cumulative distribution $F_X(x_i^*)$ at this point. Consequently, the Nataf transformation eliminates possible correlations between variables through orthogonal decomposition or Cholesky factorization.
In the space $\mathbb{Y}$, the limit state function $g_i(\mathbf{Y})$ is approximated by a hyperplane tangent to it at the design point. This point, $\boldsymbol{y}^*$, represents the most probable occurrence in the failure domain. This can be obtained from the following optimization problem:

$$\begin{aligned} &\text{Determine: } \boldsymbol{y}^* \\ &\text{That minimizes : } \|\boldsymbol{y}\| = \sqrt{\boldsymbol{y}^t\boldsymbol{y}} \\ &\text{Subject to constraint: } g(\boldsymbol{y}) = 0 \end{aligned} \tag{9}$$

In this equation, $\boldsymbol{y}$ represents the vector of random variables in the isoprobabilistic space, while $\|\boldsymbol{y}\|$ is the distance from the point to the origin. This solution provides the reliability index by Hasofer and Lind [9], denoted as β.

3 Numerical example: Flat slab with corner, edge and central columns

The example addressed in this work consists of a single-story residential building, where corner, edge, and central columns can be observed, as shown in Figure 3. It was designed and simulated at realibility analysis the spans of 6,7,8 and 9 meters. The spans were considered symmetrical in all four models. C1 to C9 are the columns, repare the position of the columns avaliated, they are described: C1, C2 and C5. The f_{ck} was considered constant at 30 MPa in all examples. In addition to the self-weight, permanent loads of approximately 1 kN/m² and occupancy loads of approximately 1.5 kN/m² were considered, according to [12].

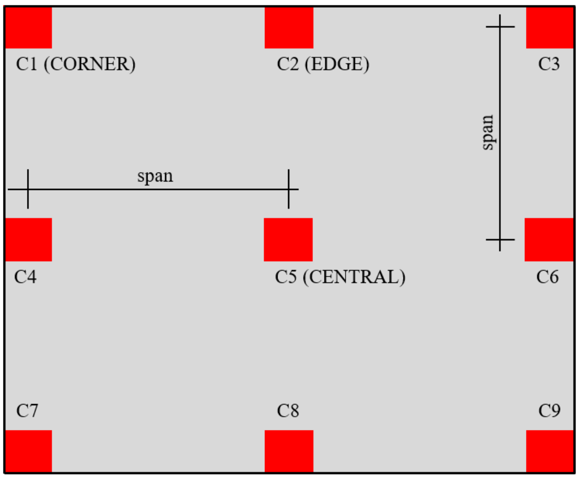

Figure 3. Executive form used as basis for design and reliability analysis

After the design, the sections of the slabs and columns were as specified in Table 1. The displacement limits adopted were those pecified in [4], and the assessment of the ultimate limit state for punching was carried out according to item 2.1. The definition of the geometric reinforcement ratio, ρ, was estimated according to the bending model implemented in the finite element method.

Table 1. Sections of slabs and columns

Flat slab's span	Slab height (cm)	Column side (cm)
6 meters	23	25
7 meters	29	35
8 meters	36	35
9 meters	40	50

The parameters of the random variables adopted in the reliability simulations are listed in Table 2.

Table 2. Random variables parameters

Random Variable	Distribution	Mean	Standard Deviation	Source
f_c	Gaussian	$1.2200f_{ck}$	$0.1830f_{ck}$	[14]
g	Gaussian	$1.0600g$	$0.1272g$	[14]
q_{50}	Gumbel	q_{50}	$0.4q_{50}$	[14]
E_{Rp}	Gaussian	0,1870	0.02	[7]

3.1 Model error results

Figure 4 shows the model errors for the normal force, N_c, bending moment in *x*, M_{cx}, and bending moment in *y*, M_{cy}, for corner, edge, and center columns, for each span type adopted. A horizontal line was highlighted in model error e1, which corresponds to the ideal model error.

