System reliability analysis of cold-formed steel structures for serviceability limit states

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Abstract. The adoption of numerical analysis in structural design practice has ensured greater control over the real behavior of structures. Design-by-analysis approaches deviate from traditional methods by directly considering the load capacity of the entire system rather than individual element analysis. These analysis models may produce results closer to the real behavior of the structure in service, enabling the safe utilization of lighter structural systems. In this context, the evaluation of serviceability limit states becomes relevant, particularly for cold-formed steel structures, typically assembled using slender elements. This paper presents the application of a reliability-based procedure to assess the compliance of cold-formed steel frames with serviceability limit state criteria for excessive displacement. Considering two structure models: warehouse portal frames and storage rack frames, the displacement limits are extracted from design standards, along with the target reliability indices for the corresponding limit state. The subsequent step evaluates the system reliability considering typical load cases for these structures and accounts for uncertainties in loads and stiffness parameters. For this purpose, a MATLAB reliability framework is presented, coupling the First Order Reliability Method to the MASTAN2 finite-element package. Conclusions regarding reliability indices and the adequacy of design practice are presented.

Keywords: design-by-analysis, structural reliability, cold-formed steel structures, serviceability limit states.

1 Introduction

In the last decade, several researches have been carried out to understand and improve design-by-analysis approaches for steel structures. Its difference when compared to the traditional design process is that the design of the elements and the connections is made simultaneously with the structural analysis. For hot-rolled steel structures, standards such as AISC 360 [1] and the Brazilian NBR 8800 [2] bring some provisions regarding the global analysis and some design-by-analysis procedures. For the case of cold-formed steel (CFS) structures, the Brazilian NBR 14762 [3] does not indicate an specific criteria for the analysis of these structures, and the AISI S100 [4] highlights the need of an rigorous second-order analysis with individual elements resistance verifications. Several researchers have been reporting the need of shell finite element analysis for the basis of the analysis and the design-by-analysis procedure for CFS structures [5]–[9], mainly because of cross-section instabilities. Few publications were found on system reliability considering serviceability limit states (SLS). Arrayago and Rasmussen [10] performed such analysis for stainless steel frames, but also using shell elements. The use of advanced beam elements might be useful for SLS analysis of small displacement structures, under a limit where no element of the structure reaches local instability loads. Its use might be advantageous when compared to shell finite elements, due to the lowest computational effort. Researches such as the presented by Rinchen, Hancock and Rasmussen [11], Liu, Gao and Ziemian [12] and Abdelrahman, Lotfy and Liu [13] presents some improved beam-column element for elements with nonsymmetric cross-section, that can be used for the system analysis of CFS structures under service loads.

This paper presents a reliability analysis of cold formed steel structures designed at its serviceability limit state for excessive displacement. The objectives are to verify the applicability of advanced beam-column elements on the SLS verification, and analyze the actual safety requirements of the design standards for this limit state, with comparisons to the obtained reliability indices for two structural systems. For this task, an improvement of the reliability framework developed by Mapa *et al.* [14] is presented, with a new extension to couple a modified MASTAN2 [15] batch model to the First Order Reliability Method (FORM) for a new limit state function for serviceability.

2 Second order elastic analysis at MASTAN2

For the identification of the structure's displacements, a second order linear analysis was performed. Since all the frames that was used for the examples in Section 4 have bars with monosymmetric cross-sections, the Liu, Gao and Ziemian line element [12] was used, to provide a proper monitoring of displacement under service loads. The assumptions made for the derivation of the finite element formulation are: local and section distortional buckling are not included; b) material is linear elastic; c) conservative loads are considered; d) element deflection can be moderately large, but strains remain small; and e) shear deformations are neglected. Figure 1 shows the local element degrees of freedom, with θ_{b1} and θ_{b2} being an additional degree of freedom at the starting and ending nodes, representing the warping deformations. Further details about the complete finite element formulation can be found at Liu, Gao and Ziemian [12] paper.

Figure 1 line element was implemented at MASTAN2 [15] in the 5.1 version, and its applicability was tested by Liu *et al*. [16]. Figure 1 also shows some examples used to test the accuracy of the proposed line-element. Under the regime where local instabilities did not occur yet, the proposed finite element worked well for Fig. 1b portal frame and the rack column lateral displacement from Fig. 1c. Liu *et al*. [16] reported that the obtained equilibrium path from Fig. 1b rack frame sightly differ from the experimental, but in a conservative way. The conclusion for the measured rotation of the the column in Fig. 1e is similar.

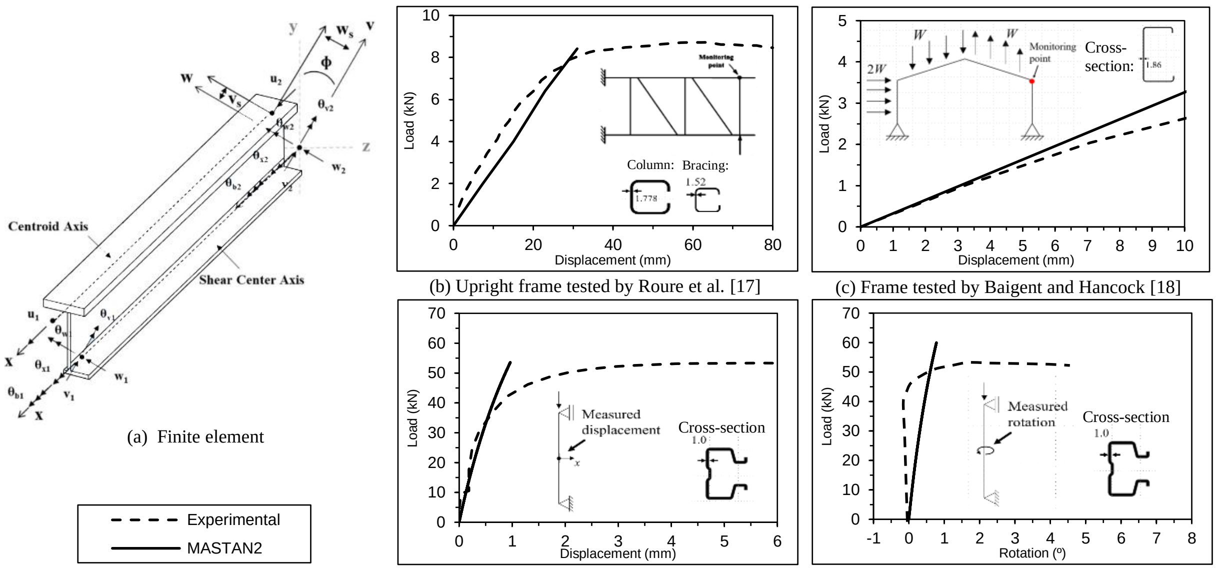


Figure 1. Advanced line-element for nonsymmetric cross-sections [12].

3 Reliability analysis

3.1 Reliability framework at MATLAB

The reliability framework used in this paper is based on a previous version developed by Mapa *et al.* [14] for ultimate limit states. The methodology used here calls the finite element solution from MASTAN2 in each iteration of the First Order Reliability Method (FORM); a different solution from that used by Arrayago and Rasmussen [10], that requires a statistical characterization of the system stiffness in the direction of the analyzed displacement. In the updated reliability program (Fig. 2), the new versions of MASTAN2 userdefined batch files are used, so as the limit state function for serviceability. Details about the reliability analysis are discussed in the next sections.