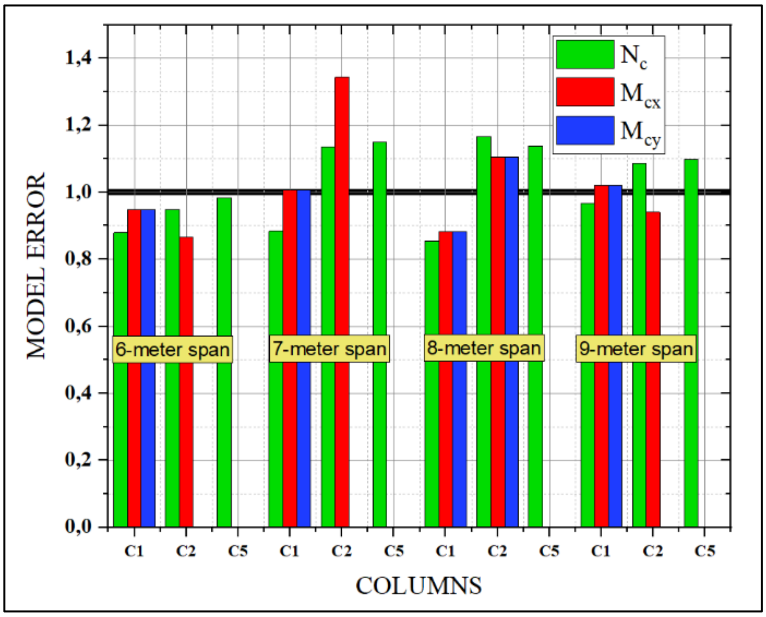

Figure 4. Model erros comparing TQS and FEM model implemented

The average error for the N_c was 1.037, for M_{cx} it was 1.0138, and for M_{cy} it was 0.82. Thus, it is possible to see that the greatest error was in M_{cy}.

3.2 Results of the reliability analysis

The reliability indices, β, for the slab-column connections under punching shear, noting that there is no punching reinforcement, are shown in Figure 5. As a comparison parameter, the target reliability indices of the [5] and [1] standards were plotted on the graph.

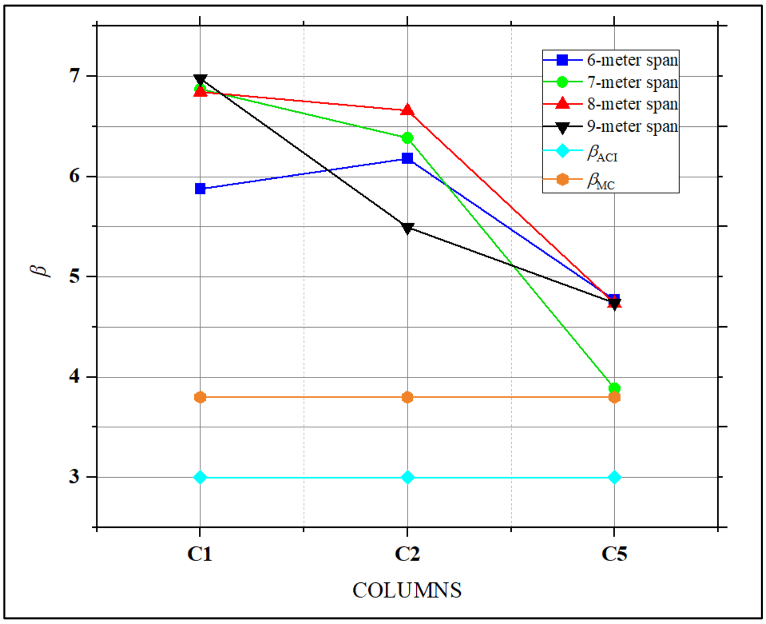

Figure 5. Punching shear reliability indices for the corner, edge, and center columns in all evaluated models.

It is possible to see that the Eurocode simplification provides reliability indices with a greater tendency towards conservatism, even in failure models that consider bending moments, which are neglected in the design. However, the corner and edge columns exceeded the target with more margin than the center columns, which actually governed the design of the slab.

4 Conclusions

It has been found that the coefficients used in the simplified punching shear design model provide structures with safety levels adequate to the target reliability indices of established standards such as [5] and [13]. Additionally, it was observed that the lowest safety margin is in the central columns, where the concentration of axial force is greater compared to other columns, even when the moment effect at the connection is near of zero, highlighting the greater significance of axial force over bending moments. Edge and corner columns, on the other hand, exhibited higher safety margins. A suggestion for future research would be to assess whether the order of magnitude of the reliability indices presented here remains consistent in structures with asymmetric columns, where the bending moment effect at the slab-column connection could be more significant.

Acknowledgements. The authors would like to thank CAPES and CNPq for financial support of this research.

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