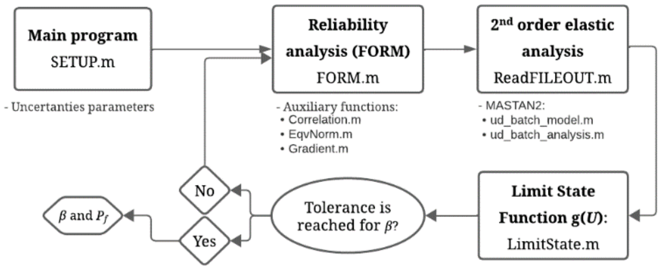
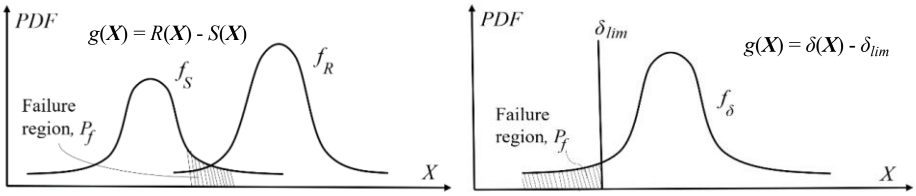


Figure 2. Reliability assessment procedure.

3.2 Limit state function for excessive displacement

For ultimate limit states, the load effect S is compared to the resistance R, and both are affected by uncertainties, so that both are also random variables. For the serviceability limit state (SLS) for excessive displacement, the actual displacement δ of an arbitrary degree of freedom is compared to a normative displacement limit δ_{lim}, but only δ is a random variable, with δ_{lim} being a deterministic value. Figure 3 shows the differece between the failure region of these two limit states. While ANSI MH16.1 [20] specifies the maximum lateral drift of $H/240$ for loaded storage rack frames, AISI S100 does not have explicit limits for maximum displacement of structures in service. In this research, the maximum lateral drift of $\delta_{lim}= H/300$ [3], [21] and the maximum vertical deflection of $\delta_{lim} = L/250$ [3], [10] were used for the portal frames.



(a) Failure region for ultimate limit state (b) Failure region for serviceability limit state

Figure 3. Failure region for different limit states.

3.3 First Order Reliability Method (FORM)

The limit state of a system is defined by $g(\mathbf{X}) = 0$, for a failure region equals to $g(\mathbf{X}) < 0$. The probability of failure (P_f) can be analytically obtained by integrations over the probability density functions (PDF), as per eq. (1):

$$P_f = \int \ldots \int_{g() = 0} f_X (x_1, x_2, \ldots, x_n) dx_1 dx_2 \ldots dx_n \tag{1}$$

The solution of the integral in eq. (1) is difficult, even if the joint probability density function is known, even for the simplest problems [22]. Then, what is commonly done to compute the probability of failure is to use approximation techniques to solve eq. (1) integral, such as first or second order reliability methods (FORM or SORM) or simulation techniques [22].

What is today known in literature as FORM is the expansion of the Hasofer-Lind technique [23] to obtain both probability of failure and reliability index (β) for limit states functions with non-normal and possibly correleted random variables. In the method, the vector $\mathbf{X}$ of random variables is converted to a vector $\mathbf{X'}$ of uncorrelated equivalent normal random variables, using the Nataf transformation [24] and Rackwitz and Fiessler [25] expressions for equivalent normal parameters. In the reduced space of uncorrelated standard normal variables, the reliability index (β) is defined by the lowest distance between the linearized failure hyperplane to the origin, and the seek for this point in the surface (design point **X'***) is an optimization process, intended to minimize the distance between the hiperplane and the origin in a point **X** subjected to $g(\mathbf{X}) = g(\mathbf{X}') = 0$. In this paper, the HLRF algorithm [25] was used to solve the optimization process and obtain the reliability index as $\beta = \beta_n = \left(\mathbf{X}_n^{'*T}\mathbf{X}_n^{'*}\right)^{1/2}$, for n iterations. The convergence criteria adopted was a difference lower than 1×10^{-2} between β_k and β_{k+1}.

$$\mathbf{X}_{k+1}^{'*} = \frac{1}{\left|\nabla g\left(\mathbf{X}_k^{'*}\right)\right|^2}\left[\nabla g\left(\mathbf{X}_k^{'*}\right)^T \mathbf{X}_{k+1}^{'*} - g\left(\mathbf{X}_k^{'*}\right)\right]\nabla g\left(\mathbf{X}_k^{'*}\right) \tag{2}$$

4 Numerical examples

To test and exemplify the implemented reliability framework, three different examples were used, changing the structural system and the load combinations. Details about the structures, the load parameters used and target reliability indices will be discussed in the following sections. Care was taken in the three examples to ensure that under the serviceability limit load, no element reached a configuration where local instabilities develop. The structures worked inside the AISI S100 [4] envelope, at eq. (4).

$$\frac{P_s}{P_{nd}} + \frac{M_{x,s}}{M_{x,nd}} + \frac{M_{y,s}}{M_{y,nd}} \leq 1 \tag{3}$$

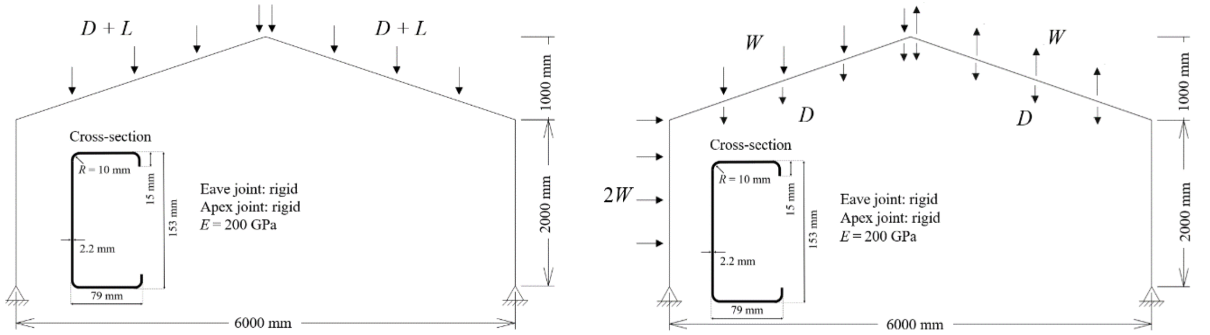
For the reliability analysis, the structures were considered designed just at its serviceability limit state for excessive displacement, so that $\delta_{lim} = \delta(\mathbf{X}_{\mathrm{nominal}})$. $\mathbf{X}_{\mathrm{nominal}}$ is the vector with the basic random variables nominal values. Based on the previous studies made by Cardoso [5] and Mapa [26], live load L, dead load D, wind load W, the Young's modulus E and the yield stress f_y was the most influential variables in frame behaviour. All these variables were considered in the reliability analysis, with the exception of f_y, since no cross-section of the frames reached the yelding plateau in the chosen load magnitude. Table 1 shows the statistical parameters of the random variables. According to Galambos and Ellingwood [27], is not reasonable to use for the serviceability criteria the same reference period than the used for the ultimate limit states (50 years). Galambos and Ellingwood [27] then suggests that the load criteria for checking deflections might be founded on the premise that the deflection limit should not be exceeded more than once, on the average, during one tenancy. The average period of tenant changes in office buildings is 8 years [27], and the parameters of the live load is attributed to this period are presented in Tab. 1. Details about the change of load parameters for different reference periods are explained by Rosowsky [28].
For the target reliability indices for serviceability limit states (SLS), Arrayago and Rassmussen [10] highlights that, since the safety is usually not generally an issue in serviceability limit states (SLS), serviceability checks do not require the use of safety factors or resistance factors and the reliability criteria for SLS generally results in lower prescribed target reliability indices β_0 than for ULS. For SLS, the authors indicate the use of target indices correspondent to an annual probability of excedence. In this case, for ASCE/SEI 7 [29], the annual probability of failure P_f = 5% associated to β_0=1.64 was used as reference. Details about the change of P_f and β_0 for different reference periods can be found at Haldar and Mahadevan [22] and Sykora and Holický [30].

Table 1. Statistical parameters of the random variables [10], [27].

Random variable	Reference period	Mean value	COV	Probability distribution
Young's modulus E	-	$1.00E_n$	0.06	Normal
Dead load D	-	1.00	0.10	Normal
Live load L	8 years	$0.65L_n$	0.32	Gumbel
Pallet live load L	8 years	$0.65L_n$	0.30	Normal
Wind load W	10 years	$0.51W_{n,50}$	0.50	Gumbel

4.1 Portal frame

The first example is a portal frame with the same geometry as the experimentally tested by Baigent and Hancock [18], and analyzed in Fig. 1. However, different than the original frame, which used a 1.86 mm thickness channel section, a 2.2 mm channel was used, to avoid local instabilities and not violate eq. (4). The load combination suggested either by ASCE/SEI 7 [29] and NBR 14762 [3] for gravity loads is $\Sigma S_d = D_n + L_n$. For the load combination considering the wind: $\Sigma S_d = D_n + W_a = D_n + 0.7W_{n,50}$ [28]. It was considered a wind-to-dead load ratio of 6, consistent with the most extreme case from Cardoso [5]. For the live-to-dead load ratio, the value of 5 was chosen. The load pattern and the structure parameters are shown in Fig. 4. The bars are positioned with the web parallel to the plane of the frame. In Fig. 4a, the analyzed serviceability limit was the vertical displacement of the apex joint, while in Fig 4b it was the horizontal displacement of the right eave joint.



(a) Gravity loads (b) Dead and wind loads

Figure 4. Portal frame.

4.2 Storage rack frame

The second example is a storage rack frame with the same geometry as the experimentally tested by Trouncer [19], but unlike the original frame, with a 1 mm thickness rack section for the columns, the frame modeled in MASTAN2 had 2.4 mm thickness section [31] to avoid local instabilities under the service loads (Fig. 5). Liu *et al.* [16] and ANSI MH16.1 [20] equations of thickness reduction was used to simulate the effect of the perforations in the columns cross-sections. For serviceability limit states, the combination $\Sigma S_d = D_n + L_n$ is consistent with ANSI MH16.1 [20]. However in storage systems non integrated with the building structure usually represents only the self-weight of the rack [5]. It represents a small contribution in the total applied load (between 1% and 2%) and can be ignored in the load combination [5], that now becomes $\Sigma S_d = L_n$. The analyzed serviceability limit here was the horizontal displacement in the down-aisle direction. Since the structure is working with gravity loads only, the P-Δ and P-δ effects are responsible for this displacement. To allow both effects to happen, in a worst scenario, an initial out-of-plumbness of δ_0 = 5 mm (H/1040) was imposed to all the 4 columns, describing a half-sine shape.

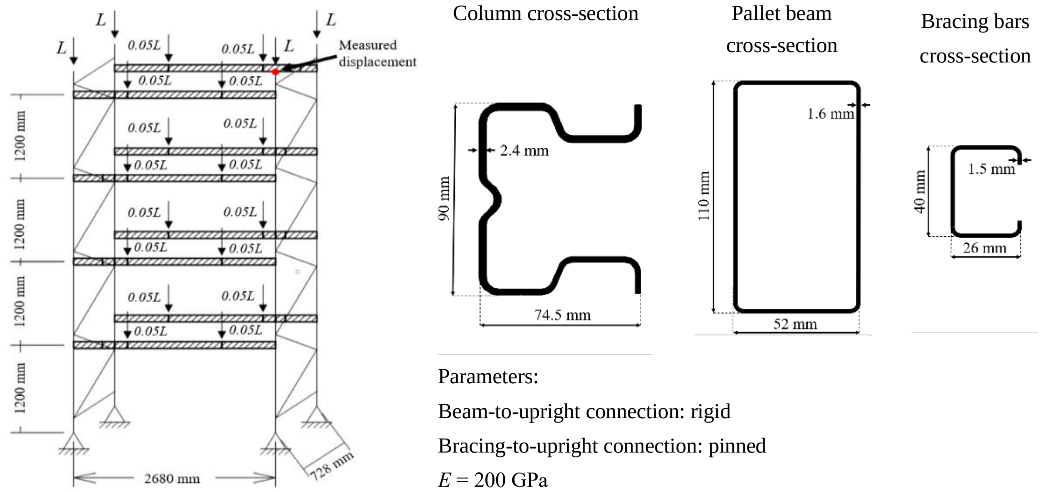


Figure 5. Storage rack frame.

4.3 Reliability indices

Table 2 shows the obtained reliability results for the examples.

Table 2. Results of the reliability analysis.

Example	Loads	Reliability index	Annual reliability index	Annual probability of failure (excedance) P_f
Portal frame	$D + L$	$\beta_{8yrs} = 1.54$	$\beta_{1yr} = 2.38$	0.86%
Portal frame	$D + W$	$\beta_{10yrs} = 0.85$	$\beta_{1yr} = 2.02$	2.18%
Storage rack frame	L	$\beta_{8yrs} = 1.78$	$\beta_{1yr} = 2.59$	0.48%

The implemented reliability framework showed convergence with 3 iterations for the FORM method to all the 3 cases studied. It highlights the efficiency of the HLRF algorithm. From Section 4, the ASCE/SEI 7 [29] target reliability index for the annual excedence of the limit displacement is β_0=1.64, associated to $P_f = 5\%$. By direct comparison, the values found for the three studied load cases are higher than the target, satisfying the safety requirement. As expected, due to the higher coefficient of variation of the wind load, the portal frame under $D + W$ load had the lowest β_{1yr}, but even though, respecting the target. Comparing to the values found by Arrayago and Rasmussen [10], the values in Tab. 2 are similar. Some differences might be attributed to differences between the stiffness of the frame, the load pattern and the adopted wind-to-dead load ratio. The behaviour of the members with monosymetric cross-sections might have influenced the results as well.

In Fig. 5, a sensitivity index $I_{X_i} = \alpha_{X_i}^2$ (α_{X_i} is the director cousine of the variable X_i) is evaluated to express the influence of each of the basic random variables in the obtained reliability results. Live and wind loads were the most influential variables in the results, specially the wind load, due to its high coefficient of variation. The dead load, when considered, showed negligible influence on the reliability results, and might be considered as deterministic to simplify the analysis.

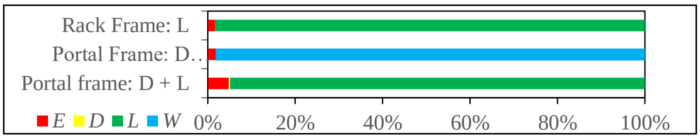


Figure 5. Storage rack frame.

5 Conclusions

In this paper, a framework was presented to assess the reliability indices of cold-formed steel frames in serviceability limit states. It was proposed the use of advanced line finite elements for this task, to avoid the need of a shell finite elements analysis.

When used to simulate the behaviour of already tested cold-formed steel structures, the MASTAN2 advanced line element showed good accuracy, when restrained to moderate displacments and stresses under the local instabilities critical stresses. The second order linear analysis from MASTAN2, when coupled to the First Order Reliability Method (FORM) worked well, with FORM showing convergence with only 3 iterations. The reliability results are consistent with the available results from the literature, and satisfied the target reliability indice from ASCE/SEI 7 for excessive displacement. Wind and live loads was the most influential variables in the reliability results, as expected by the high coefficients of variations from its stochastic models.

More frames with different geometries and load patterns could be evaluated, in order to produce a larger range of results. In the case of the rack frame, due to the importance of the P-Δ and P-δ effects for the lateral drift, the initial out-of-plumbness could also be treated as a random variable.

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